Optimal Monetary Policy with Labor Market Frictions: The Role of the Wage Channel*

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Abstract

This paper introduces right-to-manage bargaining into a labor search model with sticky prices instead of standard efficient bargaining and examines the Ramsey-optimal monetary policy. Without real wage rigidity, even when the steady state is inefficient, price stability is nearly optimal in response to technology or government shocks. Right-to-manage bargaining creates the wage channel to inflation, as there is a direct relationship between real wages and real marginal cost. In the presence of the wage channel, price markups consist of only real marginal cost, and real wages and hours per worker are determined such as in the Walrasian labor market.

Keywords: Ramsey-optimal monetary policy; Right-to-manage bargaining; Wage channel; Real wage rigidity; Wage markups

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1 Introduction

In the study of the Ramsey-optimal monetary policy in models with sticky prices and search frictions in the labor market, it is usually assumed that real wages and hours per worker are determined by efficient bargaining (EB) between firms and workers.[1] Under EB, when the steady state is efficient (i.e., the Hosios (1990) condition is met and there is a subsidy to offset the distortion stemming from monopolistic competition), price stability to technology shocks is *exactly* optimal (Thomas, 2008). The further we deviate from the efficient steady state, the more deviation from price stability becomes optimal, as endogenous price markups stemming from the net hiring cost lead to a tradeoff between inflation and unemployment (Faia, 2009).

However, in actual practice, firms have the right to manage workers; typically, hours per worker are unilaterally picked by firms rather than determined by bargaining between firms and workers. Then, real wages are determined by bargaining given the labor demand schedule of firms. Nickell and Andrews (1983) and Trigari (2006) introduced right-to-manage (RTM) bargaining into labor search models instead of EB, based on empirical backgrounds.[2]

This paper studies the Ramsey-optimal monetary policy in a labor search model with sticky prices and compares the two bargaining schemes, EB and RTM. The results under RTM contrast starkly with the ones under EB. Even when the steady state is inefficient, price stability is *nearly* optimal under RTM, as opposed to Faia (2009). This is in line with previous studies of optimal monetary policy in models with sticky prices and the Walrasian labor market.[3] Also, when the steady state is efficient, the price stability result does not exactly hold under RTM, as opposed to Thomas (2008).

Under RTM, there is a direct transmission from real wages to real marginal cost, hence leading to inflation. This is called *the wage channel to inflation* (Christoffel and Kuester, 2008). In the presence of the wage channel, price markups consist of only real marginal cost, and real wages and hours per worker are determined such as in the Walrasian labor market with variable wage markups.

Some papers have argued that real wage rigidity is important in explaining empirical regularities in labor search models.[4] With real wage rigidity, deviation from price stability becomes optimal under both EB and RTM, although the reasons are quite different. When real wages are sluggish and cannot respond immediately to a positive technology shock, firms want to increase labor input. Under RTM, as firms can adjust labor input at the intensive margin (hours per worker) and hours per worker are more volatile than real wages, wage markups become countercyclical and volatile; therefore, deviation from price stability is optimal. Under EB, firms mainly adjust labor input at the extensive margin (employment), which generates fluctuations in the net hiring cost and price markups.

Recently, a few papers have studied the Ramsey-optimal monetary policy in labor search models with sticky prices under EB. Faia (2009) used a similar setting to the one presented in this paper in terms of the timing of events and quadratic price adjustment, but with only the extensive margin. She found that deviation from price stability was optimal with the inefficient steady state. In contrast, Thomas (2008) introduced both of the intensive and extensive margins and showed that price stability is optimal with the efficient steady state. When nominal wage bargaining is staggered, the case against price stability arises. Blanchard and Galí (2010) presented a simpler framework and demonstrated the price stability result. Real wage rigidity à la Hall (2005) creates the case against price stability. Ravenna and Walsh (2011) derived the objective function of monetary policy by using second-order approximation of the household's utility and gap terms of unemployment, as well as inflation and consumption. They explicitly showed how gap terms are related to welfare costs of search and matching frictions.

