Credible Forward Guidance*

Taisuke Nakata[†]
University of Tokyo

Takeki Sunakawa[‡]
Hitotsubashi University

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Abstract

How can the central bank credibly implement a "lower-for-longer" strategy? To answer this question, we analyze a series of optimal sustainable policy problems—indexed by the duration of reputational loss—in a sticky-price model with an effective lower bound (ELB) constraint on nominal interest rates. We find that, even without an explicit commitment technology, the central bank can still credibly keep the policy rate at the ELB for an extended period—though not as extended as under the optimal commitment policy—and meaningfully mitigate the adverse effects of the ELB constraint on economic activity.

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[†]Faculty of Economics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-0033; Email: taisuke.nakata@e.u-tokyo.ac.jp

[‡]Hitotsubashi University; Email: takeki.sunakawa@gmail.com.

1 Introduction

I believe the FOMC should seriously consider pursuing a lower-for-longer or makeup strategy for setting short rates when the zero lower bound binds and should articulate its intention to do so before the next zero lower bound episode.

Janet L. Yellen, September 2018[1]

As is well known from the research literature, makeup strategies, in general, are not time consistent because when the time comes to push inflation above 2 percent, conditions at that time will not justify that action. Thus, one of the most important questions we will seek to answer in our review is whether the Fed could, in practice, attain the benefits of makeup strategies that are possible in theoretical models.

Richard H. Clarida, September 2019[2]

Developing effective strategies to manage the adverse consequences of the effective lower bound (ELB) constraint on nominal interest rates is an important task for economists and central bankers. This task is also urgent in light of the recent Covid-19 crisis that has forced many central banks to lower their policy rates to an ELB constraint. In forward-looking models with an ELB, the commitment to keeping the policy rate at the ELB for an extended period—and temporarily overshooting inflation and output targets—is known to be effective in stimulating economic activity during a deep recession, as the anticipation of an overheated economy leads foreward-looking households and firms to increase consumption and set higher prices.[3]

While the effectiveness of such overheating commitment or lower-for-longer policy in theory is widely known, central banks that recently faced, or are currently facing, the ELB constraint have not adopted this type of policy, with an exception of the Bank of Japan.[4] One key argument against overheating commitment policy is its potential time-inconsistency. *Ex ante*, it is desirable to promise to overheat the economy in the future, as the expectations of future overheating stimulate inflation and output when the economy faces headwinds and the ELB is a binding constraint. However, once the headwinds dissipate, the central bank will have an incentive to renege on the promise of overheating the economy by raising the policy rate, because the overheating is *ex post* undesirable. A number of policymakers have stated that

[1]Yellen (2018), "Comments on Monetary Policy at the Effective Lower Bound" `https://www.brookings.edu/wp-content/uploads/2018/09/ES_20180910_Yellen-ELB.pdf`.

[2]Clarida (2019a), "The Federal Reserve's Review of Its Monetary Policy Strategy, Tools, and Communication Practices" `https://www.federalreserve.gov/newsevents/speech/clarida20190926a.htm`.

[3]See, for example, Reifschneider and Williams (2000); Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006).

[4]This type of policy is also referred to as "make-up" policy. Price-level targeting, temporary price-level targeting, and average inflation targeting are alternative frameworks to implement lower-for-longer and overheating commitment policy.

this time-inconsistency problem is one reason for why commitment policy may be not as effective in reality as in theory.[5]

In this paper, we study credible overheating commitment policies in a sticky-price model with the ELB with an eye towards understanding the best allocations the central bank can credibly achieve when the optimal commitment policy is not credible. Specifically, we formulate and solve a series of *optimal sustainable policy problems* in which the central bank chooses state-contingent allocations to maximize welfare subject to not only private-sector equilibrium conditions, but also an incentive compatibility constraint—known as the sustainability constraint. The sustainability constraint requires that the continuation value associated with the chosen state-contingent allocation has to be at least as large as the continuation value associated with deviating from that allocation—and falling into a discretionary regime for N periods—at any time and after any history of shocks. Under certain conditions discussed in Nakata (2018), the sustainability constraint does not bind and the optimal sustainable policy coincides with the optimal commitment policy.[6] Our main interest is to characterize optimal sustainable policies when the sustainability constraint occasionally binds.

Our main result is that, even when optimal commitment policy is not credible, the central bank can still credibly keep the policy rate at the ELB for an extended period in the aftermath of a crisis—though not as extended as under optimal commitment policy. As in optimal commitment policy, such lower-for-longer policy generates a temporary post-crisis overheating of the economy and mitigates the declines in output and inflation in a crisis through expectations. Under reasonable assumptions regarding how long the central bank suffers from a loss of reputation after reneging on the promise of lower-for-longer, the welfare cost of the ELB constraint is substantially lower under an optimal sustainable policy than under optimal discretionary policy.

One key feature of optimal sustainable policies is that they are less history dependent than optimal commitment policy. As discussed in detail by Eggertsson and Woodford (2003), a key feature of optimal commitment policy is history dependence. In particular, under optimal commitment policy, the additional period at which to keep the policy rate at the ELB in the aftermath of a crisis—as well as the magnitude of output and inflation overshoot—increases as the realized crisis shock duration increases. When the reputational force is strong, optimal sustainable policies exhibit qualitatively similar history dependence, though the degree of his-

[5]See Appendix I of this paper or Nakata (2015) for quotes from various policymakers discussing the time-inconsistency of commitment policy.

[6]Specifically, in Nakata (2018), if the central bank were to renege on the promise of overheating the economy in the aftermath of a crisis, it would loses reputation and private-sector agents would not believe similar promises in future crises. If private-sector agents do not believe the central bank's promise to overheat the economy, future ELB episodes will be associated with large declines in inflation and output. Thus, concern for maintaining reputation gives the central bank an incentive to fulfill the promise of keeping the lower-for-longer promise. According to Nakata (2018), this incentive to maintain reputation dominates the short-run incentive to eliminate the overheating of the economy—and as a result, the optimal commitment policy is credible—if that the policy rate is expected to fall into the ELB in the future with sufficient frequency and the loss of reputation lasts for a sufficiently long duration.

tory dependence is weaker than under optimal commitment policy. When the reputational force is sufficiently weak, optimal sustainable policies do not feature any history dependence. That is, the additional ELB duration as well as the magnitude of output and inflation overshoot do not depend on the realized crisis shock duration.

Our optimal sustainable policies are of interest to central banks for two reasons. First, by construction, the optimal sustainable policies are time-consistent; thus, it is immune to the criticism that the promised overshoot of inflation and output associated with any lower-for-longer strategies may not be credible. Second, when the duration of reputational loss is sufficiently short, optimal sustainable policies are not history dependent or not as history dependent as optimal commitment policy is. Thus, it overcomes the criticism that, because the policy rate path associated with a lower-for-longer strategy is complex, it is difficult for central banks to clearly explain these strategies to the public.

Our analysis contributes to the policy debate on how to best conduct forward guidance policies in future ELB episodes. Although most central banks have refrained from using lower-for-longer policies in the past,[7] there is a growing interest in adopting this type of policy in the future (Bernanke (2017), Yellen (2018), and Williams (2018)). Our analysis suggests that it is possible for central banks to credibly adopt lower-for-longer policies, but there may be some limit on how long they can promise to keep the policy rate at the ELB. Of course, there are other factors absent in our model that may limit the effectiveness and implementability of lower-for-longer policies, such as the possibilities that lower-for-longer policies induce financial instability or the unanchoring of long-run inflation expectations (Kohn (2012) and Yellen (2018)). It would be useful to carefully examine the implications of these other factors for lower-for-longer policies in future research.

Our paper builds on the literature on optimal monetary policy in the New Keynesian model with the ELB. This literature has demonstrated the effectiveness of lower-for-longer policies in stimulating the economy at the ELB, assuming that the central bank is equipped with an explicit commitment technology (Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006); and Nakov (2008)). Our paper contributes to this body of work by characterizing optimal sustainable policies in a model with the ELB and showing that the central bank can credibly engage in lower-for-longer policies even in the absence of an explicit commitment technology.

Within the literature on optimal policy and the ELB, some authors have explored ways to implement lower-for-longer policies at the ELB in a time-consistent way. Eggertsson (2006) and Burgert and Schmidt (2014) show that in models with non-Ricardian fiscal policy, a discretionary government can provide incentives to a future government to keep the policy rate at the ELB for longer by adopting expansionary fiscal policy and raising the nominal level of government debt. Jeanne and Svensson (2007), Berriel and Mendes (2015), and Bhattarai, Eggertsson, and Gafarov (2013) show that central banks' balance sheet policies can act as

[7]See Appendix I for quotes from various policymakers expressing concerns for time-inconsistency.

a commitment device that allows the central bank to credibly implement lower-for-longer policies. Billi (2017) and Nakata and Schmidt (2019) analyze policy delegation in models with the ELB, showing that lower-for-longer policies can be implemented in a time-consistent way if the discretionary central bank's standard dual-mandate objective function is replaced by a nominal-income stabilization objective or augmented with an interest-rate smoothing objective, respectively. Unlike these papers that either introduce a new policy instrument or modify the central bank's objective function, we use reputation to achieve lower-for-longer policies in a time-consistent way.

Our paper is closely related to Nakata (2018) and Walsh (2018). Nakata (2018) has shown that optimal commitment policy in the New Keynesian model with the ELB can be made time-consistent by a particular trigger strategy capturing the reputational concern of the central bank. Our paper is different from Nakata (2018) because we study the best allocations the central bank can credibly achieve when the optimal commitment policy is not credible, whereas Nakata (2018) characterizes the conditions under which the optimal commitment policy is credible. Walsh (2018) examines credibility of simple policy rules with forward guidance—those that keeps the policy rate at the ELB for a fixed number of periods after crises—and reaches a conclusion similar to that of Nakata (2018).[8] Our paper is different from Walsh (2018) because we characterize the optimal allocation the central bank can credibly achieve subject to a sustainability constraint, whereas Walsh (2018) studies credibility of simple policy rules that may or may not be optimal.[9] It turns out that there is an interesting relationship between our optimal sustainable policies and the forward guidance policy of Walsh (2018), which will be discussed in detail in Section 4.1.[10]

This paper is also closely related to the work of Dong and Young (2019). Using the recursive method of Abreu, Pearce, and Stacchetti (1990), Chang (1998), and Phelan and Stacchetti (2001), Dong and Young (2019) characterize the entire set of sustainable plans in a fully nonlinear New Keynesian model with the ELB. They find that the commitment outcome is not sustainable in their model and that the central bank does not lower the policy rate to the ELB under the best sustainable plan. Our paper is methodologically different from theirs because we characterize a particular (countable) subset of sustainable plans that have a trigger structure and in which on-equilibrium paths are given by the solutions to a sequence of optimal sustainable policy problems indexed by the duration of reputational loss.

Finally, our paper is related to a set of papers that characterize optimal allocations in

[8]See also Sukeda (2018), which extends the analysis of Walsh (2018) to a model with a discounted Euler equation and a discounted Phillips curve.

[9]Nakata (2018) and Walsh (2018) in turn build on earlier work of reputation in macroeconomics, including Barro and Gordon (1983), Rogoff (1987), Chari and Kehoe (1990), Chang (1998), and Phelan and Stacchetti (2001), among many others. Recent contributions include Kurozumi (2008) and Loisel (2008).

[10]See also Barthélemy and Mengus (2018) who examine sustainability of optimal commitment policy in a model with the ELB constraint in which the central bank's objective function—either a benevolent or conservative kind—is unknown to private-sector agents and there is an inflationary bias. In their model, the benevolent central bank can make the optimal commitment policy sustainable by raising inflation prior to a liquidity trap and signaling its type to private-sector agents.

macroeconomic models with a sustainability constraint. Kehoe and Perri (2002) characterize the optimal allocation in an international business cycle model in which a deviation from the promised plan would push the economy to autarky. Fujiwara, Kam, and Sunakawa (2016) study the optimal sustainable policy in a two-country model in which the deviation from the promised cooperative plan would push the countries into a non-cooperative regime. Most closely related to our paper is Sunakawa (2015) who characterizes the optimal sustainable policy in a New Keynesian model with cost-push shocks but without the ELB constraint. Our paper applies the same analytical framework and methodology used in these papers to models with the ELB.

The rest of the paper is organized as follows. Section 2 describes the model and the central bank's optimization problems. Section 3 presents the results. Section 4 provides additional discussions and results. Section 5 concludes.

2 Model

2.1 Private sector

Our main model is a semi-loglinear New Keynesian model with a static Phillips curve. The private-sector equilibrium conditions of this model are given by:

$$y_t(s^t) = E_t y_{t+1}(s^{t+1}) - \sigma(i_t(s^t) - E_t\pi_{t+1}(s^{t+1}) - r^*) + s_t \tag{1}$$

$$\pi_t(s^t) = \kappa y_t(s^t) \tag{2}$$

$$i_t(s^t) \geq i_{ELB} \tag{3}$$

where y_t is output, π_t is inflation, and i_t is the policy rate. Equations (1) and (2) are the Euler equation and the static Phillips curve, respectively. Inequality (3) imposes the ELB constraint, denoted by i_{ELB}, on the policy rate. σ is the intertemporal elasticity of substitution, $r^* > 0$ is the natural rate of interest at the deterministic steady state, and κ is the slope of the static Phillips curve. s_t is a natural rate shock. s^t denotes a history of shocks up to time t. That is, $s^t := \{s_k\}_{k=1}^t$. Because there is uncertainty, allocations are state-contingent and depend on s^t. We refer to the state-contingent sequence of consumption, inflation, and the nominal interest rate, $\{y_t(s^t), \pi_t(s^t), i_t(s^t)\}_{t=1}^\infty$, as *an outcome*. Given a process for s_t, an outcome is said to be *competitive* if, for all $t \geq 1$ and $s^t \in \mathbb{S}^t$, (i) $y_t(s^t) \in \mathbb{R}$, $\pi_t(s^t) \in \mathbb{R}$, $i_t(s^t) \in \mathbb{R}$, where $\mathbb{R}$ denotes a set of real numbers, and (ii) equations (1)-(3) are satisfied.

We assume that s_t follows a two-state Markov process. $s_t = r^* > 0$ in the "high" or "normal" state, whereas $s_t = r_c < 0$ in the "low" or "crisis" state. The probability of moving from the high/normal state to the low/crisis state is denoted by p_H and will be referred to as the crisis frequency, whereas the probability of moving from the low/crisis state to the low/crisis state is denoted by p_L and will be referred to as the crisis persistence. Following

Nakata (2018), we allow p_H to be non-zero, which opens up the possibility for a reputational concern to make lower-for-longer policies credible.

The central bank's *value* at period t is given by

$$V_t(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u\left(\pi_{t+j}(s^{t+j}), y_{t+j}(s^{t+j})\right) \tag{4}$$

where the per-period objective function is given by the following function.

$$u(\pi, y) := -\frac{1}{2}\left[\pi^2 + \lambda y^2\right] \tag{5}$$

This quadratic objective function can be obtained as the second-order approximation to the household's welfare.[11] For any outcome, there is an associated state-contingent sequence of values, $\{V_t(s^t)\}_{t=1}^{\infty}$, which will be referred to as the value sequence.

We use this model with a static Phillips curve as our baseline model for a computational reason. As described in Section 2.4, we use a time-iteration method—a commonly used numerical method for nonlinear models—to solve our model. Regardless of the specification of the Phillips curve, this solution method fails to converge if the duration of reputational loss—a key parameter governing how long the central bank is prohibited from engaging in state-contingent policies after it reneges on a previously announced policy rate path—is sufficiently short. As discussed in Appendix E, we can devise alternative solution methods if the duration of reputational loss is sufficiently short in the model with a static Phillips curve, while we cannot do so in the model with a forward-looking Phillips curve. At the end of the day, we can solve the model with a static Phillips curve for any durations of reputational loss, while we cannot do so in the model with a forward-looking Phillips curve.[12] In Section 4.3, we present some select results from the model with a forward-looking Phillips curve and confirm that key features of the optimal sustainable policy in the model with the static Phillips curve survive in the model with a forward-looking Phillips curve.[13]

2.2 Central bank

We will consider three classes of competitive outcomes that differ in how the central bank sets its interest rate policy: the discretionary outcome, the commitment outcome, and the sustainable outcomes.

[11]See, for example, Woodford (2003) and Galí (2015).

[12]The instability of the time-iteration method in the presence of a sustainability constraint with low values of N is not specific to the model with ELB. The second author of this paper experienced a similar instability issue in Fujiwara, Kam, and Sunakawa (2019). We leave the task of developing a robust algorithm for models with sustainability constraints to future research.

[13]See also Bilbiie (Forthcoming) who uses a model with a static Phillips curve.

2.2.1 Discretionary outcome

At each time t, the discretionary central bank's optimization problem is to choose $\{y_t$, π_t, $i_t\}$ to maximize the value today, taking as given the value function ($W_{t+1}(\cdot)$) and policy functions for inflation and output ($\pi_{t+1}(\cdot)$ and $y_{t+1}(\cdot)$) in the next period. That is,

$$W_t(s_t) = \max_{\pi_t, y_t, i_t} \quad u(y_t, \pi_t) + \beta E_t W_{t+1}(s_{t+1}), \tag{6}$$

subject to equations (1), (2), and (3).

Let $\{W_d(\cdot)$, $\pi_d(\cdot)$, $y_d(\cdot)$, $i_d(\cdot)\}$ be the set of time-invariant value and policy functions that solve the Bellman equation above and in which the ELB binds only in the crisis state.[14] They are functions of today's shock realization, s_t. The discretionary outcome is defined as, and denoted by, the state-contingent sequence of output, inflation, and the policy rate, $\{y_{d,t}(s^t)$, $\pi_{d,t}(s^t)$, $i_{d,t}(s^t)\}_{t=1}^{\infty}$ such that $y_{d,t}(s^t) = y_d(s_t)$, $\pi_{d,t}(s^t) = \pi_d(s_t)$, and $i_{d,t}(s^t) = i_d(s_t)$ and the discretionary value sequence is defined as, and denoted by, $\{V_{d,t}(s^t)\}_{t=1}^{\infty}$ such that $V_{d,t}(s^t) = W_d(s_t)$. We will also refer to the discretionary outcome as the outcome under the optimal discretionary policy (ODP).

2.2.2 Commitment outcome

At the beginning of time one, the central bank with commitment technology chooses a state-contingent allocation, $\{y_t(s^t)$, $\pi_t(s^t)$, $i_t(s^t)\}_{t=1}^{\infty}$, to maximize the time-one value. That is,

$$V_{c,1}(s_1) = \max_{\{y_t(s^t), \pi_t(s^t), i_t(s^t)\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t(s^t), \pi_t(s^t)), \tag{7}$$

subject to equations (1), (2), and (3) for all $t \geq 1$ and after all histories of shocks s^t. The commitment outcome, or the Ramsey outcome, is defined as the solution to this optimization problem. In other words, the commitment outcome is a competitive outcome with the highest time-one value. We denote the commitment outcome by $\{y_{c,t}(s^t)$, $\pi_{c,t}(s^t)$, $i_{c,t}(s^t)\}_{t=1}^{\infty}$. The value sequence associated with the commitment outcome is denoted by $\{V_{c,t}(s^t)\}_{t=1}^{\infty}$ and will be referred to as the commitment value sequence. We will also refer to the commitment outcome as the outcome under the optimal commitment policy (OCP).

2.2.3 Sustainable outcomes

At the beginning of time one, the central bank chooses a state-contingent allocation, $\{y_t(s^t)$, $\pi_t(s^t)$, $i_t(s^t)\}_{t=1}^{\infty}$, to maximize the time-one value:

[14] There also exists a time-invariant solution to this discretionary government's problem in which the ELB binds in both states. See Armenter (2017), Nakata (2018), and Nakata and Schmidt (Forthcoming) for extensive analyses of such deflationary Markov-perfect equilibrium.

$$V_{s,1}(s_1) = \max_{\{y_t(s^t),\pi_t(s^t),i_t(s^t)\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t, \pi_t), \tag{8}$$

subject to equations (1), (2), and (3), and the following sustainability constraint,

$$E_t \sum_{k=0}^{\infty} \beta^k u(y_{t+k}(s^{t+k}), \pi_{t+k}(s^{t+k})) \geq W_d^N(s_t), \tag{9}$$

for all $t \geq 1$ and after all histories of shocks, s^t. The left-hand side of the sustainability constraint is the continuation value of implementing a chosen state-contingent allocation at time t after s^t. The right-hand side, $W_d^N(s_t)$, is the continuation value if the central bank deviates from the chosen state-contingent allocation, with N indicating how many periods it takes for the central bank to restore its lost reputation ("punishment" duration). During the periods of reputational loss, the central bank cannot engage in state-contingent policies. That is, the central bank has to act under discretion.

$W_d^N(s_t)$ is recursively defined as follows. For $N = 0$,

$$W_d^0(s) := V_{s,1}(s), \quad \pi_d^0(s) := \pi_{s,1}(s), \quad y_d^0(s) := y_{s,1}(s).$$

In this case with $N = 0$, the punishment duration is zero and the central bank is not allowed to deviate from the sustainable outcome. For any $N > 0$,

$$W_d^N(s) = \max_{\pi,y,i} \quad u(\pi, y) + \beta E[W_d^{N-1}(s')|s]$$

where the maximization is subject to the private-sector equilibrium conditions, taking as given the value and policy functions for the next period (that is, $W_d^{N-1}(\cdot)$, $\pi_d^{N-1}(\cdot)$, and $y_d^{N-1}(\cdot)$).

Note that the sustainability constraint has to be respected each period and for each history of shocks, just as the Euler equation, the Phillips curve, and the ELB constraint have to be respected each period and for each history of shock. *The sustainable outcome with N-period reputational loss* is defined as the solution to this infinite-horizon optimization problem. We will also refer to the sustainable outcome with N-period reputational loss as the outcome under the optimal sustainable policy (OSP) with N-period reputational loss.

Note that the punishment value, $W_d^N(s_t)$, is determined jointly with the sustainable outcome, except when $N = \infty$. When $N = \infty$, the punishment lasts forever and its value is given by the discretionary value, $W_d(s)$, which is independent of the sustainable outcome. For any finite N, the central bank eventually restores its reputation and the economy returns to the allocations consistent with the sustainable outcome. Thus, the punishment value and the sustainable outcome are not independent of each other. All else equal, an increase (decrease) in the value associated with the sustainable outcome implies an increase (decrease) in the punishment value.

As described in detail in Appendix A, once the sustainable outcome is computed from the optimization problem above, we can construct a plan—a pair of central bank and private-sector strategies—that induces the sustainable outcome and that has a trigger-type structure. In particular, we can construct a revert-to-discretion plan in which (i) the economy follows the sustainable outcome as long as the central bank has never deviated from the policy rate path consistent with the sustainable outcome in the past, and (ii) the economy follows the discretionary outcome, or a temporary deviation to a discretionary regime, otherwise. By construction, such a revert-to-discretion plan is credible, meaning that neither the central bank nor private-sector agents have incentives to deviate from the instructions given by the strategies. The central bank does not have an incentive to deviate from the policy rate path consistent with the sustainable outcome because the sustainability constraint is imposed on the central bank's optimization problem, ensuring that the continuation value under the sustainable outcome is at least as large as the punishment continuation value. Private-sector agents do not have incentives to deviate from the private-sector strategy because the Euler equation and the Phillips curves are satisfied, meaning that the output and inflation are consistent with their optimizing behaviors given the central bank strategy. Even though the deviation does not occur in equilibrium, the specification of what would happen if the central bank were to deviate from the sustainable outcome does affect what happens under the sustainable outcome.

If the sustainability constraint does not bind at any time t and after any histories of shocks, the sustainable outcome coincides with the commitment outcome. Also, if the sustainability constraint always binds—which happens, for example, when the punishment length (N) is zero or when the crisis frequency (p_H) is zero—the sustainable outcomes coincides with the discretionary outcome. Our main interest is those cases in which the sustainability constraint occasionally binds.

2.3 Parameter values

Table 1 shows the baseline parameter values. The quarterly frequency of crises is set to 0.5/100 (=2/400). This choice is motivated by the fact that, in the United States, there have been two large crises that pushed the short-term nominal interest rate to the ELB over roughly the last 100 years (400 quarters) since the creation of the Federal Reserve System. The crisis shock persistence is set to 3/4, which implies the expected duration of the crisis shock of 4 quarters. σ is set to 1. r_c is chosen so that output declines 7 percent in the crisis state under the optimal discretionary policy. Conditional on the value of r_c, the value of κ is chosen so that inflation declines 1 percentage point (annualized) in the crisis state under the optimal discretionary policy. This severity of the crisis is consistent with that considered in Boneva, Braun, and Waki (2016) and Nakata (2018), and is intended to capture the severity of the Great Recession of 2007-2009 in the United States.

We consider three values for the duration of reputational loss (20, 60, and ∞), which are chosen to cover qualitatively distinct cases that can arise. To put these values into perspective,

Table 1: Baseline Parameter Values

Parameter	Description	Values
β	Discount factor	0.9925
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips Curve	0.25/7
i_{ELB}	Effective lower bound on the policy rate	0
p_H	Frequency of the crisis state	0.5/100
p_L	Persistence of the crisis state	3/4
r^*	Natural rate in the normal state	3/400
r_c	Natural rate in the crisis state	**Chosen so that $y_d(s_t = r_c) = -0.07$
N	Duration of reputational loss	[20, 60, ∞]

note that the assumption that the central bank can restore its reputation after a finite number of periods can be motivated by the fact that the tenure of governorship at central banks is finite as well as the possibility that reputation may be specific to the leader of the central bank, as opposed to the institution. As shown in Appendix H, the average tenure of the governorship in central banks in economies that have recently faced, or are currently facing, the ELB ranges from about 5 years (20 quarters) in the Bank of Japan to about 10 years (40 quarters) for the Bank of Canada. The maximum tenure duration exceeds 15 years (60 quarters) at several central banks (the Federal Reserve, Bank of Canada, Bank of England, and Sveriges Riksbank).

2.4 Solution method

The model is highly nonlinear, featuring two inequality constraints—the ELB constraint and the sustainability constraint—and cannot be solved analytically. Following Kehoe and Perri (2002) and Sunakawa (2015), we recursify the infinite-horizon optimization problem of the central bank into a saddle-point functional equation using the Lagrange multiplier on the Euler equation as a pseudo-state variable. We then apply a time-iteration method to find the set of time-invariant policy functions that solve the saddle-point functional equation. Appendix C describes the details of the solution method as well as its accuracy.[15]

3 Results

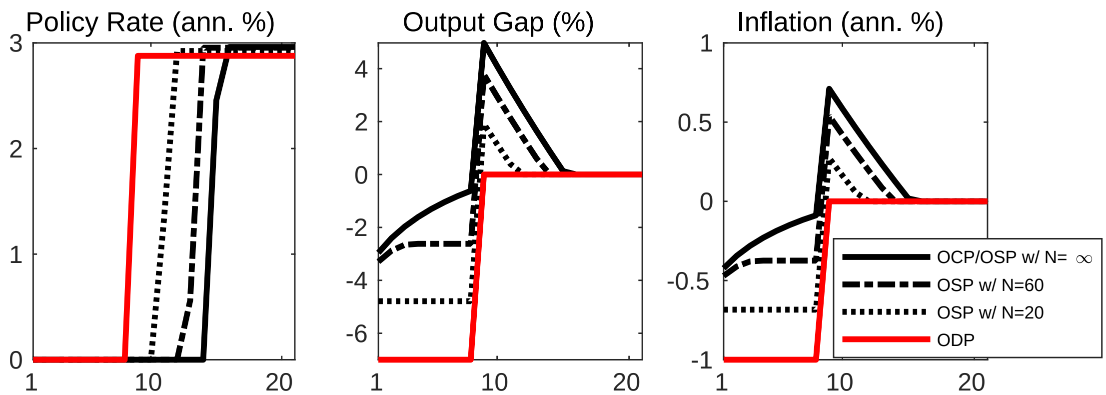
3.1 Dynamics

Figure 1 shows the dynamics of the economy under the ODP, the OCP, and OSPs with $N = [20, 60, \infty]$. In this figure, the crisis shock hits the economy at time 1 and stays there

[15] As discussed earlier, the time-iteration method fails to converge when N is sufficiently small. For those small values of N under which the time-iteration method fails, we use alternative solution methods described in Appendix E.

until time 8. The crisis shock disappears at time 9 and the economy is in the normal state from that point on.

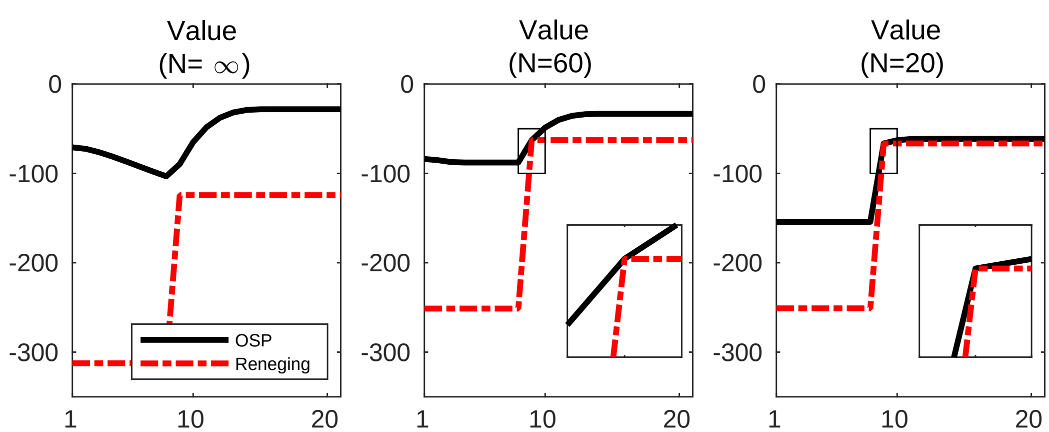
Figure 1: Dynamics



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

Under the ODP—shown by the solid red lines—the central bank keeps the policy rate at the ELB as long as the crisis shock persists and raises the policy rate immediately after the crisis shock disappears. Under the OCP—shown by the solid black lines—the central bank keeps the policy rate at the ELB even after the crisis shock disappears, engineering the overshooting of inflation and output above their targets. Since households are forward looking, the anticipation of high inflation and high output in the aftermath of the crisis stimulates economic activity during the crisis. The declines in inflation and output are substantially smaller under the OCP than under the ODP.

Figure 2: Value of fulfilling versus reneging on the promised allocations



Note: OSP stands for optimal sustainable policy.

The allocations under the OSP with $N = \infty$ are identical to those under the OCP in this crisis scenario. As shown in the left panel of Figure 2, the value under the OSP with $N = \infty$—shown by the solid black—is always above the value in case the central bank deviates from the OSP with $N = \infty$—shown by the dashed red line. That is, the sustainability constraint does not bind. In our calibration, the crisis shock is sufficiently frequent so that the cost of being unable to use lower-for-longer policies in the future forever outweighs the benefit of eliminating the temporary overshooting of inflation and output targets. This result is consistent with the finding of Nakata (2018) that a very small probability of being hit by the crisis shock suffices to make the OCP credible.

When the loss of reputation is not as long, the cost of reneging on the lower-for-longer promise in the aftermath of the crisis shock is smaller. In other words, the continuation value in case of deviation is higher with a smaller N. The middle panel of Figure 2 shows the value of the OSP with $N = 60$ and the value of deviating from the OSP with $N = 60$ (solid black and dashed red lines, respectively). According to the panel, the sustainability constraint binds right after the crisis shock disappears, limiting the magnitude of the overshooting in the aftermath of the crisis. The smaller overshoot means that inflation and output decline by more during the crisis under the OSP with $N = 60$ than under the OCP and the OSP with $N = \infty$. However, the declines in inflation and output are still much smaller under the OSP with $N = 60$ than under the ODP. Similarly, the sustainability constraint binds right after the crisis shock disappears under the OSP with $N = 20$, as can be seen in the right panel of Figure 2, limiting the magnitude of the overshoot. The overshoot in the aftermath of the crisis is smaller—and as a result, the declines in inflation and output are larger—under the OSP with $N = 20$ than under the OSP with $N = 60$. Even with $N = 20$, the declines in inflation and output are still much smaller under the OSP than under the ODP.

Reflecting the less severe crisis under the OSP with $N = 20$ and $N = 60$, the welfare cost of the ELB—shown in Table 2—is substantially lower under these OSPs than under the ODP. With $N = 60$, welfare cost of the ELB is about 20 percent of that under the ODP and is only slightly larger than under the OCP. Even with $N = 20$, welfare cost of the ELB is only about half of the ODP.

Table 2: Welfare Cost of the ELB

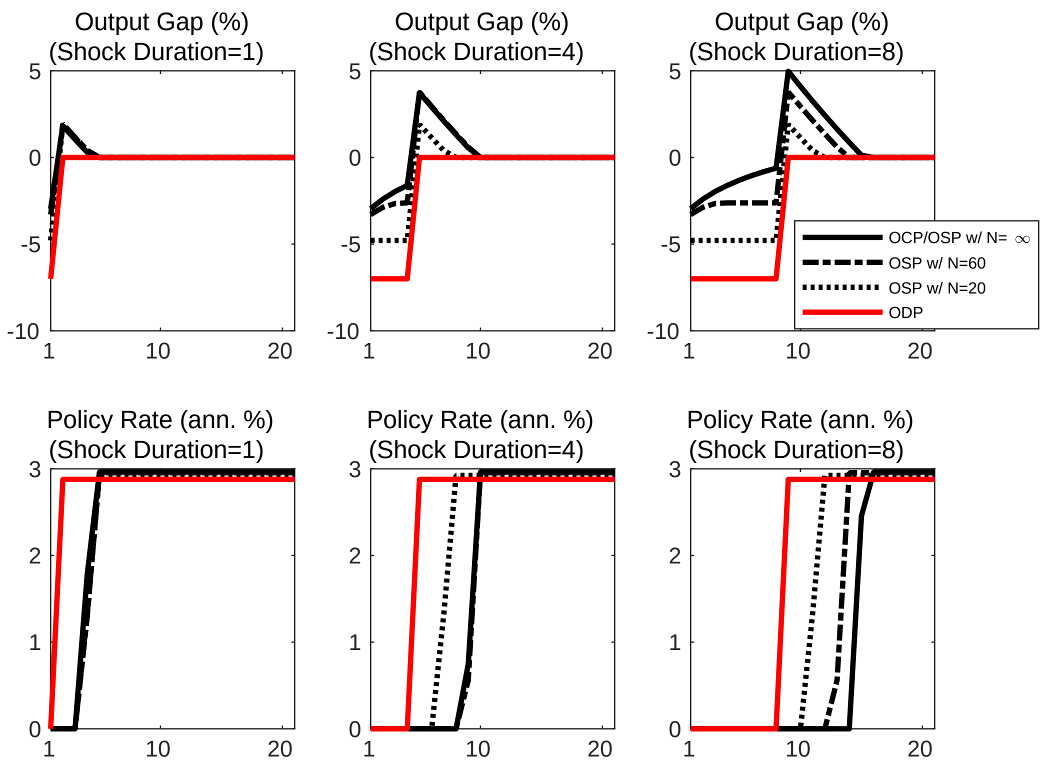
	abs($E[V]$)
Optimal commitment policy	29.5 (0.23)
Optimal sustainable policy	
with $N = \infty$	29.5 (0.23)
with $N = 60$	34.6 (0.27)
with $N = 20$	63.3 (0.49)
Optimal discretionary policy	128.1 (1.00)

Note: Numbers in parentheses are the welfare cost of the ELB relative to that under the optimal discretionary policy.

3.2 History Dependence

Figure 3 shows the dynamics of output and the policy rate—displayed in the top and bottom rows, respectively—under three alternative realized durations of the crisis shock. The first, second, and third columns are for the realized crisis shock duration of 1, 4, and 8 quarters, respectively.