This paper examines both cases with an inefficient and an efficient steady state, and compares different methodologies to compute the Ramsey-optimal monetary policies: the Lagrange and linear-quadratic (LQ) methods. The Lagrange method can be applied even when the steady state is inefficient, whereas the LQ method, with first-order approximation of the equilibrium conditions and second-order approximation of the household's utility, is usually used only when the steady state is efficient.[5] Faia (2009) used the Lagrange method, whereas Thomas (2008); Blanchard and Galí (2010); Ravenna and Walsh (2011) used the LQ method. This paper is consistent with the previous studies; under EB, when the steady state is efficient, price stability is exactly optimal, and the Lagrange and LQ methods yield exactly the same result up to first order. The further we deviate from the efficient steady state, the more volatile optimal inflation is, and the larger the numerical error between the Lagrange and LQ methods. Under RTM, however, the optimal volatility of inflation is not zero nor minimized even when the steady state is efficient.

In the next section, the labor search model and the two types of bargaining, EB and RTM, are introduced. The relationship between the labor market allocation and the wage channel is also examined. The Ramsey-optimal monetary policies are computed by the Lagrange and LQ methods, and calibration is discussed in Section 3. Section 4 shows the results. The case of the inefficient steady state is first considered in Section 4.1, whereas the case of the efficient steady state is examined in Section 4.2. Section 5 offers a conclusion. Calculations of the steady state and proofs of propositions are shown in the appendix.

2 Model

In this section, the labor search model used for monetary policy analysis is presented. Quadratic price adjustment costs, as in Rotemberg (1982), two different types of bargaining (EB and RTM), as in Trigari (2006), and a simple form of real wage rigidity, as in Hall (2005), are introduced. The relationship between labor market allocation and the different types of wage bargaining is also discussed.

2.1 Timing of Events

Time is discrete and infinite: $t = 0, 1, 2, ..., \infty$. The timing of events is as follows:

1. $1-n_{t-1}$ is the measure of unemployed workers at the end of the last period. A fraction of the employed workers ρn_{t-1} are exogenously separated as

$$u_t = 1 - n_{t-1} + \rho n_{t-1}, \tag{1}$$

   where u_t workers seek jobs in the current period.

2. Firms post vacancies v_t to match with the unemployed workers u_t by a matching function $m(u_t, v_t)$, which exhibits constant returns to scale. The labor market tightness is given by

$$\theta_t = v_t/u_t. \tag{2}$$

   Each worker and firm are small, in the sense that they take as given the labor market tightness at the aggregate level θ_t. The job finding rate for workers, $p(\theta_t) = m(u_t, v_t)/u_t = m(1, \theta_t)$, and the job filling rate for firms, $q(\theta_t) = m(u_t, v_t)/v_t = m(\theta_t^{-1}, 1)$, are also given for them.

3. The number of the newly employed workers is given by

$$n_t = (1-\rho)n_{t-1} + m(u_t, v_t). \tag{3}$$

   Note that the newly employed workers can work immediately.[6]

4. The employed workers enter into production:

$$y_t = a_t f(h_t) n_t, \tag{4}$$

   where a_t is the technology shock. Each employed worker works h_t hours and produces $a_t f(h_t)$, which is the decreasing returns to scale in terms of hours per worker, so that firms earn a positive profit period by period. Total production is linear in n_t.