Figure 3: History Dependence (I)
—Dynamics with Alternative Realized Crisis Shock Durations—

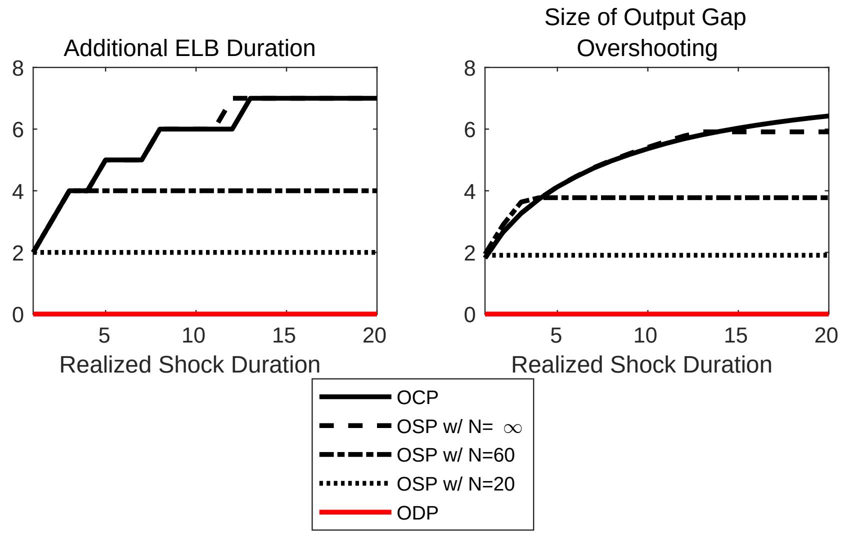


Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent.

Under the OCP—shown by the solid black lines—the additional ELB duration is 2, 4, and 6 quarters when the realized crisis shock duration is 1, 4, and 8 quarters, respectively, as can be seen in the bottom panels of Figure 3. The magnitude of the output overshoot is 2 percentage points, 4 percentage points, and 5 percentage points when the realized crisis shock duration is 1, 4, and 8 quarters, respectively, as can be seen in the top panels of Figure 3. Thus, both the additional ELB duration and the size of the inflation and output overshoot depend on the realized crisis shock duration. This dependence can be seen in Figure 4, which shows how the additional ELB duration and the size of the output gap overshoot—shown in the left and right panels, respectively—vary with the realized crisis shock duration.[16]

[16]In computing the additional ELB duration and the size of the output gap overshoot, we assume that, prior to the crisis shock, the economy has been in the normal state for some time and the Lagrange multiplier

Figure 4: History Dependence (II)



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The output gap is expressed in percent.

The history dependence of the OCP is in sharp contrast with the lack of history dependence in the ODP. Under the ODP—shown by the solid red lines in Figure 3 and 4—the additional ELB duration and the size of output gap overshoot do not depend on the realized crisis shock duration: they are 0 quarter and 0 percent, regardless of the realized crisis shock duration.

OSPs with sufficiently large Ns exhibit qualitatively similar history dependence to that of the OCP. With $N = \infty$, the dynamics of the economy under the OSP—shown by the dashed lines in Figure 3 and 4—are identical to those under the OCP when the realized crisis duration is sufficiently small. However, according to the right panel of Figure 4, when the realized crisis duration is sufficiently long—longer than 12 quarters—the size of the output gap overshoot does not increase further with an increase in the realized crisis shock duration, because the sustainability constraint binds and limits the magnitude of the output overshoot. Because the crisis persistence is 0.75, the probability that the crisis shock lasts for more than 12 quarters is very small, but it is not zero. Thus, the OSP with $N = \infty$ is less history dependent than the OCP.

Under the OSP with $N = 60$, the additional ELB duration and the size of the output gap overshoot increase with the realized crisis duration when the realized crisis duration is short, as can be seen by the dash-dotted lines in Figure 3 and 4. In particular, when the realized crisis duration is longer than 3 quarters, they do not increase further with the realized crisis duration, as the sustainability constraint binds and limits the magnitude of the output overshoot. The OSPs with $N = 60$ is history dependent but is less history dependent than the OCP or the OSP with $N = \infty$.

on the Euler equation is zero in the period right before the crisis shock materializes.

OSPs are not history dependent at all when the duration of reputational loss is sufficiently short. As can be seen by the dotted lines in Figure 3 and 4, with $N = 20$, the additional ELB duration and the size of the output gap overshoot do not depend at all on the realized crisis shock duration: they are 2 quarters and 2 percentage points. That is, the OSP with $N = 20$ is history independent.

Note that the magnitude of overshooting is slightly larger under the OSP with $N = \infty$ than under the OCP when the realized crisis shock duration is 9 to 13 quarters—at or slightly below the shortest crisis shock duration associated with the binding sustainability constraint under the OSP with $N = \infty$. Consistent with this pattern, the post-crisis path of the policy rate is slightly lower under the OSP with $N = \infty$ than under the OCP when the realized crisis shock duration is 9 to 13 quarters (not shown). In fact, the post-crisis ELB duration is one period longer under the OSP with $N = \infty$ than under the OCP when the realized crisis shock duration is 12 quarters. Similarly, the magnitude of overshooting is slightly larger under the OSP with $N = 60$ than under the OCP or the OSP with $N = \infty$ when the realized crisis shock duration is 2 to 4 quarters—at or slightly below the lowest crisis shock duration associated with the binding sustainability constraint under the OSP with $N = 60$.

How can the magnitude of overshooting be slightly larger with a smaller N for certain realized crisis shock durations? Consider the following situation: the crisis shock has persisted for a long time, and if the economy were to return to the normal state in the next period, the magnitude of post-crisis overshooting will not be limited by the sustainability constraint. In this situation, suppose that the magnitude of post-crisis overshooting will be limited by the sustainability constraint dowa the road in case the crisis lasts for longer. Then, the possibility of being constrained by the sustainability constraint down the road exerts downward pressures on output and inflation now through expectations. In this situation, because the sustainability constraint will not be binding if the economy returns to the normal state in the next period, the central bank can counteract that force by promising a larger post-crisis overshooting in case the economy were to return to the normal state in the next period—larger than what it would promise if the sustainability constraint would not bind down the road.

3.3 On "reasonable" duration of reputational loss

When the loss of reputation lasts for a long time, the power of reputation is strong and OSPs resemble the OCP. When the loss of reputation lasts for a short time, the power of reputation is weak and OSPs resemble the ODP. A natural question that arises is what reasonable values of the duration of reputational loss are.

One way to think about the reasonable duration of reputational loss is to hypothetically ask how long it might take for a central bank to restore its reputation once it loses it. The reasonable value of the duration of reputational loss based on this thought experiment may depend on various factors: whether one believes the central bank's reputation is individual-specific or institution-specific and the tenure duration of the central bank's governors or chairs, if one believes in the individual specific nature of reputation.

Another way to think about the reasonable duration of reputational loss is to theoretically refine the concept of sustainability. In models in which the commitment and discretionary outcomes are different, there are multiple—typically infinitely many—sustainable plans. In this paper, we study infinitely many (countable) sustainable plans indexed by the duration of lost reputation. By imposing further restrictions on the set of sustainable plans and thus refining the concept of sustainability, we can select one of these sustainable plans as being more reasonable than others.

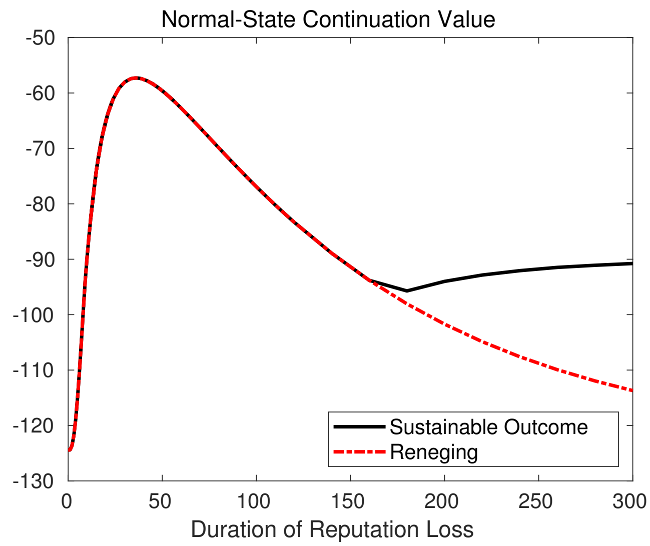
One refinement concept for any sequential equilibria—a sustainable plan in our setup—developed by game theorists in the context of two-player games is renegotiation proofness. Roughly speaking, renegotiation proofness requires that, even if the deviation from an equilibrium were to occur hypothetically, two players would have no incentives to renegotiate the contract—strategies in our setup—that they have initially agreed on. According to one definition of renegotiation-proofness proposed by Pearce (1987), a sustainable plan is renegotiation-proof if the punishment value associated with that plan is higher than the punishment value of any other sustainable plans.[17]

To apply this concept of renegotiation proofness to our model, the dash-dotted red line in Figure 5 shows the punishment value associated with deviating from the OSPs with different values for N. According to the figure, the punishment continuation value is non-monotonic. When N is large, a reduction in the punishment duration increases the punishment value: all else equal, it is good to stay in the discretionary regime for a shorter duration, as the value under the discretionary regime is lower than the value under the OSP. However, when the punishment duration is sufficiently short and the sustainability constraint binds, a shorter punishment duration lowers the value associated with the OSPs. This non-monotonicity arises because a shorter punishment duration limits the size of the overshoot in the aftermath of crises and lowers the value associated with OSPs. As a result, when the punishment regime ends, the economy will return to a sustainable outcome that is not as good as the sustainable outcome with a longer punishment duration. When N is sufficiently small, this second effect dominates the first effect, and a shorter punishment duration lowers the punishment value.

According to Figure 5, in our model, the sustainable outcome with the least severe punishment value is the sustainable outcome with $N = 36$. The dash-dotted red lines in Figure 6 show the dynamics of the economy under the OSP with $N = 36$, together with those under the OCP and the ODP. The post-crisis ELB duration is 3 quarters under the OSP with $N = 36$, 3 quarters shorter than that under the OCP. The size of the post-crisis overshooting is smaller—and output and inflation decline by more—under the OSP with $N = 36$ than

[17] Farrell and Maskin (1989) proposed an alternative definition of renegotiation-proofness whereby a sustainable plan is renegotiation proof if there is no Pareto-improving move to another sustainable plan after deviating from the on-equilibrium path at any point in time. As pointed out by Matsuyama (1997), in models with benevolent government where the government's objective function and the private-sector's objective function coincide, this definition rules out any chance for the economy to achieve allocations better than the ODP.

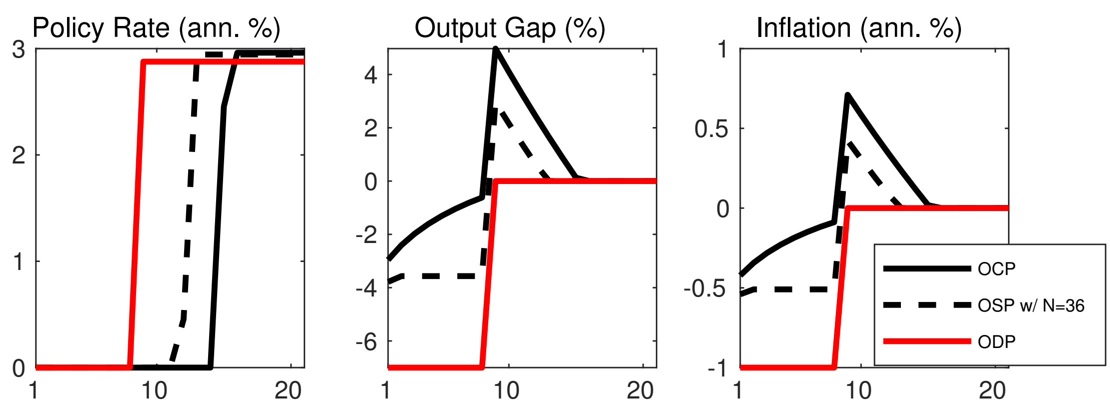
Figure 5: Values in case of reneging



Note: Continuation value under the sustainable outcome after a crisis that has lasted for 8 quarters.

under the OCP. However, the declines in inflation and output are much smaller under the OSP with $N = 36$ than under the ODP. The welfare cost of the ELB constraint is about one-third of that under the ODP.

Figure 6: Dynamics



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

4 Additional results and discussion

4.1 Relation to the simple forward guidance policies of Walsh (2018)

We have shown that, when N is sufficiently small, the OSPs are not history dependent; the policy rate path after the crisis shock disappears does not depend on the realized duration of the crisis shock. Thus, the OSPs bear some resemblance to the simple forward guidance policies considered by Walsh (2018). Under the simple forward guidance policies of Walsh (2018), the central bank keeps the policy rate at the ELB for a fixed number of periods after the crisis shock disappears, regardless of the realized duration of the crisis shock, and lets the policy rate return to the steady-state level immediately thereafter. The only (minor) difference is that, under the optimal sustainable policy, the policy rate does not return to the steady state level immediately after liftoff. Instead, there is typically one period after liftoff in which the policy rate is still below the steady-state level.

The similarity between the simple forward guidance policies and OSPs with small Ns points to one benefit of OSPs over the OCP; it may be easier for central banks to explain these OSPs to the public than the OCP. One key criticism against the OCP is that it is complex. As Walsh (2018) argues, because of its complexity, it may be difficult for the central bank in practice to steer the private-sector agents' expectations in a way consistent with the OCP. One dimension of complexity is history dependence. The OSPs have an advantage over the OCP because they are less history dependent and thus simpler.

Note that, in our model, the ODP and the OSPs with small Ns are history independent but state-contingent. They are state-contingent because the policy rate path—in particular the liftoff date—depends on the realized crisis shock duration. Thus, these policies are different from so-called calendar-based forward guidance that specifies the likely liftoff date, if that guidance were to be narrowly or mistakenly interpreted as a non-state-contingent commitment to raising the policy rate from the ELB at a particular date regardless of the evolution of the economy.[18]

4.2 Relation to the loose commitment approach of Bodenstein, Hebden, and Nunes (2012)

Under OSPs, the central bank achieves crisis-state allocations that are "in between" that under the ODP and that under the OCP. This feature of OSPs is reminiscent of the optimal policy obtained in a loose commitment approach in which the central bank reoptimizes with a constant probability every period regardless of the incentive to renege on the prior

[18] Even when central banks indicate a likely date of liftoff from the ELB, they typically emphasize that the liftoff date will depend on the evolution of the economic outlook. That is, if the economy were to recover faster or more slowly than in the baseline economic projection, the central bank will raise the policy rate from the ELB earlier or later than the most likely liftoff date under the baseline projection. In practice, it is unlikely that any central bank will ever engage in non-state-contingent forward guidance, though market participants may not interpret the forward guidance specifying the likely date of liftoff as state-contingent as the central bank intends.

commitment.[19] While these two approaches differ from each other in many ways, both approaches share the same spirit that they are intended to shed light on what the central bank may be able to achieve when no explicit commitment technology is available. Indeed, recent work by Fujiwara, Kam, and Sunakawa (2019) shows that, when using a model without the ELB, the allocations under the loose commitment approach with an appropriately chosen re-optimization probability can approximate the allocation under the OSP with N-period punishment reasonably well for any N. While we believe their result is likely to extend to the model with ELB, it would be useful to verify the validity of their claim in our model in future research.

4.3 Results from the model with a forward-looking Phillips curve

As discussed earlier, we have focused on the model with a static Phillips curve instead of the model with a forward-looking Phillips curve, because the range of durations of reputational loss under which we can solve the model is wider in the model with a static Phillips curve. A natural question is whether our key results thus far would extend to the model with a forward-looking Phillips curve. In this section, we discuss some select results from the model with a forward-looking Phillips curve.[20]

The private sector equilibrium conditions of the model with a forward-looking Phillips curve are characterized by the Euler equation given by equation (1), the forward-looking Phillips curve,

$$\pi_t(s^t) = \kappa y_t(s^t) + \beta E_t \pi_{t+1}(s^{t+1}) \tag{10}$$

and the ELB constraint given by inequality (3). The central bank's objective function is the same as that in the model with a static Phillips curve. The ODP, the OCP, and the OSPs are defined in ways that are similar to how they are defined in the model with the static Phillips curve. The parameter values used are shown in Table 3. The values for β, σ, p_H, and s_H are the same as in the previous section. κ is set to 0.005. p_L is set to 0.5, implying the expected duration of the crisis state of 2 quarters. λ is set to 1/16, a value consistent with equal weights on the volatility of the output gap and the volatility of the annualized rate of inflation. A high value of λ and a low value of p_L increase a range of the duration of reputational loss values under which we can solve the model. With these parameter values, we could solve the model for $N \geq 75$. We show the dynamics of the model under the OSPs with three values of $N = [80, 160, \infty]$.

Figure 7 shows the dynamics of the economy under the OCP, the ODP, and OSPs with the three values of N listed above.

[19] See Bodenstein, Hebden, and Nunes (2012) for an analysis of optimal monetary policy under loose commitment in the model with ELB

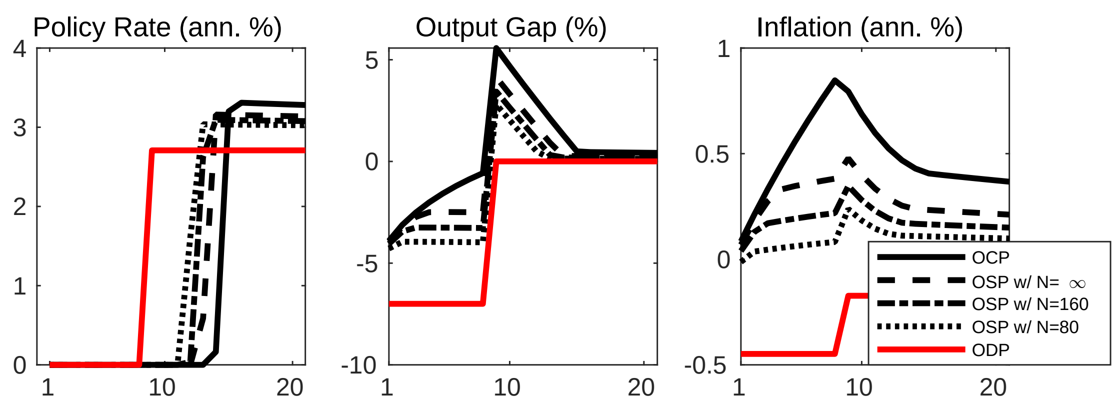
[20] In addition, we also considered a model with a static Euler equation and a forward-looking Phillips curve and a model with a discounted Euler equation and a static Phillips curve. The main results of our paper also hold in these two models.

Table 3: Parameter Values
—Model with the Forward-Looking Phillips Curve

Parameter	Description	Parameter Value
β	Discount rate	$\frac{1}{1+0.0075} \approx 0.9925$
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips curve	0.005
λ	Relative weight on output volatility	[1/16]
p_H	Crisis shock frequency	0.5/100
p_L	Crisis shock persistence	0.5
r^*	Natural rate in the normal state	3/400
r_c	Natural rate in the crisis state	-0.0125
N	Punishment length	[80, 160, ∞]

The dynamics of the economy under the OCP and the ODP are consistent with those in the model with a static Phillips curve as well as those in the existing studies. Under the OCP—shown by the solid black lines—the central bank keeps the policy rate at the ELB even after the crisis shock disappears and engineers the overshooting of inflation and output above their targets. Under the ODP—shown by the solid red lines—the central bank keeps the policy rate at the ELB as long as the crisis shock continues and raises the policy rate immediately once the crisis shock disappears.

Figure 7: Dynamics
—Model with Forward-Looking Phillips Curve—



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

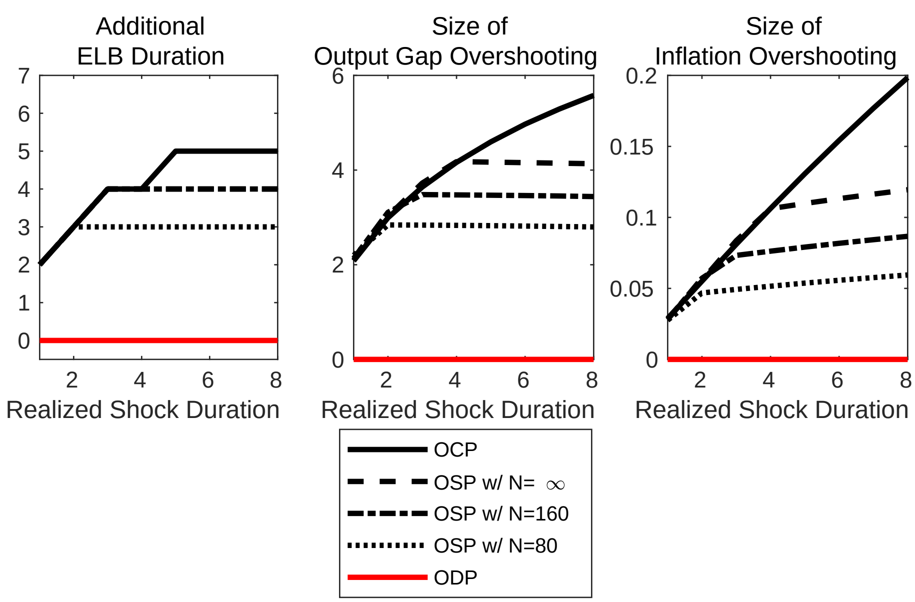
Under the OSPs—shown by the dashed, dash-dotted, and dotted black lines for $N = \infty$, $N = 160$, and $N = 80$, respectively—the central bank keeps the policy rate at the ELB after the crisis shock disappears, but not as long as it would do under the OCP. The magnitudes of inflation and output overshoots are smaller under the OSPs than under the OCP, with the magnitudes smaller when N is smaller. Consistent with the magnitudes of the overshoot in the aftermath of the crisis, the paths of output and inflation during the crisis state are lower when N is smaller. While the OSPs are not as stimulative as the OCP, they still substantially reduce the welfare cost of the ELB constraint relative to the ODP, which can be seen in Table 4.

Table 4: Welfare Cost of ELB
—Model with Forward-Looking Phillips Curve—

	abs($E[V]$)
Optimal commitment policy	26.8 (0.39)
Optimal sustainable policy	
with $N = \infty$	27.1 (0.39)
with $N = 160$	28.0 (0.40)
with $N = 80$	29.9 (0.43)
Optimal discretionary policy	68.9 (1.00)

Note: Numbers in parentheses are the welfare cost of the ELB relative to that under the optimal discretionary policy.

Figure 8: History Dependence
—Model with Forward-Looking Phillips Curve—



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The inflation rate are expressed in annualized percent. The output gap is expressed in percent.

One key feature of OSPs in the model with a static Phillips curve is that they are less history dependent than the OCP. To examine whether this feature holds in the model with a forward-looking Phillips curve, Figure 8 shows how the additional ELB duration, the size of the output gap overshoot, and the size of the inflation overshoot vary with the realized crisis shock duration. In the figure, the solid black and red lines are for the OCP and the ODP, respectively, whereas the dashed, dash-dotted, and dotted black lines are for the OSPs with $N = \infty$, $N = 160$, and $N = 80$, respectively. According to the figure, the additional ELB duration, the size of the output gap overshoot, and the size of the inflation overshoot are less sensitive to the realized crisis shock duration under the OSPs than under the OCP. That is, the OSPs are less history dependent than the OCP. Also, the OSPs are less history dependent with smaller Ns.

All told, qualitatively, the key insights from the model with a static Phillips curve carry over to the model with a forward-looking Phillips curve.

5 Conclusion

In this paper, we have characterized OSPs in models with the ELB constraint. We find that, even when the OCP is not credible, the central bank can still credibly commit to keeping the policy rate at the ELB in the aftermath of a crisis—though not as long as under the OCP—and meaningfully mitigate the adverse consequences of the ELB constraint on economic activity in crises.

By construction, our OSPs are time-consistent and thus overcome the criticism that the temporary overheating of the economy associated with lower-for-longer strategies is not credible. When the loss of reputation is sufficiently short-lived, these OSPs are not history dependent or not as history dependent as the OCP. Thus, it overcomes the criticism that the implied policy rate path is too complex for the central bank to be able to explain to the public, making the OSPs even more attractive.

Although we focus on the time-consistency aspect of lower-for-longer policies in this paper, there are other aspects of these policies that could make them less attractive in reality than in theory. For example, the public may not understand the temporary nature of the inflation overshooting, resulting in unanchoring of the long-run inflation expectations (Kohn (2012) and Yellen (2018)). The overheating of the economy may be less desirable for policymakers in reality than what's implied by our model if the overheating of the economy leads to financial instability (Yellen (2018)). It would be useful to formally analyze how these factors affect the effectiveness and implementability of lower-for-longer strategies. We leave such analysis to future research.

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Technical Appendix for Online Publication

This technical appendix is organized as follows:

- Appendix A defines some key concepts.
- Appendix B describes the equilibrium conditions characterizing the sustainable outcome in the model with a static Phillips curve in detail.
- Appendix C describes the numerical solution method and reports the solution accuracy.
- Appendix D analyzes the dynamics of the model with a static Phillips curve in detail through policy and value functions.
- Appendix E describes solution methods for the model with a static Phillips curve when N is small.
- Appendix F describes additional results in the model with a static Phillips curve and discounted Euler equations.
- Appendix G describes the equilibrium conditions characterizing the sustainable outcome in the model with a static Euler equation in detail.
- Appendix H documents the average tenure of chairpersons/governors in select central banks.
- Appendix I collects policymakers' speeches in which the time-inconsistency problem of the lower-for-longer policy is discussed.

A Definition of a plan and credibility

This section defines a plan, credibility, and the revert-to-discretion plan. The definitions closely follow Chang (1998) and Nakata (2018).

A.1 Plan

A government strategy, denoted by $\sigma_g := \{\sigma_{g,t}\}_{t=1}^{\infty}$, is a sequence of functions that maps a history of the nominal interest rates up to the previous period and a history of states up to today into today's nominal interest rate. Formally, $\sigma_{g,t}$ is given by $\sigma_{g,1} : \mathbb{S} \to \mathbb{R}_{\geq 0}$ and $\sigma_{g,t} : \mathbb{R}_{\geq 0}^{t-1} \times \mathbb{S}^t \to \mathbb{R}_{\geq 0}$ for all $t \geq 2$.[21] Given a particular realization of $\{s_t\}_{t=1}^{\infty}$, a sequence of nominal interest rates will be determined recursively by $i_1 = \sigma_{g,1}(s_1)$ and $i_t = \sigma_{g,t}(i^{t-1}, s^t)$ for all $t > 1$ and for all $s^t \in \mathbb{S}^t$. A government strategy is said to *induce* a sequence of the nominal

[21] The first period is a special case, as there is no previous policy action.

interest rates. *A private-sector strategy*, denoted by $\sigma_p := \{\sigma_{p,t}\}_{t=1}^{\infty}$, is a sequence of functions mapping a history of nominal interest rates up to today and a history of states up to today into today's consumption and inflation. Formally, $\sigma_{p,t}$ is given by $\sigma_{p,t} : \mathbb{R}^t \times \mathbb{S}^t \rightarrow (\mathbb{R}, \mathbb{R})$ for all t.

Given a government and private-sector strategy, a sequence of consumption and inflation will be determined recursively by $(y_t, \pi_t) = \sigma_{p,t}(i^t, s^t)$ for all $t \geq 1$ and for all $s^t \in \mathbb{S}^t$. A private sector strategy, together with a government strategy, is said to *induce* a sequence of consumption and inflation.[22] *A plan* is defined as a pair of government and private sector strategies, (σ_g,σ_p). Notice that a plan induces an outcome—a state-contingent sequence of consumption, inflation, and the nominal interest rate. As discussed earlier, there is a value sequence $\{w_t(s^t)\}_{t=1}^{\infty}$, associated with any outcome.

A.2 Credibility

Let us use $CE_t^I(s)$ to denote a set of state-contingent sequences of the nominal interest rate consistent with the existence of a competitive equilibrium when $s_t = s$. Formally, for each $s \in \mathbb{S}$, $CE_t^I(s) := \{\boldsymbol{i}_t(s) \in \mathbb{I}^{\infty} | \ \exists \ (\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s)) \text{ s.t. } (\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s), \boldsymbol{i}_t(s)) \in CE_t(s)\}$. σ_g is said to be *admissible* if, after any history of policy actions, i^{t-1}, and any history of states, s^t, $\boldsymbol{i}_t(s)$ induced by the continuation of σ_g belongs to $CE_t^I(s_t)$.

A plan, (σ_g, σ_p), is *credible* if (i) σ_g is admissible, (ii) after any history of policy actions, i^t, and any history of states, s^t, the continuation of σ_p and σ_g induce a $(\boldsymbol{y}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t)) \in CE_t(s_t)$, and (iii) after any history i^{t-1} and s^t, $\boldsymbol{i}_t(s_t)$ induced by σ_g maximizes the government's objective over $CE_t^I(s_t)$ given σ_p. In plain languages, a plan is said to be credible if neither the private sector nor the government has incentive to deviate from the strategies associated with it.

An outcome is said to be credible if there is a credible plan that induces it. When a certain plan A is credible and the plan A induces a certain outcome α, we say that the outcome α can be made credible, or time-consistent, by the plan A.

A.3 The revert-to-discretion plan

I now define a key object of this paper, the revert-to-discretion plan, and discuss the condition under which this plan is credible.

The revert-to-discretion plan, $(\sigma_g^{rtd}, \sigma_p^{rtd})$, consists of (i) the following government strategy: $\sigma_{g,1}^{rtd} = i_{s,1}(s_1)$ for any $s_1 \in \mathbb{S}$, $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{s,t}(s^t)$ if $i_j = i_{s,j}(s^j)$ for all $j \leq t-1$, and $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{d,t}(s^t)$ otherwise, and (ii) the following private-sector strategy: $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{s,t}(s^t), \pi_{c,t}(s^t))$ if $r_j = i_{c,j}(s^j)$ for all $j \leq t$, $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{br}(s_t, i_t), \pi_{br}(s_t, i_t))$ otherwise,[23] where

$$y_{br}(s_t, r_t) = E_t y_{d,t+1}(s^{t+1}) - \sigma\Big[\big[i_t - E_t \pi_{d,t+1}(s^{t+1}) - r^*\big] + s_t\Big] \tag{11}$$

$$\pi_{br}(s_t, r_t) = \kappa y_{br}(s_t, r_t) + \beta E_t \pi_{d,t+1}(s^{t+1}) \tag{12}$$

[22] Note that, while the nominal interest rate today depends on the history of nominal interest rates up to the previous period, consumption and inflation today depend on the history of nominal interest rates up to today. The implicit within-period-timing protocol behind this setup is that the government moves before the private sector does.

[23] Subscript *br* stands for *b*est *r*esponse.

The government strategy instructs the government to choose the nominal interest rate consistent with the sustainable outcome, but chooses the interest rate consistent with the discretionary outcome if it has deviated from the sustainable outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the sustainable outcome as long as the government has never deviated from the sustainable outcome. If the government has ever deviated from the nominal interest rate consistent with the sustainable outcome, the private sector strategy instructs the household and firms to choose output and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the discretionary outcome. By construction, the revert-to-discretion plan induces the sustainable outcome, and the implied value sequence is identical to the sustainable value sequence.

It is relatively straightforward to show that the revert-to-discretion plan is credible. By construction, $V_{s,t}(s^t) \geq V_{d,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$, making sure that the government does not have an incentive to deviate from the instruction given by the government strategy after any history i^{t-1} and s^t in which the optimal sustainable policy has been followed.

The revert-to-discretion plan that induces the optimal sustainable outcome with a finite period punishment is defined in a similar way (see Nakata (2018) for rigorous exposition). It is also straightforward to show that such a plan is credible.

B Model with a static Phillips Curve

The policymaker maximizes

$$V_0 = -E_0 \sum_{t=0}^{\infty} \beta^t y_t^2,$$

subject to

$$y_t = (M_{ee} + \sigma^{-1}\kappa) E_t y_{t+1} - \sigma\left(i_t - r^*\right) + s_t, \tag{13}$$

$$i_t \geq i_{ELB}, \tag{14}$$

$$V_t = -E_t \sum_{j=0}^{\infty} \beta^j y_{t+j}^2 \geq W(s_t), \tag{15}$$

for all $t \geq 0$, where we used a relationship based on the static Phillips curve, $\pi_t = \kappa y_t$ to obtain equation 13. $M_{ee} \leq 1.0$ is a parameter whose value is less than one to diminish the degree of forward-lookingness in the case of a discounted Euler equation. The shock, s_t, follows two-state Markov chain, $s_t \in \{s_H, s_L\}$ where $s_H > s_L$. Transition probability matrix is given as $P = \begin{bmatrix} 1-p_H & p_H \\ 1-p_L & p_L \end{bmatrix}$, where p_H is the frequency of the crisis and p_L is the persistence of the crisis. $W(s_t)$ is the value under the optimal discretionary policy.

The analytical solution for the discretionary outcome: The ZLB is slack in the high/normal state, $\phi_H = 0$, and the ZLB is binding in the low/crisis state, $\phi_L = \phi > 0$.

Thus, the equilibrium conditions become

$$
\begin{aligned}
&y_H = 0, \\
&y_L = -\phi, \\
&y_H + \sigma(i_H - r^* - \sigma^{-1}s_H) - (M_{ee} + \sigma\kappa)(p_H y_L + (1 - p_H) y_H) = 0, \\
&y_L + \sigma(i_{ELB} - r^* - \sigma^{-1}s_L) - (M_{ee} + \sigma\kappa)(p_L y_L + (1 - p_L) y_H) = 0, \\
&V_H = -y_H^2 + \beta(p_H V_L + (1 - p_H) V_H), \\
&V_L = -y_L^2 + \beta(p_L V_L + (1 - p_L) V_H).
\end{aligned}
$$

The solution to the system of equations is given by the following:

$$
\begin{aligned}
y_H &= 0, & y_L &= -\phi = \frac{-i_{ELB} + r^* + \sigma^{-1}s_L}{\sigma^{-1}(1 - (M_{ee} + \kappa\sigma)p_L)}, \\
i_H &= r^* + \sigma^{-1}s_H + \sigma^{-1}(M_{ee} + \kappa\sigma)p_H y_L, & i_L &= i_{ELB}, \\
V_H &= \frac{-\beta p_H y_L^2}{(1 - \beta)(1 - \beta(p_L - p_H))}, & V_L &= \frac{-y_L^2 + \beta(1 - p_L)V_H}{(1 - \beta p_L)}.
\end{aligned}
$$

Solving for the sustainable equilibrium: The Lagrangean is given by

$$
\begin{aligned}
\mathcal{L} &\equiv E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -y_t^2 + 2\phi_t \left(y_t + \sigma \left(i_t - r^* \right) - s_t - (M_{ee} + \sigma\kappa) E_t y_{t+1} \right) \\
&\quad + \psi_t \left(-E_t \sum_{j=0}^{\infty} \beta^j y_{t+j}^2 - W(s_t) \right) \Big\}, \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\Psi_t y_t^2 + 2\phi_t \left(y_t + \sigma \left(i_t - r^* \right) - s_t \right) - \frac{1}{\beta} 2\phi_{t-1}(M_{ee} + \sigma\kappa) y_t - \psi_t W(s_t) \right\},
\end{aligned}
$$

where $\Psi_t = \psi_{-1} + \psi_0 + ... + \psi_t < \infty$ is the sum of the Lagrange multipliers on the sustainability constraint. The FOCs are given by

$$
\begin{aligned}
\partial y_t : &\quad -\Psi_t y_t - \phi_t + \beta^{-1}(M_{ee} + \sigma\kappa)\phi_{t-1} = 0, \\
\partial \phi_t : &\quad y_t + \sigma \left(i_t - r^* \right) - s_t - (M_{ee} + \sigma\kappa) E_t y_{t+1} = 0.
\end{aligned}
$$

Normalizing the first equation by Ψ_t, we have

$$
-y_t - \tilde{\phi}_t + \beta^{-1}(M_{ee} + \sigma\kappa) z_t \tilde{\phi}_{t-1} = 0,
$$

where $\tilde{\phi}_t = \phi_t / \Psi_t$ and $z_t = \Psi_{t-1}/\Psi_t \in (0, 1]$. The Karush-Kuhn-Tucker conditions (KKTCs) must be satisfied as well

$$
\begin{aligned}
&\phi_t \left(i_t - i_{ELB} \right) = 0, \\
&\phi_t \geq 0, \\
&\psi_t \left(V_t - W(s_t) \right) = 0, \\
&\psi_t \geq 0.
\end{aligned}
$$

The initial conditions on the Lagrange multipliers are such that $\phi_{-1} = 0$ and $\psi_{-1} = 1$, which implies $\tilde{\phi}_{-1} = 0$ and $\Psi_{-1} = 1$.