2.2 Households

A family of households chooses consumption c_t, one-period nominal bonds B_t that pay gross interest rate R_t, and the number of the employed workers n_t so as to maximize the life-time utility:

$$\begin{aligned}
&V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - g(h_t)n_t + \beta E_t V_{t+1}, \\
&\text{subject to} \\
&c_t + \frac{B_t}{P_t} \leq w_t h_t n_t + b(1-n_t) + \frac{R_{t-1}B_{t-1}}{P_t} + \frac{T_t}{P_t}, \\
&n_t = (1-\rho)n_{t-1} + (1 - n_{t-1} + \rho n_{t-1})p(\theta_t).
\end{aligned}$$

A fraction of households n_t are employed and earn wage bill $w_t h_t$, and $1-n_t$ are unemployed at the end of the period and receive unemployment benefits b. w_t is real wage, and h_t is hours per worker, which are determined by bargaining between workers and firms. T_t is the sum of transfers from firms and the government. P_t is the aggregate price index. $\beta \in (0,1)$ is a discount factor. $\sigma > 0$ is a parameter for the degree of risk aversion. $g(h_t)$ is an increasing function that measures labor disutility. The job finding rate $p(\theta_t)$ is taken as given by each worker. Consumption is pooled among households to insure against the unemployment risk; therefore, households consume the same amount regardless of their employment status (Merz, 1995; Andolfatto, 1996).

By the first order necessary conditions (FONCs) of c_t and B_t, the standard consumption Euler equation is derived as

$$1 = \beta R_t E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{5}$$

The FONC of n_t and the envelope theorem yields

$$S_t = w_t h_t - g(h_t)c_t^{\sigma} - b + \beta(1-\rho)E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} (1 - p(\theta_{t+1}))S_{t+1} \right\}, \tag{6}$$

where S_t is the ratio of Lagrange multipliers of the budget constraint and the law of motion for n_t. S_t is also the matching value of a marginal worker for the family of households.

2.3 Firms

There are two types of firms: a final-good producing firm and differentiated intermediate-good producing firms. The final-good producing firm purchases each intermediate good y_{it} at price P_{it} for $i \in [0,1]$ and minimizes its expenditure $P_t y_t = \int_0^1 P_{it} y_{it} di$ subject to the Dixit-Stiglitz (1977) aggregator $y_t = \left(\int_0^1 y_{it}^{(\varepsilon_p-1)/\varepsilon_p} di\right)^{\varepsilon_p/(\varepsilon_p-1)}$. It yields the demand curve with the elasticity parameter $\varepsilon_p > 1$, $y_{it} = (P_{it}/P_t)^{-\varepsilon_p} y_t$.

Each intermediate-good producing firm i produces y_{it} in monopolistic competition and sells it to the final-good producing firm. The intermediate-good producing firm i chooses prices and allocations P_{it}, n_{it}, and v_{it} so as to maximize the discounted sum of future profits

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t}{c_0}\right)^{-\sigma} \left\{ \frac{(1+\tau)P_{it}}{P_t} y_{it} - w_{it} h_{it} n_{it} - c_v v_{it} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 \right\},$$

subject to

$$y_{it} = (P_{it}/P_t)^{-\varepsilon_p} y_t = a_t f(h_{it}) n_{it},$$

$$n_{it} = (1-\rho) n_{it-1} + v_{it} q(\theta_t),$$

where $\beta^t (c_t/c_0)^{-\sigma}$ is a stochastic discount factor for the firm. The firm i sets the intermediate-good price P_{it} subject to the downward-sloping demand curve and quadratic price adjustment costs with the parameter $\psi > 0$. $\tau \geq 0$ is a subsidy to firms to offset the distortion stemming from monopolistic competion.[7] The firm i also chooses the amount of employment and vacancies n_{it} and v_{it} and pays the total wage bill $w_{it} h_{it} n_{it}$ and vacancy costs $c_v v_{it}$ with the parameter $c_v > 0$. The job filling rate $q(\theta_t)$ is taken as given by each firm. The FONCs are

$$\begin{aligned} \partial P_{it}: \quad & \psi \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} = \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \psi \left(\frac{P_{t+1}}{P_t} - 1\right) \frac{P_{t+1}}{P_t} \right\} \\ & + y_t (1-\varepsilon_p) \left((1+\tau) - \frac{\varepsilon_p}{\varepsilon_p - 1} \varphi_t \right), \qquad (7) \\ \partial n_{it}: \quad & J_t = \varphi_t a_t f(h_t) - w_t h_t + \beta(1-\rho) E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} J_{t+1} \right\}, \\ \partial v_{it}: \quad & J_t = \frac{c_v}{q(\theta_t)}. \qquad (8) \end{aligned}$$