C Time-iteration method

We explain details of the time-iteration method for the model with a static Phillips curve. The method is extended to solve other models considered in this paper—the model with a static Euler equation and the standard New Keynesian model. We explicitly consider a vector of policy and value functions $\varsigma(\xi_j) = [y(\xi_j), i(\xi_j), \tilde{\phi}'(\xi_j), z(\xi_j), V(\xi_j)]'$ as functions of the state variables $\xi_j = (\tilde{\phi}, s_j)$ for $j = 1, ..., N$. We have the following a system of functional equations:

$$
\begin{aligned}
e_1(\xi_j) \equiv & - y(\xi_j) - \phi'(\xi_j) + \beta^{-1}(M_{ee} + \sigma\kappa) z(\xi_j)\tilde{\phi} = 0, \\
e_2(\xi_j) \equiv & - y(\xi_j) - \sigma\left(i(\xi_j) - r^*\right) + s_j + (M_{ee} + \sigma\kappa) \sum_{k=1}^{N} p(s_k|s_j) y(\tilde{\phi}'(\xi_j), s_k) = 0, \\
e_3(\xi_j) \equiv & - V(\xi_j) - y(\xi_j)^2 + \beta \sum_{k=1}^{N} p(s_k|s_j) V(\tilde{\phi}'(\xi_j), s_k) = 0.
\end{aligned}
$$

Note that we also have two occasionally binding constraints

$$
\begin{aligned}
& i(\xi_j) \geq i_{ELB}, \\
& V(\xi_j) \geq W(s_j).
\end{aligned}
$$

Algorithm The time iteration method takes the following steps:

1. Make an initial guess for the policy function $\varsigma^{(0)}(\xi_j)$ for $j = 1, ..., N$.

2. For $n = 1, 2, ...$ (n is an index for the number of iteration), given the policy function previously obtained $\varsigma^{(n-1)}(\xi_j)$ for each j, solve

$$
\begin{aligned}
& - y - \tilde{\phi}' + \beta^{-1}(M_{ee} + \sigma\kappa) z\tilde{\phi} = 0, \\
& - y - \sigma\left(i - r^*\right) + s_j + (M_{ee} + \sigma\kappa) \sum_{k=1}^{N} p(s_k|s_j) y^{(n-1)}(\tilde{\phi}', s_k) = 0, \\
& - V - y^2 + \beta \sum_{k=1}^{N} p(s_k|s_j) V^{(n-1)}(\tilde{\phi}', s_k) = 0, \\
& i \geq i_{ELB}, \\
& V \geq W(s_j),
\end{aligned}
$$

   for $(y, r, \tilde{\phi}', z, V)$.

3. Update the policy function by setting $y = y^{(n)}(\xi_j)$, $i = i^{(n)}(\xi_j)$, $\tilde{\phi}' = \tilde{\phi}'^{(n)}(\xi_j)$, $z = z^{(n)}(\xi_j)$, $V = V^{(n)}(\xi_j)$ for $j = 1, ..., N$.

4. Repeat 2-3 until $\left\|\varsigma^{(n)}(\xi_j) - \varsigma^{(n-1)}(\xi_j)\right\|$ is small enough.

In Step 2, there are four patterns of binding constraints due to the KKTCs.

(i) $i > i_{ELB}$ and $V > W(s_j)$: We immediately know $\tilde{\phi}' = 0$ and $z = 1$. Note that, taking the policy functions in the previous iteration, $y^{(n-1)}(\tilde{\phi}', s_k)$ and $V^{(n-1)}(\tilde{\phi}', s_k)$ as given, the expected values of the next period's variables are obtained by evaluating the policy functions at $\tilde{\phi}' = 0$. Then we obtain

$$
\begin{aligned}
y &= \beta^{-1}(M_{ee} + \sigma\kappa)\tilde{\phi}, \\
i &= r^* + (\sigma^{-1}M_{ee} + \kappa)\sum_{k=1}^{N} p(s_k|s_j)y^{(n-1)}(0, s_k) - \sigma^{-1}y + \sigma^{-1}s_j, \\
V &= -y^2 + \beta\sum_{k=1}^{N} p(s_k|s_j)V^{(n-1)}(0, s_k).
\end{aligned}
$$

(ii) $i > i_{ELB}$ and $V \leq W(s_j)$: $\tilde{\phi}' = 0$ and $z \in (0, 1)$. The sustainability constraint is binding, $V = W(s_j)$. Then we obtain

$$
\begin{aligned}
y &= \left(-W(s_t) + \beta\sum_{k=1}^{N} p(s_k|s_j)V^{(n-1)}(0, s_k)\right)^{\frac{1}{2}}, \\
i &= r^* + (\sigma^{-1}M_{ee} + \kappa)\sum_{k=1}^{N} p(s_k|s_j)y^{(n-1)}(0, s_k) - \sigma^{-1}y + \sigma^{-1}s_j, \\
z &= \beta(M_{ee} + \sigma\kappa)^{-1}y/\tilde{\phi}.
\end{aligned}
$$

(iii) $i \leq i_{ELB}$ and $V > W(s_j)$: $\tilde{\phi}' > 0$ and $z = 1$. The ZLB is binding, $i = i_{ELB}$. $(y, \tilde{\phi}')$ are obtained by solving the following nonlinear equations:

$$
\begin{aligned}
y &= -\tilde{\phi}' + \beta^{-1}(M_{ee} + \sigma\kappa)\tilde{\phi}, \\
i_{ELB} &= r^* + (\sigma^{-1}M_{ee} + \kappa)\sum_{k=1}^{N} p(s_k|s_j)y^{(n-1)}(\tilde{\phi}', s_k) - \sigma^{-1}y + \sigma^{-1}s_j.
\end{aligned}
$$

(iv) $i \leq i_{ELB}$ and $V \leq W(s_j)$: $\tilde{\phi}' > 0$ and $z \in (0, 1)$. Both constraints are binding, $i = i_{ELB}$ and $V = W(s_j)$. $(y, \tilde{\phi}')$ are obtained by solving the following nonlinear equations:

$$
\begin{aligned}
W(s_k) &= -y^2 + \beta\sum_{k=1}^{N} p(s_k|s_j)V^{(n-1)}(\tilde{\phi}', s_k), \\
i_{ELB} &= r^* + (\sigma^{-1}M_{ee} + \kappa)\sum_{k=1}^{N} p(s_k|s_j)y^{(n-1)}(\tilde{\phi}', s_k) - \sigma^{-1}y + \sigma^{-1}s_j.
\end{aligned}
$$

Note that these equations do not depend on $\tilde{\phi}$. Having $(y, \tilde{\phi}')$ at hand, $z = \beta(M_{ee} +$

$\sigma\kappa)^{-1}(y+\tilde{\phi}')/\tilde{\phi}$ is also obtained.

In solving the problem, we discretize the state space of $\tilde{\phi} \in [0, \bar{\tilde{\phi}}]$. We use 401 points for each state variable. The upper bound $\bar{\tilde{\phi}}$ is endogenously set so that the sequence of the Lagrange multipliers $\{\tilde{\phi}_t\}$ under consecutive negative shocks $s_t = s_L$ for $t = 1, 2, ..., T_1$ are bounded above.[24] We divide the state space by log-spaced grid points so that more grid points are toward the origin. We use piecewise-linear functions to approximate the policy functions off the grid points.

Figure 9 shows the impulse response of the residual functions $\{e_{1,t}, e_{2,t}, e_{3,t}\}$ under the OSP with $N = 60$[25]. As in the impulse responses in Figure 1 and 2 in the main text, the crisis shock hits the economy at time 1 and stays until time 8. We can see that all the values of the residual functions are reasonably small. When the sustainability constraint binds at time 9, the value of $e_{1,t}$ increases as $z_t < 1$ and the equation becomes nonlinear. Also, the values of $e_{3,t}$ increase when the values of V_t are in transition from one value to another (i.e., in each of the first few periods after time 1 and after time 9).

Figure 9: Residuals (under the OSP with $N = 60$)

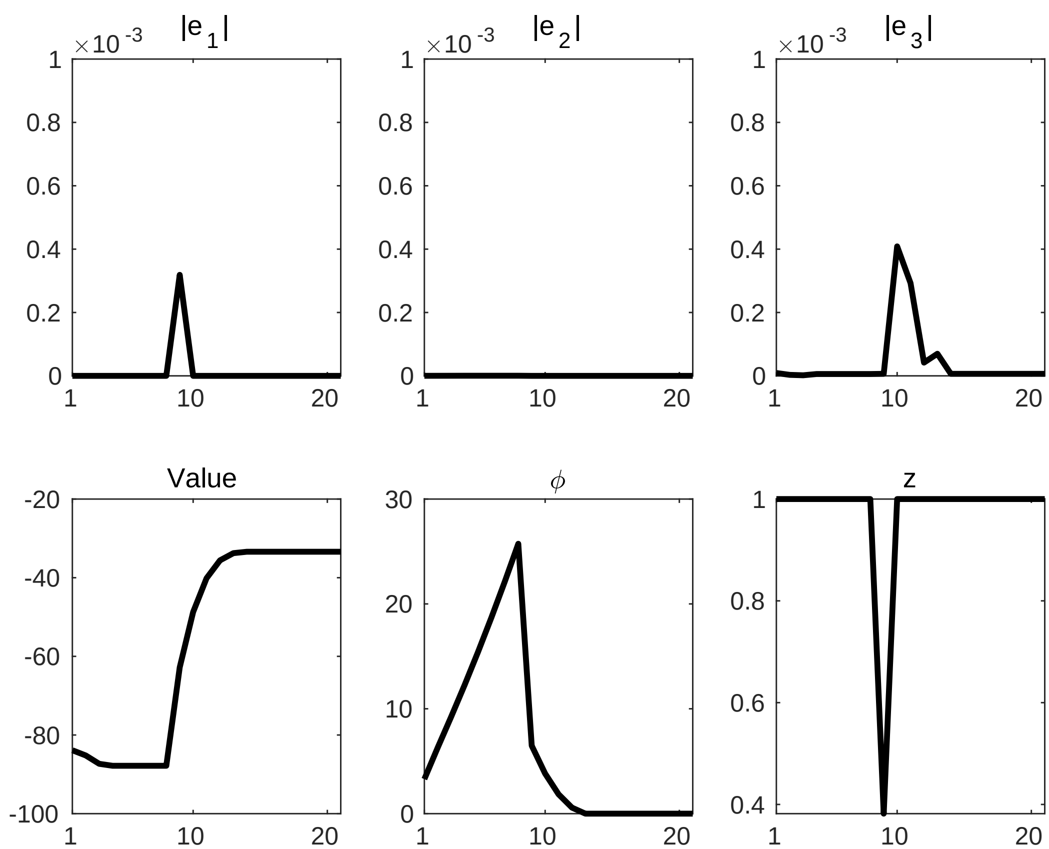


Table 5 shows the the average and maximum of the residuals in absolute terms (log 10 units) based on a 100,000-period stochastic simulation under the OCP, the OSP with $N = \infty$ and the OSP with $N = 60$. The values of e_3 tend to be worse than others as we use linear

[24]Taking as given the cutoff probability $\bar{p} = 0.005$, T_1 is set to the highest integer that satisfies $p_L^{T_1} > \bar{p}$.

[25]We cannot calculate the Euler equation errors under the OSP with $N = 20$ as the only analytical solutions are available.

interpolation, whereas the value functions are at least quadratic. Note that the equation for e_1 holds with equality (up to the machine precision) under the OCP, as the equation is linear (i.e., $z = 1$) and we use the equation to substitute variables other than the ones we solve for with the other equations.

Table 5: Residuals

	$\lvert e_1\rvert$		$\lvert e_2\rvert$		$\lvert e_3\rvert$	
	L_1	L_∞	L_1	L_∞	L_1	L_∞
OCP	-16.59	-14.15	-7.55	-3.48	-4.83	-2.36
OSP						
with $N = \infty$	-7.43	-3.38	-7.27	-3.66	-4.84	-2.28
with $N = 60$	-6.22	-3.46	-8.01	-3.94	-4.99	-2.66

Notes: L_1 is the average and L_∞ is the maximum of the residuals in absolute terms (log 10 units) based on a 100,000-period stochastic simulation.

D A detailed account of the model with a static Phillips curve

In this section, we provide a detailed account of the dynamics of the model with a static Phillips curve from the vantage point of the policy functions. There are two purposes. First, seeing the dynamics of the economy in this way helps us better understand what's happening in the model. Second, it helps us to understand why the dynamics of the economy are history independent when N is sufficiently small and why it is possible to solve the model in this case in an alternative way that we later describe in Appendix E.

We can accomplish the first with the baseline calibration of the main text, but cannot accomplish the second because the time-iteration does not converge for any small values of N consistent with history independent dynamics. Accordingly, we will use an alternative calibration in which the time-iteration method converges for some values of N consistent with history independent dynamics. In this alternative calibration, $p_H = 0.2/100$ and $p_L = 0.5$. The values for other parameters are the same as the baseline values from the main text.

We first describe the dynamics of the economy under the optimal commitment policy (OCP). We then describe the dynamics of the economy under optimal sustainable policies (OSPs) with $N = \infty$ and $N = 120$.

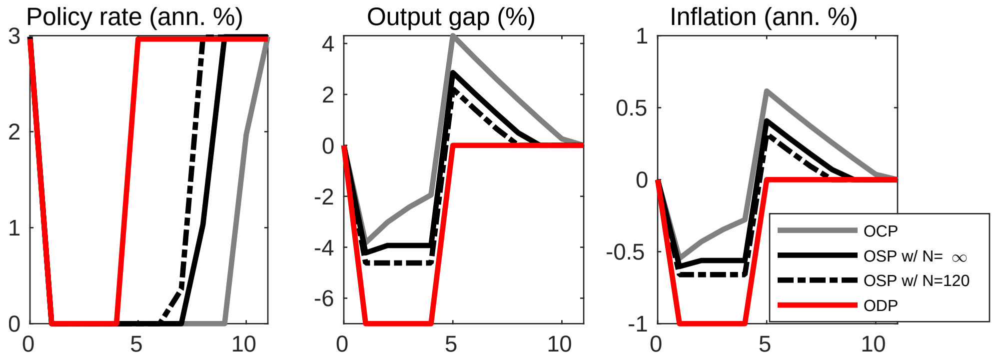
D.1 Optimal commitment policy

Suppose that the economy is initially at its risky steady state at time 0 where the policy rate is positive and the Lagrange multiplier is zero. The economy falls into the crisis state at time 1 and stays there until time 4. The economy is back in the normal state from time 5 on.

The economy's dynamics under the OCP in this recession scenario are shown by the solid grey lines in Figure 10. Consistent with what we saw in the main body of the paper, the central bank keeps the policy rate at the ELB even after the crisis shock disappears and engineers an overheating of the economy.

The solid black lines in Figure 11 are the value and policy functions associated with the OCP. The black dots in Figure 11 trace the dynamics of the economy in this recession scenario along the value and policy functions. The dynamics of the economy from time 1 to time 4

Figure 10: Dynamics with an alternative calibration



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

are governed by the crisis-state value and policy functions shown in the right panel. As the economy stays in the crisis state, the Lagrange multiplier—the sole endogenous state variable of the model—increases. The pace of the increase decelerates the longer the economy is in the crisis state. If the economy were to stay in the crisis state forever, the Lagrange multiplier converges to a finite value. The evolution of output in the crisis state mirrors that of the Lagrange multiplier. As the Lagrange multiplier increases, output increases. If the economy were to stay in the crisis state forever (which is a zero probability event), output converges to a finite value, just as the Lagrange multiplier converges to a finite value.

Once the economy is back in the normal state, the dynamics of the economy are governed by the normal-state value and policy functions shown in the left panel. In the first period back in the normal state, the lagged Lagrange multiplier is positive, which implies positive output at time 1. As the time continues, the Lagrange multiplier gradually declines, eventually returns to zero. Together with the Lagrange multiplier, output declines as the normal state continues, eventually returning to zero.

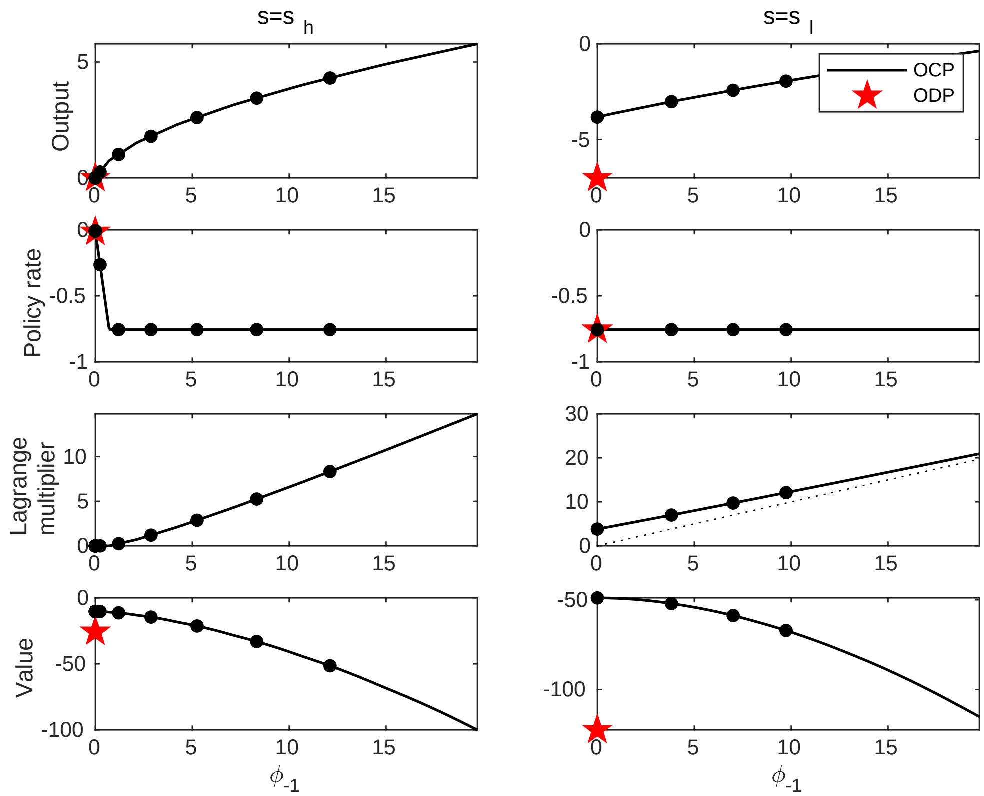
D.2 Optimal sustainable policies

The economy's dynamics under the OSPs with $N = \infty$ and $N = 120$ in this recession scenario are shown by the solid and dash-dotted black lines in Figure 10. For the set of parameter values considered in this section, the sustainability constraint binds right after the crisis state ends even under the OSP with $N = \infty$, shortening the ELB duration and limiting the size of output and inflation overshoots. With $N = 120$, the ELB duration is shorter and the size of the overshoot is smaller than with $N = \infty$.

The value/policy functions associated with the OSPs with $N = \infty$ and $N = 120$ are shown by the solid black lines in Figure 12 and 13, respectively. The dynamics of the economy under the OSPs in this recession scenario are traced by the black dots in these figures.

Under the OSP with $N = \infty$, the crisis-state Lagrange multiplier increases as the crisis state persists, as seen in the right column of Figure 12. The crisis-state output increases with the Lagrange multiplier, but only when the lagged Lagrange multiplier is very small. Otherwise, the crisis-state policy function for output is flat. Once the economy returns to the normal state, the dynamics of the economy are governed by the normal-state value and policy functions shown in the left column of Figure 12. The sustainability constraint binds if

Figure 11: Value/policy functions
—OCP and ODP—



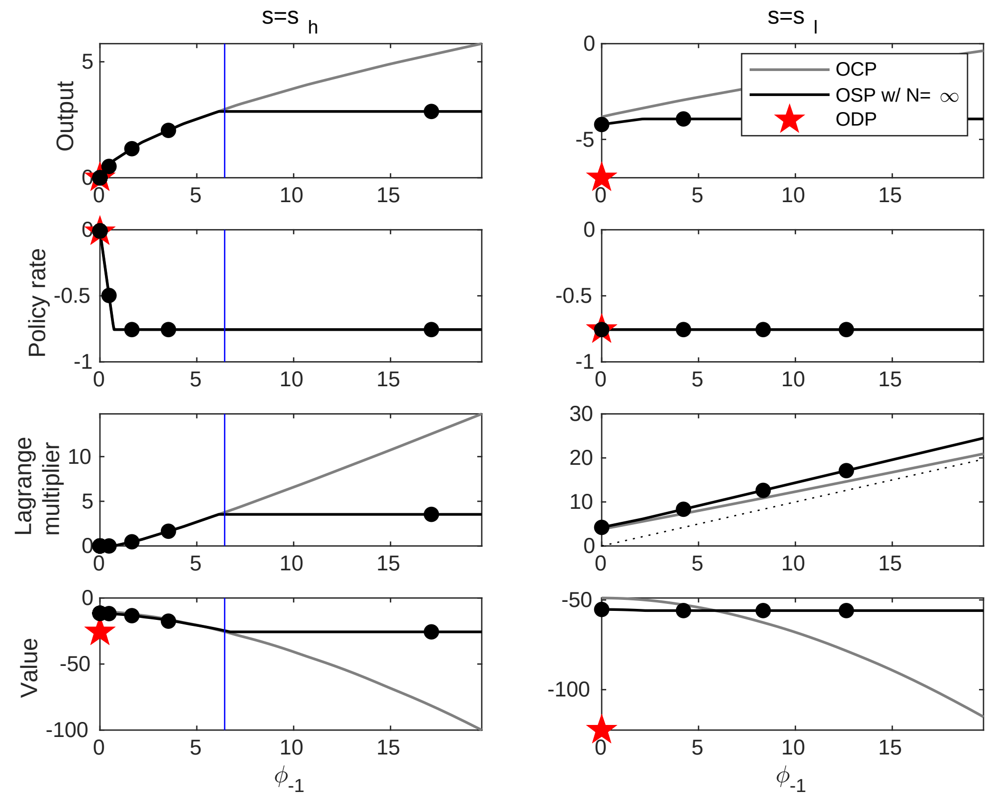
Note: ODP and OCP stand for optimal discretionary policy and optimal commitment policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

the lagged Lagrange multiplier is sufficiently large. For those values of the lagged Lagrange multiplier, the value and policy functions are flat. As a result, the Lagrange multiplier takes a low positive value in the first period of the normal state. Thus, the Lagrange multiplier declines quickly under the OSP with $N = \infty$ than under the OCP.

Under the OSP with $N = 120$, the crisis-state Lagrange multiplier increases as the crisis state persists, as seen in the right column of Figure 13. However, the policy function for output is flat so that output is constant in the crisis state. When the economy returns to the normal state, the dynamics of the economy are governed by the normal-state value and policy functions shown in the left column of Figure 12. The normal-state value and policy functions are qualitatively similar to those under the OSP with $N = \infty$. However, one interesting feature of the economy under the OSP with $N = 120$ is that the value of the lagged Lagrange multiplier in the first period of the normal state does not depend on the realized crisis shock duration. No matter how long the crisis shock lasts, the Lagrange multiplier takes a certain value given by the flat part of the normal-state policy function for the Lagrange multiplier. As a result, post-crisis dynamics do not depend on the realized crisis shock duration. In other words, the OSP with $N = 120$ is history independent. In this case, the dynamics of the economy in the normal state can be fully characterized by a finite sequence of output, policy rate, Lagrange multiplier, and the value.

Note that the key difference between the history dependent and history independent cases

Figure 12: Value/policy functions
—OSP with $N = \infty$—



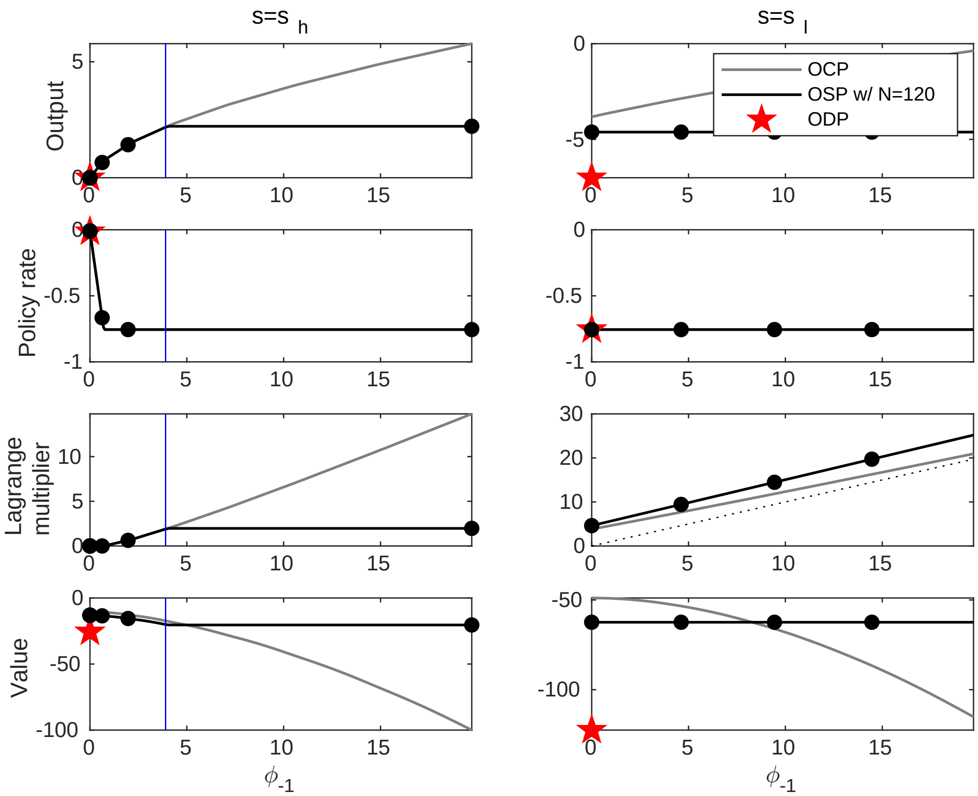
Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

is whether the value of the Lagrange multiplier in *the first period of the crisis state* is above or below the value of the lagged Lagrange multiplier above which the sustainability constraint binds in *the normal state*. This threshold value of the lagged Lagrange multiplier is indicated by thin vertical lines in the left panels of Figure 12 and 13.

If the value of the Lagrange multiplier in *the first period of the crisis state* is above the threshold value of the lagged Lagrange multiplier, then the value of the Lagrange multiplier in *any period of the crisis state* is also above the threshold value of the lagged Lagrange multiplier because the Lagrange multiplier increases as the crisis persists. In this case, when the economy returns to the normal state, output, inflation, value, policy rate, and the Lagrange multiplier in the first period of the normal state will be given by the value of the normal state policy function in the flat region where the sustainability constraint binds, regardless of how long the crisis lasts. Thus, the allocations in the first period back in the normal state do no depend on the realized crisis shock duration. If the Lagrange multiplier in the first period of the normal state does not the realized crisis duration, the dynamics of the economy will not depend on the realized crisis shock duration.

If the value of the Lagrange multiplier in *the first period of the crisis state* is below the threshold value of the lagged Lagrange multiplier, then the allocations in the first period back in the normal state depend will differ depending on whether the realized crisis duration is one period or longer. This is because the normal-state value and policy functions are not in

Figure 13: Value/policy functions
—OSP with $N = 120$—



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

the flat region. In this case, the economy's dynamics exhibit history dependence. The higher the threshold value of the lagged Lagrange multiplier is relative to the value of the Lagrange multiplier in the first period of the crisis state, the more history dependent the economy's dynamics become.

E Two alternative solution methods when N is low

The previous section describes why the dynamics are history independent when N is sufficiently small and suggests that the dynamics of the economy are fully characterized by a vector of scalars satisfying a system of certain nonlinear equations and inequality constraints. As a result, it is not necessary to rely on the time-iteration method to compute the dynamics of the model. In this section, we provide the details of the solution algorithm we use when the economy's dynamics are history independent.

We also show that even if the dynamics are history dependent for some low N, we can exploit the fact that the policy functions are flat above the threshold of the Lagrange multiplier. This complements the standard numerical method with time iteration explained in Section C and the analytical method to be explained below. As a result, we can solve the model for all

$N \in \mathbb{R}_+$ by using either of the solution methods. We confirmed that these solution methods yield the same allocations when multiple solution methods are available for the same N.

Analytical method

The big picture of the solution algorithm is as follows. Assuming that the additional ELB duration, denoted by τ, is k, compute the allocation satisfying the model's equilibrium conditions that are captured by equality constraints. Check (i) whether the model's inequality constraints (ELB and sustainability constraints) are satisfied and (ii) whether the history independence assumption is indeed valid. If these two requirements are met, then we have found the solution. If not, continue to search for the value of τ that satisfies all the equilibrium conditions, both those represented by equality and inequality constraints.

As discussed in Section 2, this alternative solution method works in the model with a static Phillips curve, but not in the model with a forward-looking Phillips curve. As analyzed in Nakata and Schmidt (Forthcoming), in the model with a forward-looking Phillips curve, when the crisis shock is recurring, inflation and the output gap in the normal state are not zero even after the effect of a past crisis dissipated. However, in the model with a static Phillips curve, inflation and the output gap converge to zero in the normal state once the effect of a past crisis shock dissipates (because there is no trade-off between inflation and output stabilization). The alternative solution method we describe below takes advantage of this feature of the model with a static Phillips curve.[26]

Below, we first describe the set of equality and inequality constraints the solution has to satisfy given τ. We then describe how to verify that the history independence assumption is satisfied.

Case 1: $\tau = 0$ (liftoff occurs immediately after the crisis shock disappears)

We describe how to find the solution to the model if the economy's dynamics are history independent and the policy rate is above the ELB right after the crisis shock disappears. In this case, the dynamics of the economy are fully characterized by $\{i_H, i_M, i_L, y_H, y_M, y_L, v_H, v_M, v_L\}$ where the subscript denote one of the three states of this economy:

- In L state, the crisis shock is present. Only the ELB constraint is binding.
- In M state, the crisis shock is absent. Only the sustainability constraint is binding. This state follows L state.
- In H state, the crisis shock is absent. No constraint is binding.

By definition of this case, the following equality and inequality constraints must be satisfied:

$$i_H > -r^*$$
$$i_M > -r^*$$
$$i_L = -r^*$$

[26] Note that we could also use this alternative solution method to the model with perfectly sticky prices—in which the inflation rate is zero at any time and after any history of shocks—. In the model with perfectly sticky prices, there is no inflation-output trade-off and the output gap eventually converges to zero once the effect of a past crisis dissipates.

and

$$
\begin{aligned}
v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

where $v_{Punish,H}(v_H, v_L, y_H, y_L)$ and $v_{Punish,L}(v_H, v_L, y_H, y_L)$ are given by:

$$
\begin{aligned}
v_{D,H}^{i-1} &= \beta \left[(1-p_H)v_{D,H}^i + p_H v_{D,L}^i\right], \\
v_{D,L}^{i-1} &= -(y_{D,L}^{i-1})^2 + \beta \left[(1-p_L)v_{D,H}^i + p_L v_{D,L}^i\right], \\
y_{D,L}^{i-1} &= -\sigma\left(-r^* - s_L\right) + \left(M_{ee} + \sigma\kappa\right)\left[(1-p_L)y_{D,H}^i + p_L y_{D,L}^i\right],
\end{aligned}
$$

for $i = 1, ..., K$, where $(v_{D,H}^K, v_{D,L}^K, y_{D,H}^K, y_{D,L}^K) = (v_H, v_L, y_H, y_L)$ and $v_{Punish,H} = v_{D,H}^0$. K is the length of punishment. In this case, the equilibrium conditions are given by:

$$
\begin{aligned}
y_H &= \left(M_{ee} + \sigma\kappa\right)\left[(1-p_H)y_H + p_H y_L\right] - \sigma\left(i_H - s_H\right), \\
y_M &= \left(M_{ee} + \sigma\kappa\right)\left[(1-p_H)y_H + p_H y_L\right] - \sigma\left(i_M - s_H\right), \\
y_L &= \left(M_{ee} + \sigma\kappa\right)\left[(1-p_L)y_M + p_L y_L\right] - \sigma\left(-r^* - s_L\right), \\
v_H &= -y_H^2 + \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= -y_M^2 + \beta\left[(1-p_H)v_H + p_H v_L\right] = v_{Punish,H}(v_H, v_L, y_H, y_L), \\
v_L &= -y_L^2 + \beta\left[(1-p_L)v_M + p_L v_L\right],
\end{aligned}
$$

where we used $i_L = -r^*$. Because the central bank only cares about output, $y_H = 0$. Thus,

$$
\begin{aligned}
i_H &= \sigma^{-1}(M_{ee} + \sigma\kappa)p_H y_L + s_H, \\
i_M &= \sigma^{-1}(M_{ee} + \sigma\kappa)p_H y_L + s_H - \sigma y_M, \\
y_L &= \left(M_{ee} + \sigma\kappa\right)\left[(1-p_L)y_M + p_L y_L\right] - \sigma\left(-r^* - s_L\right), \\
v_H &= \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= -y_M^2 + \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_L &= -y_L^2 + \beta\left[(1-p_L)v_M + p_L v_L\right], \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

There are 7 equations and 7 unknowns: $(i_H, i_M, y_M, y_L, v_H, v_M, v_L)$.
Guess (y_L, v_L). Then,

- From the first three equations, we can determine (i_H, i_M, y_M).
- From the fifth and sixth equations, we can determine (v_H, v_M).

We need to check whether the fourth and seventh equations hold:

$$
\begin{aligned}
v_H &= \beta\left[(1-p_H)v_H + p_H v_L\right], \\
v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

That is, we can basically reduce the system to two-unknowns in two equations. Once you solve the system of equations, we need to verify the following four inequalities:

$$
\begin{aligned}
i_H &> -r^* \\
i_M &> -r^* \\
v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

When $\tau = 0$, the Lagrange multiplier is zero in the first period after the crisis shock disappears. Thus, the dynamics of the economy cannot depend on the realized duration of the shock. That is, the economy's dynamics are history independent by construction.

Case 2: $\tau > 0$ (liftoff occurs at least two periods after the crisis shock disappears)

In this section, we discuss how to solve for cases in which the policy rate stays at the ZLB for at least one period after the crisis shock disappears:

- In L state, the crisis shock hits the economy. Only the ZLB constraint is binding.
- In M_0 state, the crisis shock is absent. Both the ZLB and sustainability constraints are binding. This state follows L state.
- In M_i state from $i = 1$ to $i = \tau - 1$, the crisis shock is absent. Only the ZLB constraint is binding. This state follows M_{i-1} state.
- In M_τ state, the crisis shock is absent. No constraint is binding. This state follows $M_{\tau-1}$ state.
- In H state, the crisis shock is absent. No constraint is binding. And, output gap is zero.