In equilibrium, all of the firms choose the same prices and allocations, so index i is dropped in the resulting conditions. The FONC of P_{it} is a nonlinear version of the Phillips curve. φ_t is the Lagrange multiplier on the production function and measures real marginal cost. φ_t is also the price of intermediate goods relative to final goods.[8]

In the FONC of n_{it}, J_t is the Lagrange multiplier on the law of motion for employment and measures the matching value of a marginal job for each firm. The firm's profit period by period is given by $\varphi_t a_t f(h_t) - w_t h_t > 0$, and the firm continues to match in the next period with the probability $(1-\rho)$. Note that the value of a vacancy is zero under the assumption of free entry, and J_t is equal to the expected cost of vacancy in the FONC of v_t. The last two equations are summarized into the hiring condition:

$$\frac{c_v}{q(\theta_t)} = \varphi_t a_t f(h_t) - w_t h_t + \beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\frac{c_v}{q(\theta_{t+1})}\right\}. \tag{9}$$

The expected cost of vacancy is equal to the discounted sum of future profits per worker. The net hiring cost

$$x_t \equiv \frac{c_v}{q(\theta_t)} - \beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\frac{c_v}{q(\theta_{t+1})}\right\} \tag{10}$$

is also defined; employment is a stock, and the net hiring cost includes the benefit of stock hoarding in the next period.

2.4 Wage Contracts

As shown in Sections 2.2 and 2.3, matching values of workers and firms are given by (6) and (8). A wage bill set $[\underline{\omega}_t, \overline{\omega}_t]$, which is compatible with their incentive to continue the matching, is derived from the conditions $S_t \geq 0$ and $J_t \geq 0$. $\underline{\omega}_t$ is the workers' reservation wage bill, and $\overline{\omega}_t$ is the firms' reservation wage bill. As long as the wage bill $w_t h_t$ is in the set $[\underline{\omega}_t, \overline{\omega}_t]$, workers and firms continue their matching. To pin down the equilibrium real wages w_t and hours per worker h_t within the set, two different types of wage contracts between workers and firms are introduced.

2.4.1 Efficient Bargaining

Under EB, real wages and hours per worker (w_t, h_t) are determined by Nash bargaining between worker and firm to maximize the joint surplus $S_t^{\eta} J_t^{1-\eta}$ subject to the equations (6) and (8), where $\eta \in (0,1)$ is a parameter for workers' actual bargaining power. The FONCs are

$$\partial h_t: \quad \varphi_t a_t f'(h_t) = g'(h_t) c_t^{\sigma}, \tag{11}$$

$$\partial w_t: \quad \eta J_t = (1-\eta) S_t. \tag{12}$$

Equation (11) shows that the marginal rate of substitution is equal to the real marginal cost times the marginal product of labor. Equation (12) shows that worker's surplus from bargaining is simply a fraction η of the total surplus, $S_t = \eta(S_t + J_t)$. Plugging the matching values for workers and firms, S_t and J_t given by (6) and (8) into (12), we have

$$\begin{aligned} w_t h_t &= \eta \varphi_t a_t f(h_t) + (1-\eta)\left(g(h_t)c_t^{\sigma} + b\right) \\ &+ \eta\beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} c_v \theta_{t+1}\right\}. \end{aligned} \tag{13}$$

The wage bill is equal to a weighted average of the workers' reservation wage and the firms' reservation wage, $w_t h_t = \eta\overline{\omega}_t + (1-\eta)\underline{\omega}_t$ with a constant weight η.

2.4.2 Right-to-Manage Bargaining

Under RTM, there are two stages in the bargaining in each period. First, real wages w_t are determined by Nash bargaining between workers and firms so as to maximize the joint surplus $S_t^{\eta} J_t^{1-\eta}$. Second, firms choose the level of hours per worker h_t so as to solely maximize the matching value for firms J_t. In the second stage, given w_t, the FONC of h_t is

$$w_t = \varphi_t a_t f'(h_t). \tag{14}$$

Real wages are equal to the real marginal cost times the marginal product of labor. Equation (14) implies that firms have labor demand schedule on real wages, $h_t = f'^{-1}(w_t/(\varphi_t a_t)) \equiv h(w_t)$. In the first stage, taking the wage schedule into account, workers and firms bargain over real wages w_t so as to maximize the joint surplus. The FONC of w_t is

$$\delta_t = [(1-\eta)/\eta] S_t/J_t, \tag{15}$$

where $\delta_t \equiv -(\partial S_t/\partial w_t)/(\partial J_t/\partial w_t)$ is the marginal gain for workers (relative to marginal loss for firms) by incremental real wages. By plugging the matching values for workers and firms S_t and J_t into (15), we have

$$\begin{aligned} w_t h_t &= \chi_t \varphi_t a_t f(h_t) + (1-\chi_t)\left(g(h_t)c_t^{\sigma} + b\right) \\ &+ \chi_t\beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(1 - (\delta_{t+1}/\delta_t)[1 - s(\theta_{t+1})]\right)\frac{c_v}{q(\theta_{t+1})}\right\}, \end{aligned} \tag{16}$$

where $\chi_t = \eta\delta_t/(1-\eta(1-\delta_t))$ is workers' *effective* bargaining power. The wage bill is equal to a weighted average of the workers' reservation wage and the firms' reservation wage, $w_t h_t = \chi_t\overline{\omega}_t + (1-\chi_t)\underline{\omega}_t$ with a time-varying weight χ_t.

2.4.3 Real Wage Rigidity

Under both EB and RTM, workers and firms bargain over real wages period by period. Real wage rigidity is introduced into the model in the form of Hall's (2005) real wage norm

$$w_t = (1-\gamma_w)w_t^* + \gamma_w w_{t-1}, \tag{17}$$

where w_t is the actual wage and w_t^* is the target wage determined by bargaining. γ_w is a parameter for the degree of real wage rigidity. Although the wage norm itself is an ad-hoc assumption and not considered when the target wage is determined, the framework encompasses various sources of wage rigidity at a more primitive level, such as in Hall and Milgrom (2008) and Gertler and Trigari (2009). This type of wage rigidity is also robust to the Barro's (1977) critique; as long as the actual wage bill is inside the wage bill set $[\underline{\omega}_t, \overline{\omega}_t]$, both the workers and firms have an incentive to continue the matching.

2.5 Closing the Model

The resource constraint, including posting vacancies and price adjustment costs and unemployment benefit, is given by

$$c_t + g_t + c_v v_t + \frac{\psi}{2}\pi_t^2 = y_t + (1-n_t)b, \tag{18}$$

where g_t is the government expenditure. Note that all firms set the same price so that $\int_0^1 \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}}-1\right)^2 di = (\psi/2)\pi_t^2$, where $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate. To close the model, the Ramsey-optimal policy is given in Section 3.

Competitive equilibrium is defined by the equilibrium conditions (1)-(9), (11), and (13) under EB or (14) and (16) under RTM, (17) and (18) under the Ramsey-optimal policy such that (i) households maximize their utility, (ii) firms maximize their profits, (iii) wages and hours per worker are determined via bargaining between firms and workers, and (iv) the markets clear.