By construction, the following equality and inequality constraints must be satisfied:

$$
\begin{aligned}
i_H &> -r^* \\
i_{M,\tau} &> -r^* \\
i_{M,\tau-1} &= -r^* \\
&\dots \\
i_{M,1} &= -r^* \\
i_{M,0} &= -r^* \\
i_L &= -r^*
\end{aligned}
$$

and

$$
\begin{aligned}
v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_{M,\tau} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_{M,\tau-1} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
&... \\
v_{M,1} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_{M,0} &= v_{Punish,H}(v_H, v_L, y_H, y_L) \\
v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

where $v_{Punish,H}(v_H, v_L, y_H, y_L)$ and $v_{Punish,L}(v_H, v_L, y_H, y_L)$ are given by:

$$
\begin{aligned}
v_{D,H}^{i-1} &= \beta\left[(1-p_H)v_{D,H}^{i} + p_H v_{D,L}^{i}\right], \\
v_{D,L}^{i-1} &= -(y_{D,L}^{i-1})^2 + \beta\left[(1-p_L)v_{D,H}^{i} + p_L v_{D,L}^{i}\right], \\
y_{D,L}^{i-1} &= -\sigma\left(-r^* - s_L\right) + (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{D,H}^{i} + p_L y_{D,L}^{i}\right],
\end{aligned}
$$

for $i = 1, ..., K$, where $(v_{D,H}^K, v_{D,L}^K, y_{D,H}^K, y_{D,L}^K) = (v_H, v_L, y_H, y_L)$ and $v_{Punish,H} = v_{D,H}^0$. K is the length of punishment. In this case, the equilibrium conditions are given by:

$$
\begin{aligned}
&y_H = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma\left(i_H - s_H\right), \\
&y_{M,\tau} = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_H + p_H y_L\right] - \sigma\left(i_{M,\tau} - s_H\right), \\
&y_{M,\tau-1} = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_{M,\tau} + p_H y_L\right] - \sigma\left(-r^* - s_H\right), \\
&... \\
&y_{M,k} = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_{M,k+1} + p_H y_L\right] - \sigma\left(-r^* - s_H\right), \\
&... \\
&y_{M,1} = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_{M,2} + p_H y_L\right] - \sigma\left(-r^* - s_H\right), \\
&y_{M,0} = (M_{ee} + \sigma\kappa)\left[(1-p_H)y_{M,1} + p_H y_L\right] - \sigma\left(-r^* - s_H\right), \\
&y_L = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,0} + p_L y_L\right] - \sigma\left(-r^* - s_L\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&v_H = -y_H^2 + \beta \left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau} = -y_{M,N}^2 + \beta \left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau-1} = -y_{M,\tau-1}^2 + \beta \left[(1-p_H)v_{M,\tau} + p_H v_L\right], \\
&... \\
&v_{M,k} = -y_{M,k}^2 + \beta \left[(1-p_H)v_{M,k+1} + p_H v_L\right], \\
&... \\
&v_{M,1} = -y_{M,1}^2 + \beta \left[(1-p_H)v_{M,2} + p_H v_L\right], \\
&v_{M,0} = -y_{M,0}^2 + \beta \left[(1-p_H)v_{M,1} + p_H v_L\right], \\
&v_L = -y_L^2 + \beta \left[(1-p_L)v_{M,0} + p_L v_L\right], \\
&v_{M,1} = v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

Because the central bank only cares about output stabilization, $y_H = 0$. Using $y_H = 0$, we obtain

$$
\begin{aligned}
&i_H = \sigma^{-1}(M_{ee} + \sigma\kappa)p_H y_L + s_H, \\
&i_{M,\tau} = \sigma^{-1}(M_{ee} + \sigma\kappa)p_H y_L + s_H - \sigma^{-1} y_{M,\tau}, \\
&y_{M,\tau-1} = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,\tau} + p_L y_L\right] - \sigma\left(-r^* - s_H\right), \\
&... \\
&y_{M,k} = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,k+1} + p_L y_L\right] - \sigma^{-1}\left(-r^* - s_H\right), \\
&... \\
&y_{M,1} = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,2} + p_L y_L\right] - \sigma\left(-r^* - s_H\right), \\
&y_{M,0} = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,1} + p_L y_L\right] - \sigma\left(-r^* - s_H\right), \\
&y_L = (M_{ee} + \sigma\kappa)\left[(1-p_L)y_{M,0} + p_L y_L\right] - \sigma\left(-r^* - s_L\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&v_H = \beta \left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau} = -y_{M,\tau}^2 + \beta \left[(1-p_H)v_H + p_H v_L\right], \\
&v_{M,\tau-1} = -y_{M,\tau-1}^2 + \beta \left[(1-p_H)v_{M,\tau} + p_H v_L\right], \\
&... \\
&v_{M,k} = -y_{M,k}^2 + \beta \left[(1-p_H)v_{M,k+1} + p_H v_L\right], \\
&... \\
&v_{M,1} = -y_{M,1}^2 + \beta \left[(1-p_H)v_{M,2} + p_H v_L\right], \\
&v_{M,0} = -y_{M,0}^2 + \beta \left[(1-p_H)v_{M,1} + p_H v_L\right], \\
&v_L = -y_L^2 + \beta \left[(1-p_L)v_{M,0} + p_L v_L\right], \\
&v_{M,1} = v_{Punish,H}(v_H, v_L, y_H, y_L)
\end{aligned}
$$

We are solving for $(y_L, i_H, i_{M,\tau})$, $\{y_{M,i}\}_{i=0}^{\tau}$, (v_H, v_L) and $\{v_{M,i}\}_{i=0}^{\tau}$. $2(\tau+1)+5$ unknowns in $2(\tau+1)+5$ equations.

Guess (y_L, v_L). Then,

- From the first $\tau+3$ equations, we can determine $(i_H, i_{M,\tau})$ and $\{y_{M,i}\}_{i=0}^{\tau}$.
- From the equations for $v_{M,i}$ and v_L (τ+2 equations), we can determine v_H and $\{y_{M,i}\}_{i=0}^{\tau}$.

Then, we need to check whether the following two equations hold:

$$\begin{aligned} v_H &= \beta\left[(1-p_H)v_H + p_H v_L\right], \\ v_M &= v_{Punish,H}(v_H, v_L, y_H, y_L) \end{aligned}$$

That is, we can basically reduce the system to two unknowns in two equations. Once you solve the system of equations, we need to verify the following $\tau+4$ inequalities:

$$\begin{aligned} i_H &> -r^* \\ i_{M,\tau} &> -r^* \\ v_H &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_{M,\tau} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ &\ldots \\ v_{M,1} &> v_{Punish,H}(v_H, v_L, y_H, y_L) \\ v_L &> v_{Punish,L}(v_H, v_L, y_H, y_L) \end{aligned}$$

If these inequality constraints are satisfied, then we have the right τ and proceed to verify the history independence assumption. Otherwise, we adjust τ and solve the system of equations described above again.

Alternative *numerical* method

Next, we modify the time-iteration method to solve the model numerically for some low Ns when the solution is not history-independent. We use the fact that the policy functions are flat above the threshold of the Lagrange multiplier $\tilde{\phi}$ (which is the only endogenous state variable in the model) for each state. Roughly speaking, the alternative numerical method works as follows. First, taking the policy functions as given, we solve for numbers of the output and value $(\bar{y}_H, \bar{v}_H, \bar{y}_L, \bar{v}_L)$ and the threshold $(\tilde{\phi}_H^*, \tilde{\phi}_L^*)$ when the policy function is flat and the current period's state variable $\tilde{\phi}$ is irrelevant for each state. Then, we use the equilibrium conditions to update the policy functions only for $\tilde{\phi} < \tilde{\phi}^*$. We iterate this process until the policy functions converge.

Followings are the details. First of all, we use the fact that the following equations hold in the low state $s = s_L$ for a sufficiently high Lagrange multiplier $\tilde{\phi}$:

$$\begin{aligned} \bar{y}_L &= (M_{ee} + \kappa\sigma)\left(p_L \bar{y}_L + (1-p_L)\bar{y}_H\right) + \sigma(r^* + s_L), \\ \bar{V}_L &= -\bar{y}_L^2 + \beta p_L \bar{V}_L + \beta(1-p_L)\bar{V}_H. \end{aligned}$$

That is, the output and value $y_L = \bar{y}_L$ and $V_L = \bar{V}_L$ are constant in the current period regardless of the value of $\tilde{\phi}$. This is the case when the next period is in the high state $s = s_H$ and the sustainability constraint will be binding. Then, the next period's output and value in the high state are also constant, and given by

$$\bar{y}_H = (M_{ee} + \kappa\sigma)\left(p_H y(\tilde{\phi}^{**}, s_L) + (1 - p_H) y(\tilde{\phi}^{**}, s_H)\right) + \sigma(r^* + s_H),$$
$$\bar{V}_H = W(s_H) = -\bar{y}_H^2 + \beta p_H V(\tilde{\phi}^{**}, s_L) + \beta(1 - p_H) V(\tilde{\phi}^{**}, s_H),$$

where $\tilde{\phi}' = \tilde{\phi}^{**}$ is the value of the (two periods ahead) Lagrange multiplier after the sustainability constraint is binding in the next period, which is also known to be constant. These equations can be solved for $(\bar{y}_H, \bar{V}_H, \tilde{\phi}^{**})$ taking the policy functions $y(\phi, s)$ and $V(\phi, s)$ as given. Once we have $(\bar{y}_H, \bar{V}_H)$, we can also compute $(\bar{y}_L, \bar{V}_L)$.

Then, let $\tilde{\phi}_H^*$ the threshold of the Lagrange multiplier above which the output and value in the high state is constant. Taking the policy function $\tilde{\phi}'(\tilde{\phi}, s)$ as given, such a threshold must satisfy $\tilde{\phi}^{**} = \tilde{\phi}'(\tilde{\phi}_H^*, s_H)$. Also, let $\tilde{\phi}_L^*$ be the threshold of the Lagrange multiplier above which the output and value in the low state are constant. As the ELB always binds in the low state, we have the following equation from the first-order condition

$$\tilde{\phi}_H^* = -\bar{y}_L + \beta^{-1}(M_{ee} + \kappa\sigma)\tilde{\phi}_L^*.$$

Therefore we obtain $\tilde{\phi}_L^* = \frac{\beta(\tilde{\phi}_H^* + \bar{y}_L)}{M_{ee} + \kappa\sigma}$. Then we use the equilibrium conditions to update the policy functions at each grid point of $\tilde{\phi}$, but we do so only for $\tilde{\phi} < \tilde{\phi}_H^*$ in the high state and for $\tilde{\phi} < \tilde{\phi}_L^*$ in the low state. The ELB may or may not bind in the high state (Case (i) or (ii)) whereas the ELB always binds in the low state (Case (ii)). We also know that the sustainability constraint is always slack, which is greatly helpful to get a convergent result. When $\tilde{\phi}$ is above the threshold, we set $y = \bar{y}_H$, $\tilde{\phi}' = \tilde{\phi}^{**}$, and $V = \bar{V}_H$ for $\tilde{\phi} \geq \tilde{\phi}_H^*$ in the high state, and $y = \bar{y}_L$, $\tilde{\phi}' = -\bar{y}_L + \beta^{-1}(1 + \kappa\sigma)\tilde{\phi}$, and $V = \bar{V}_L$ for $\tilde{\phi} \geq \tilde{\phi}_L^*$ in the low state.[27] Once we update the policy functions $y(\tilde{\phi}, s)$, $\tilde{\phi}'(\tilde{\phi}, s)$, and $V(\tilde{\phi}, s)$, we also renew the value of reneging $W(s)$.

With the parameter values we use except for N, the altenative numerical solutions are available for $12 \leq N \leq 59$ whereas the original numerical solutions are computed for $N \geq 39$. Below $N < 12$, only the history-independent analytical solutions are available. Note that the analytical solutions are valid only when N is low, as the assumption of history independence is violated otherwise. The gap between the analytical solution and the numerical soluiton is at most 0.002% for $N \leq 24$. Also, the gap between two numerical solutions is at most 0.001% for $39 \leq N \leq 59$.

[27]Note that $\tilde{\phi}_L^*$ can be below zero. If so, $y_L = \bar{y}_L$ regardless of $\tilde{\phi}$ in the low state and the solution is history-independent.

F Results in the model with a static Phillips curve and discounted Euler equation

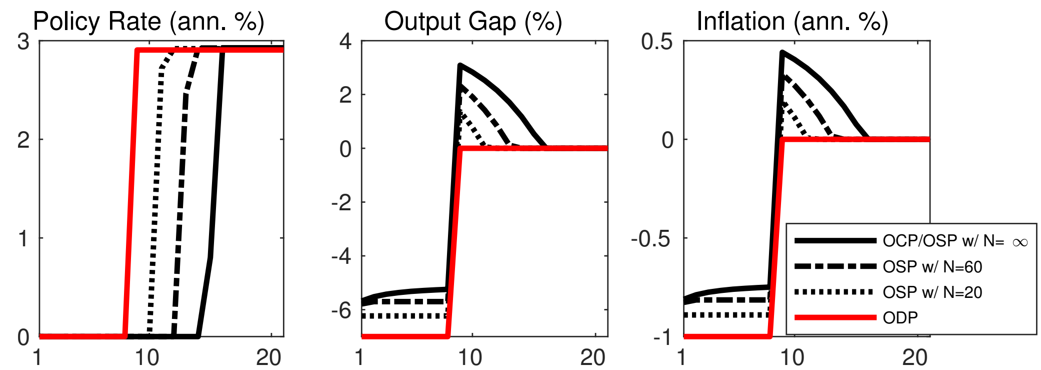
Figure 14 shows the dynamics of the economy under the ODP, the OCP, and OSPs with $N = [20, 60, \infty]$ and discounted Euler equations for the cases with $M_{ee} = [0.8, 0.5, 0.2]$. As in the baseline case with $M_{ee} = 1.0$, the responses of the policy rate, the output gap, and inflation under OSPs are in between those under the ODP and those under the OCP. As M_{ee} gets lower, the commitment effect is diminished and the degree of the overshooting under the OCP is smaller. Under OSPs, the overshoot is even smaller with lower Ns.[28]

Also, OSPs show history independence. The highest value of N under which the OSP exhibits history independence is increasing as the future output gap is more discounted.

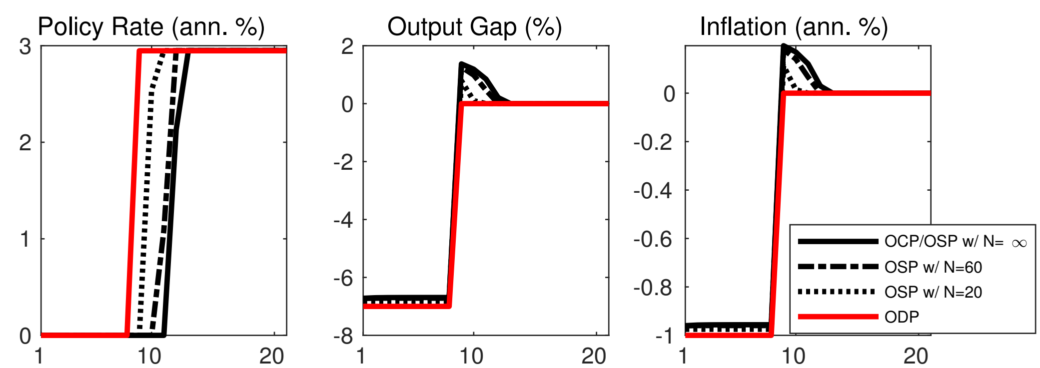
[28] As in the baseline case, the standard time iteration method does not converge with some low Ns. We use the alternative numerical method (when the solution is history-dependent) or the analytical method (when the solution is history-independent) to solve for the allocations.

Figure 14: Dynamics
—Model with discounted Euler equations—

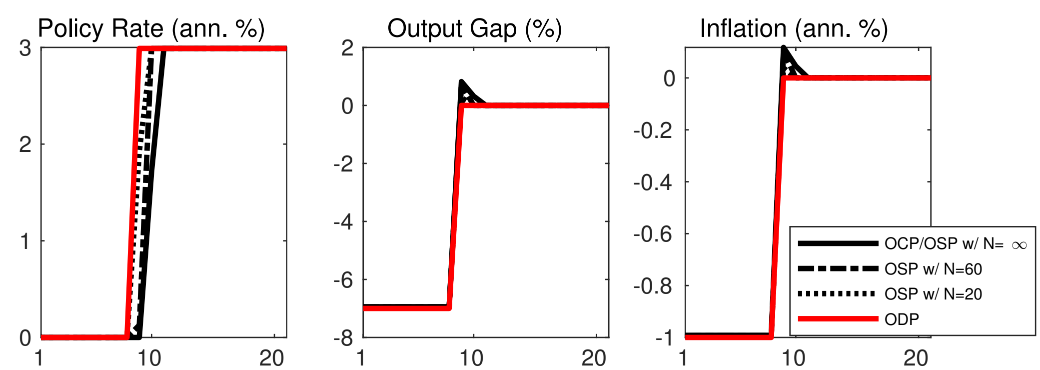
(i) $M_{ee} = 0.8$



(ii) $M_{ee} = 0.5$



(iii) $M_{ee} = 0.2$



Notes: Parameter values are $\beta = 0.9925$, $\sigma = 1.0$, $\kappa = 0.25/7$, $p_H = 0.005$, and $p_L = 0.75$. s_L is chosen so that $y_L = -7.0$.