2.6 Labor Market Allocation and the Wage Channel

Labor market allocation is characterized by equations (11) and (13) under EB or (14) and (16) under RTM. In (14), real wages have a direct link to real marginal cost and hence inflation. As firms choose the level of hours per worker, real wages are equal to the real marginal cost times the marginal product of labor. Christoffel and Kuester (2008) called this transmission mechanism *the wage channel to inflation*. Real wage rigidity is irrelevant to inflation persistence under EB (Krause and Lubik, 2007), whereas it is relevant under RTM (Christoffel and Linzert, 2010).

In this section, it is assumed that elasticity of output to hours per worker is given by $\alpha = f'(h)h/f(h) \in (0,1)$, and elasticity of match to unemployment is given by $\xi = (\partial m(v,u)/\partial u)(u/m) \in (0,1)$. Under RTM, the marginal gain for workers (relative to marginal loss for firms) by incremental real wages can be written as

$$\begin{aligned} \delta_t &= -\frac{\partial S_t/\partial w_t}{\partial J_t/\partial w_t}, \\ &= 1 - \varepsilon_w + \varepsilon_w g'(h_t)c_t^{\sigma}/w_t, \end{aligned}$$

where $\varepsilon_w = -h'(w)w/h(w) = 1/(1-\alpha) > 1$ is elasticity of demand for hours per worker to real wages. This equation can be solved for

$$w_t = \mu_{wt} g'(h_t)c_t^{\sigma}, \tag{19}$$

where $\mu_{wt} = \varepsilon_w/(\varepsilon_w - 1 + \delta_t)$ is wage markup over the marginal rate of substitution. Real wages and hours per worker are determined by demand (14) and supply (19), as if they are in the Walrasian labor market with variable wage markups.

If the marginal product of labor $mpl_t \equiv a_t f'(h_t)$ is equal to the marginal rate of substitution $mrs_t \equiv g'(h_t)c_t^{\sigma}$, hours per worker is at the efficient level. Galí, Gertler, and Lopez-Salido (2007) called the wedge between the marginal product of labor and the marginal rate of substitution the inefficiency gap.[9] The inefficiency gap is decomposed into price and wage markups. In the present paper, even with search and matching frictions and labor adjustment at the extensive margin, the inefficiency gap at the intensive margin consists of price and wage markups. Price and wage markups are defined by $\mu_{pt} \equiv mpl_t/w_t$ and $\mu_{wt} \equiv w_t/mrs_t$, and it is assumed that there is no real wage rigidity for the sake of exposition. It can then be proven that

Proposition 1. *The inefficiency gap is given by*

$$\frac{mpl_t}{mrs_t} \equiv \mu_{pt}\mu_{wt} = \begin{cases} \varphi_t^{-1}, & \textit{under EB,} \\ \varphi_t^{-1}\frac{\varepsilon_w}{\varepsilon_w - 1 + \delta_t}, & \textit{under RTM,} \end{cases}$$

where price and wage markups are

$$\mu_{pt} = \begin{cases} \varphi_t^{-1}\alpha\left(1+\frac{x_t}{w_t h_t}\right), & \textit{under EB,} \\ \varphi_t^{-1}, & \textit{under RTM,} \end{cases} \tag{20}$$

$$\mu_{wt} = \begin{cases} \alpha^{-1}\left(1+\frac{x_t}{w_t h_t}\right)^{-1}, & \textit{under EB,} \\ \frac{\varepsilon_w}{\varepsilon_w-1+\delta_t}, & \textit{under RTM.} \end{cases} \tag{21}$$

Proof. See Appendix A.2.1. □

Under EB, firms adjust labor input only at the extensive margin, as hours per worker is determined by bargaining, and $\mu_{pt}^{\mathrm{EB}} = \varphi_t^{-1}\alpha\left(1+\frac{x_t}{w_t h_t}\right)$ holds. This includes endogenous price markups related to the net hiring cost of labor adjustment at the extensive margin (Faia, 2009). Under RTM, firms adjust labor input not only at the extensive margin but also at the intensive margin; firms choose the level of hours per worker and smooth out the net hiring cost; $\alpha\left(1+\frac{x_t}{w_t h_t}\right) = 1$ holds under RTM. This implies $\mu_{pt}^{\mathrm{RTM}} = \varphi_t^{-1}$; price markups consist of only real marginal cost.

Under EB, wage markups just offset price markups related to the net hiring cost, $\mu_{wt}^{\mathrm{EB}} = \alpha^{-1}\left(1+\frac{x_t}{w_t h_t}\right)^{-1}$ so that the inefficiency gap consists of only real marginal cost measuring inefficiency in the goods market. Under RTM, the inefficiency gap also includes endogenous wage markups $\mu_{wt}^{\mathrm{RTM}} = \varepsilon_w/(\varepsilon_w - 1 + \delta_t)$ regarding inefficiency in the labor market.

Under RTM, the allocation in the decentralized economy is not efficient, even when the standard Hosios condition is satisfied (Sunakawa, 2012). It can be proven that

Proposition 2. *The allocation in the decentralized economy is efficient in the steady state if and only if $\varphi = 1$ and*

$$\eta = \xi, \qquad \textit{under EB,}$$
$$\eta = \xi = \chi, \qquad \textit{under RTM.}$$

Also, the allocation in the efficient steady state is common between EB and RTM.

Proof. See Appendix A.2.2. □

The efficiency condition under RTM imposes more parameter restriction than the standard Hosios condition under EB, as the effective workers' bargaining power, $\chi = \eta\delta/(1 - \eta(1-\delta))$, depends on δ and hence other variables. Therefore, from the viewpoint of its empirical relevance, the case of the inefficient steady state is first considered in Section 4.1, whereas the case of the efficient steady state is also examined in Section 4.2.

3 Optimal policy and calibration

The equilibrium conditions in the previous section are summarized into (c.f., Schmitt-Grohé and Uribe (2004)):

$$E_t \{f_i(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{x}_t)\} = 0,$$

where f_i is the (possibly nonlinear) equilibrium conditions for $i = 1, ..., n-1$, $\mathbf{x}_t$ is a $n_x \times 1$ vector of the state variable, $\mathbf{y}_t$ is a $n_y \times 1$ vector of the jump variable, and $n = n_x + n_y$. There are $n-1$ equilibrium conditions and n variables. The Ramsey-optimal policy pins down the equilibrium (if it exists and is unique) $\mathbf{y}_t = g^*(\mathbf{x}_t; \sigma_\varepsilon)$ and $\mathbf{x}_{t+1} = h^*(\mathbf{x}_t; \sigma_\varepsilon) + \eta_\varepsilon \sigma_\varepsilon \varepsilon_{t+1}$.

3.1 Ramsey-Optimal Policies

The Ramsey-optimal policy yields the constrained efficient equilibrium, which can be obtained by maximizing the representative household's utility subject to the equilibrium conditions of the economy. There are two different methodologies to compute the Ramsey-optimal policies: the Lagrange and LQ methods.

In the Lagrange method, the Ramsey-optimal policy is computed by using the Lagrangean

$$\mathcal{L}_0 = \min_{\{\Lambda_t\}_{t=0}^{\infty}} \max_{\{\Xi_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t E_t \left\{ U(\mathbf{x}_t, \mathbf{y}_t) + \sum_{i=1}^{n-1} \lambda_{it} f_i(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{x}_t) \right\} \right\},$$

where Λ_t is the set of Lagrange multipliers, Ξ_t is the set of endogenous variables, and U is the household's utility period by period. The Lagrangean yields n first-order necessary conditions and $n-1$ Lagrange multipliers; therefore, there are $2n-1$ equilibrium conditions and unknowns.