G Model with a static Euler equation

The policymaker maximizes

$$V_0 = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda y_t^2\right),$$

subject to

$$\pi_t = \kappa y_t + \tilde{\beta} E_t \pi_{t+1}, \tag{16}$$

$$\tilde{r}_t = \tilde{\beta}^{-1}\pi_t - \tilde{\beta}^{-1}(\kappa + \sigma^{-1}\tilde{\beta}) y_t \geq -r^* - \sigma^{-1} s_t, \tag{17}$$

$$V_t = -E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2\right) \geq W(s_t), \tag{18}$$

for all $t \geq 0$. $M_{pc} \leq 1.0$ is a parameter whose value is less than one to diminish the degree of forward-lookingness. Note that the right-hand side of Equation (7) is obtained by the static version of the Euler equation, $-\sigma^{-1} y_t = \tilde{r}_t - E_t \pi_{t+1}$ and Equation (6), where we define $\tilde{r}_t \equiv i_t - \sigma^{-1} s_t$ and $\tilde{\beta} \equiv \beta M_{pc}$. The shock, s_t, follows two-state Markov chain, $s_t \in \{s_H, s_L\}$ where $s_H > s_L$. Transition probability matrix is given as $P = \begin{bmatrix} 1-p_H & p_H \\ 1-p_L & p_L \end{bmatrix}$, where p_H is the frequency of the crisis and p_L is the persistence of the crisis. $W(s_t)$ is the value under the optimal discretionary policy.

The analytical solution for the discretionary outcome: The ZLB is slack in the high/normal state, $\phi_H = 0$, and the ZLB is binding in the low/crisis state, $\phi_L = \phi > 0$ and $\tilde{r}_L = -r^* - s_L$. Thus, the equilibrium conditions become

$$\begin{aligned}
&\lambda y_H + \kappa \pi_H = 0, \\
&\pi_H = \kappa y_H + \tilde{\beta}\left((1-p_H)\pi_H + p_H \pi_L\right), \\
&\pi_H = (\kappa + \sigma^{-1}\tilde{\beta}) y_H + \beta \tilde{r}_H, \\
&\lambda y_L + \kappa \pi_L + \sigma^{-1}\tilde{\beta}\phi = 0, \\
&\pi_L = \kappa y_L + \tilde{\beta}\left((1-p_L)\pi_H + p_L \pi_L\right), \\
&\pi_L = (\kappa + \sigma^{-1}\tilde{\beta}) y_L + \beta \tilde{r}_L.
\end{aligned}$$

These equations can be solved for $(y_H, \pi_H, \tilde{r}_H, y_L, \pi_L, \phi)$.

Solving for the sustainable equilibrium: The Lagrangean is given by

$$
\begin{aligned}
\mathcal{L} \equiv\; & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\pi_t^2 - \lambda y_t^2 \\
& + 2\phi_{1t} \left(-\kappa y_t + \pi_t - \tilde{\beta} E_t \pi_{t+1} \right) + 2\phi_{2t} \left(\pi_t - (\kappa + \sigma^{-1}\tilde{\beta}) y_t \right) \\
& + \psi_t \left(-E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2 \right) - W(s_t) \right) \Big\}, \\
=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\Psi_t \left(\pi_t^2 + \lambda y_t^2 \right) + 2\phi_{1t} \left(-\kappa y_t + \pi_t \right) - 2M_{pc}\phi_{t-1}\pi_t + 2\phi_{2t} \left(\pi_t - (\kappa + \sigma^{-1}\tilde{\beta}) y_t \right) - \psi_t W(s_t) \right\},
\end{aligned}
$$

where $\Psi_t = \psi_{-1} + \psi_0 + ... + \psi_t < \infty$ is the sum of the Lagrange multipliers on the sustainability constraint. The FOCs are given by

$$
\begin{aligned}
\partial y_t : \quad & -\lambda \Psi_t y_t - \kappa \phi_{1t} - (\kappa + \sigma^{-1}\tilde{\beta}) \phi_{2t} = 0, \\
\partial \pi_t : \quad & -\Psi_t \pi_t + \phi_{1t} - M_{pc} \phi_{1t-1} + \phi_{2t} = 0.
\end{aligned}
$$

Normalizing the equations by Ψ_t, we have

$$
\begin{aligned}
& -\lambda y_t - \kappa \tilde{\phi}_{1t} - (\kappa + \sigma^{-1}\tilde{\beta}) \tilde{\phi}_{2t} = 0, \\
& -\pi_t + \tilde{\phi}_{1t} - z_t M_{pc} \tilde{\phi}_{1t-1} + \tilde{\phi}_{2t} = 0,
\end{aligned}
$$

where $\tilde{\phi}_{1t} = \phi_{1t}/\Psi_t$, $\tilde{\phi}_{2t} = \phi_{2t}/\Psi_t$, and $z_t = \Psi_{t-1}/\Psi_t \in (0, 1]$. The Kraush-Kuhn-Tucker conditions (KKTCs) must be satisfied as well

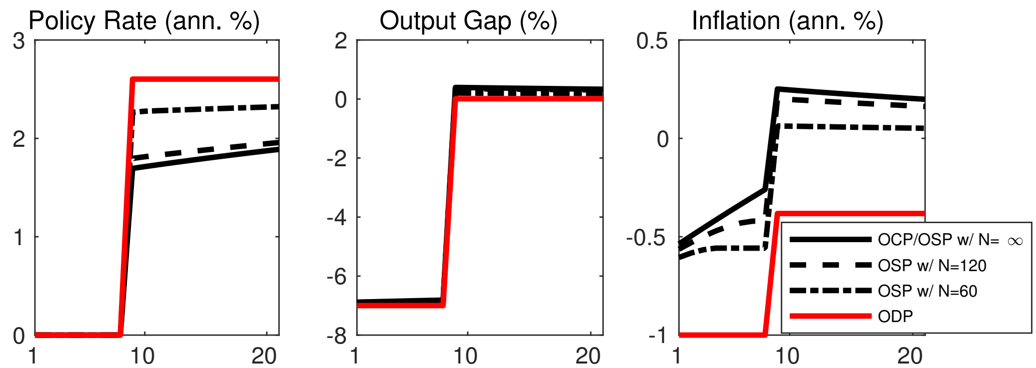
$$
\begin{aligned}
& \phi_{2t} \left(i_t - i_{ELB} \right) = 0, \\
& \phi_{2t} \geq 0, \\
& \psi_t \left(V_t - W(s_t) \right) = 0, \\
& \psi_t \geq 0.
\end{aligned}
$$

The initial conditions on the Lagrange multipliers are such that $\phi_{1,-1} = \phi_{2,-1} = 0$ and $\psi_{-1} = 1$, which implies $\tilde{\phi}_{1,-1} = \tilde{\phi}_{2,-1} = 0$ and $\Psi_{-1} = 1$.

Figure 15 shows the dynamic of the economy under the OCP, the ODP, and OSPs with $N = [60, 120, \infty]$. The dynamics of the economy under the OCP and the ODP are consistent with those in the model with a static Phillips curve. However, we can see more differnce in inflation and the policy rate, whereas the output gap difference is small. The lift off is immediately after the crisis shock is gone even under the OCP, although the steady-state policy rate is lower under the OCP than under the ODP. This makes inflation higher in the normal state, which also mitigates the inflation drop in the crisis state through expectation under the OCP.

Under the OSPs (except for $N = \infty$), the policy maker wants to raise the policy rate more than under the OCP but less than under the ODP. This makes inflation lower under the OSPs than under the OCP. The lower N gets, the lower inflation is under the OSPs.

Figure 15: Dynamics
—Model with a static Euler equation—



Notes: Parameter values are $\beta = 0.9925$, $\sigma = 1.0$, $\kappa = 0.057$, $\lambda = 0.0625$, $s_L = -7.5443$, $p_H = 0.005$, and $p_L = 0.75$. (κ, s_L) is chosen so that $y_L = -7.0$ and $\pi_L = -1.0/4$.

H Tenure duration of leadership at central banks

Table 6: Average Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Year of foundation	No. of leaders since foundation	No. of leaders since 1946	Average tenure since foundation	Average tenure since 1946
Federal Reserve System	1914	16	10	6.9	8.1
European Central Bank	1998	3	3	6.5	6.5
Bank of Canada	1934	9	8	9.9	9.7
Bank of Japan	1882	31	15	4.4	4.9
Bank of England	1694	120	9	2.7	8.5
Sveriges Riksbank	1901	14	11	8.1	6.1
Swiss National Bank	1907	14	10	8.1	7.4

I Time-inconsistency of the commitment policy in the words of policymakers

The time-inconsistency of the commitment policy at the ELB is not a mere theoretical curiosity. Policymakers in many central banks have pointed out the potential time-inconsistency of the commitment-type forward guidance policy. Some have argued that the

Table 7: Maximum Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Max duration since foundation	Max duration since 1946
Federal Reserve System	18 yrs and 10 months (Martin)	18 yrs and 10 months (Martin)*
European Central Bank	8 (Trichet)	8 (Trichet)
Bank of Canada	20 yrs and 4 months (Towers)	14 (Boey)
Bank of Japan	8 yrs and 6 months (Ichimada)	8 yrs and 6 months (Ichimada)
Bank of England	24 (Norman)	12 (Cobbold)
Sveriges Riksbank	19 (Rooth)	18 (Asbrink)
Swiss National Bank	14 (Bachmann)	11 (Leutwiler)

Note: The tenure of Alan Greenspan lasted for 18 years and 6 months.

time-inconsistency is one key reason for why most central banks refrained from making the overheating commitment. Below are some examples:

I.1 Bean (2013)

"In particular, we signalled our intention not to countenance tightening policy until unemployment has fallen to at least 7 percent."

"**This guidance is intended primarily to clarify our reaction function and thus make policy more effective, rather than to inject additional stimulus by pre-committing to a time-inconsistent 'lower for longer' policy path in the manner of Woodford (2012). While such a time-inconsistent policy may be desirable in theory, in an individualistic committee like ours, with a regular turnover of members, it is not possible to implement a mechanism that would credibly bind future members in the manner required.**"

I.2 Bullard (2013)

"The New Keynesian, sticky price literature has been influential in U.S. monetary policymaking. The literature has been led by Michael Woodford. This line of research argues that policy accommodation can be provided even when the policy rate is near zero. The extra accommodation comes from a promise to maintain the near zero policy rate into the future, beyond the point when ordinary policymaker behavior would call for an increase in the policy rate. This promise must be credible to have an impact.

The "Woodford period" approach to forward guidance relies on a credible announcement made today that future monetary policy will deviate from normal. The central bank does not actually behave differently today. One might argue that such an announcement is unlikely to be believed. Why should future monetary policy deviate from normal once the economy is growing and inflation is rising? But if the announcement is not credible, then the private sector will not react with more consumption and investment today. That is, any effects would be minimal."

I.3 Carney (2012)

"Today, to achieve a better path for the economy over time, a central bank may need to commit credibly to maintaining highly accommodative policy even after the economy and, potentially, inflation picks up. **Market participants may doubt the willingness of an inflation-targeting central bank to respect this commitment if inflation goes temporarily above target. These doubts reduce the effective stimulus of the commitment and delay the recovery.**"

I.4 Clarida (2019b)

"The benefits of the makeup strategies rest heavily on households and firms believing in advance that the makeup will, in fact, be delivered when the time comes–for example, that a persistent inflation shortfall will be met by future inflation above 2 percent. **As is well known from the research literature, makeup strategies, in general, are not time consistent because when the time comes to push inflation above 2 percent, conditions at that time will not warrant doing so. Because of this time inconsistency, any makeup strategy, to be successful, would have to be understood by the public to represent a credible commitment. That important real-world consideration is often neglected in the academic literature, in which central bank "commitment devices" are simply assumed to exist and be instantly credible on decree. Thus, one of the most challenging questions is whether the Fed could, in practice, attain the benefits of makeup strategies that are possible in models.**"

I.5 Cœuré (2013)

"Most notably, the central bank may try to convince markets that it would keep interest rates low, even if this would imply inflation well above its previous objective, at least temporarily. The promise of higher future inflation, if credible, induces private agents to substitute future for current consumption, hence providing additional stimulus today. This type of forward guidance is closer to the academic concept of forward guidance in the strict sense—as discussed, for example, in Woodford (2012).

The main challenge of such guidance is its inherent inconsistency over time and thus lack of credibility. When the time comes, the central bank may be tempted to deviate from its prior commitment: once the benefits of higher inflation expectations in terms of front-loaded spending have been reaped, the central bank may not be willing to pay the bill in terms of higher inflation afterward. If the public foresees this temptation, expectations might remain unaffected in the first instance and the desired inter-temporal substitution of spending might not materialise. This is a possible explanation why, in practice, central banks have refrained from using forward guidance in a way that implies a major change in strategy. Therefore, central banks' forward guidance has rather aimed at providing greater clarity on the reaction function and the assessment of future economic conditions."

I.6 Dudley (2013)

"With respect to forward guidance, it is important to distinguish between two specific forms that this guidance may take. In the first form the central bank provides its forecast for the future path of the policy rate and, possibly, some sense of the degree of uncertainty around this path. In the second, the central bank pre-commits to a specific future path for its policy rate.

Providing a forecast for the policy rate by itself does not create any budget or reputational risk for the Federal Reserve, so I generally do not see the first form of forward guidance as posing much risk to central bank independence.

The second form of forward guidance—pre-commitment to a policy rate path—could create more risk for the central bank. In particular, consider a scenario in which the central bank decided to increase monetary accommodation by committing to maintain a low short-term interest rate for a long time even if this commitment resulted in inflation overshooting the central bank's objective in the future. I could see how this could create a potential threat to the central bank's independence. That is because the commitment could force the central bank in the future to conduct monetary policy in a way that was inconsistent with the inflation portion of its mandate.

Although this second form of forward guidance could create greater risk for the central bank with respect to its future independence, this is not a policy that has been adopted by the Federal Reserve. There are implementation challenges with this approach. **In particular, it is difficult for a monetary policy committee today to institutionally bind future monetary policy committees to follow actions that could conflict with their objectives in the future. Without such a credible forward commitment, such policies would likely be ineffective in affecting expectations in the manner needed to provide additional monetary policy accommodation.**"

I.7 George (2019)

"Third, a price-level targeting strategy is time inconsistent unless policymakers can credibly commit to following it. If the goal is to have inflation of 2 percent on average, a period of below 2 percent inflation would require an equal period of inflation above 2 percent. **But once inflation has moved up to 2 percent, policymakers might be tempted to renege on their prior commitment and not allow inflation to go higher. This would undermine the future credibility of the price-level targeting strategy. To the extent the public understood this time inconsistency problem, price-level targeting would not be credible to begin with, absent a commitment device. With regular turnover among members of the FOMC, it would be difficult for one Committee to commit a future Committee to a particular course of action.**"

I.8 Lacker (2013)

"Designing such conditional guidance involves trade-offs, however. Credibility requires consistency, over time, between a central bank's statements and its actual subsequent actions. A central bank's statements will have greater immediate effect on the public's expectations the more they are seen as limiting the central bank's future choices. Yet there are likely to be circumstances, ex post, in which the central bank feels constrained by past statements. Yielding to the temptation to implicitly renege by reworking decision criteria or citing unforeseen economic developments may have short-term appeal, but widely perceived discrepancies between actual and foreshadowed behavior will inevitably erode the faith people place in future central bank statements. So central banks face an ex ante trade-off, as well, between the short-run value of exercising discretion and the ability to communicate effectively and credibly in the future."

I.9 Plosser (2013)

"Note, however, that the central bank's ability to influence the public's belief about the future path of policy and the economy depends critically on the bank's commitment to that policy path and the credibility of that commitment in the eyes of the public. **The public must believe that even after the economy begins to strengthen, the central bank will hold rates lower than it otherwise might have found desirable to do had it not been at the zero bound in the past.**"

I.10 Powell (2019)

"By the time of the crisis, there was a well-established body of model-based research suggesting that some kind of makeup policy could be beneficial. In light of this research, one might ask why the Fed and other major central banks chose not to pursue such a policy. The answer lies in the uncertain distance between models and reality. For makeup strategies to achieve their stabilizing benefits, households and businesses must be quite confident that the "makeup stimulus" is really coming. This confidence is what prompts them to raise spending and investment in the midst of a downturn. **In models, confidence in the policy is merely an assumption. In practice, when policymakers considered these policies in the wake of the crisis, they had major questions about whether a central bank's promise of good times to come would have moved the hearts, minds, and pocketbooks of the public. Part of the problem is that when the time comes to deliver the inflationary stimulus, that policy is likely to be unpopular–what is known as the time consistency problem in economics.**"

I.11 Ueda (2013)

"If Max (the Taylor rule rate, zero) describes the usual central bank's reaction function to the macroeconomic environment, the central bank can generate easing effects by offering a new reaction function to the market with a promise of a longer period at the zero rate than the above rule suggests. **To the extent that the Taylor rule represents an optimal response of the central bank to macroeconomic environment, however, this forward guidance strategy amounts to "irresponsible" central bank behaviour. In other words, the strategy is time-inconsistent. This means that when the economy no longer requires a zero rate, it is better to raise the interest rate, reneging on the promise made. If people foresaw this ex ante, however, the strategy would become ineffective.** Thus, the central bank would be sending a confusing signal if it was using forward guidance in this sense and insisted that it was still behaving in a "responsible" way. Also, the central bank does not seem to get much mileage out of a vague promise, such as the maintenance of a low policy rate "for an extended period," unless there is much confusion in the market as to where the policy rate would go in the short term.

The BOJ seems to have faced the time-inconsistency problem in 2000."

I.12 Williams (2012)

"Although forward policy guidance has proven to be a very useful policy tool, it's not a perfect substitute for the kind of monetary stimulus that comes from lower interest rates. One issue is that, for the forward guidance policy to work as desired, the public has to believe that the FOMC will really carry out the policy as it says it will. But, the Fed doesn't have the ability to tie its hands that way. This point was made by Finn Kydland and Edward Prescott in the late 1970s. Let me explain. For forward policy guidance to have its maximum effect, the Fed must commit to keeping the short-term policy rate lower than it otherwise would to compensate for the fact that the short-term interest rate cannot be lowered today. **But when the time comes to carry out the commitment made in its forward guidance, it may no longer want to do so. For instance, it might be hard to resist raising rates earlier than promised to head off an increase in inflation. So, even when central bankers say they will keep rates unusually low for a set time, the public may worry that the central bank will raise rates earlier to fight budding inflation pressures.**"