In the LQ method, the household's utility U is approximated up to second order, and the equilibrium conditions f_is are approximated up to first order in the Lagrangean shown above. Benigno and Woodford (2012, Proposition 1) showed that, if the household's utility is correctly approximated, the Lagrange and LQ methods are exactly the same up to first order. However, computing the correct LQ approximation can be very tedious. All the linear terms in the approximated household's utility are substituted out by using second-order approximation of the equilibrium conditions so that the household's utility is purely quadratic. When the steady state is efficient, such a computation is relatively easier as follows:

Proposition 3. *When the steady state is efficient, under both EB and RTM, the discounted* *sum of the household's utility is approximated as*

$$\begin{aligned} V_0 - V \approx & -\frac{c^{1-\sigma}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\psi}{c}\pi_t^2 + \sigma \hat{c}_t^2 - \frac{y}{c}(\hat{y}_t^2 - \hat{n}_t^2) \right. \\ & -\frac{\alpha y/c}{1+\phi}\left(\hat{n}_t^2 - \left[\hat{n}_t + (1+\phi)\hat{h}_t\right]^2\right) \\ & \left. +\frac{c_v v/c}{1-\xi}\left(\xi \hat{u}_t^2 + (1-\xi)\hat{v}_t^2 - \hat{m}_t^2\right) \right\}, \end{aligned}$$

where $\hat{x}_t = \ln(x_t/x)$ is the log deviation from the steady state.

Proof. See Appendix A.2.3. □

Note that the efficient steady state is common between EB and RTM, and so is the correct LQ approximation. The correct LQ approximation is used to compute the Ramsey-optimal policies in the case of the efficient steady state in Section 4.2.

3.2 Calibration

Functional forms are assumed as follows. The production function is $f(h_t) = h_t^\alpha$, where $\alpha \in (0,1)$ is elasticity of production to hours per worker. The labor disutility is $g(h_t) = \kappa h_t^{1+\phi}/(1+\phi)$, where $\phi > 0$ is the inverse of Frisch elasticity and $\kappa > 0$ is a normalization parameter. The matching function is $m(u_t, v_t) = m u_t^\xi v_t^{1-\xi}$, where $\xi \in (0,1)$ is elasticity of matches to unemployment and $m > 0$ is a normalization parameter. Unemployment benefits are given by $b = b_w wh$, where $b_w \in (0,1)$ is the ratio of unemployment benefit to the wage bill.

Parameter values are calibrated following previous studies based on the postwar U.S. data and summarized in Table 1.[10] Steady state unemployment after matching $1-n = 0.4$ and filling vacancy rates for firms $q = 0.7$ are given as the calibration targets.[11] $\beta = 0.99$ implies that the annual interest rate is 4%. The risk aversion parameter is set to $\sigma = 2$. $\varepsilon_p = 6$ implies that the steady state price markup is 20%. The ratio of government expenditure to output is 25%. These values are standard and in line with the postwar U.S. data. The price adjustment cost parameter $\psi = 20$ implies that the elasticity of inflation to real marginal cost in the log-linearized Phillips curve is $(\varepsilon_p - 1)y/\psi = 0.15$. The job destruction rate $\rho = 0.1$ and elasticity of matches to unemployment $\xi = 0.4$ are also standard. The ratio of unemployment benefit to wage bill is $b_w = 0.6$. Elasticity of production to hours per worker is $\alpha = 0.9$, which also implies elasticity of labor demand to real wages at the intensive margin $\varepsilon_w = 1/(1-\alpha) = 10$. The inverse of Frisch elasticity is $\phi = 2$. The shock parameters are assumed $\rho_a = 0.95$ and $\sigma_a = 0.008$, following the evidence on the Solow residual from the postwar U.S. data.

[Table 1 is inserted here]

In the case of the inefficient steady state in Section 4.1, $\eta > \xi = 0.4$, and the Hosios condition is not met under EB. Given the values of η and ξ, the marginal gain for workers by incremental real wages is $\delta = 0.26 < 1$ and the wage markup is $\mu_w = w/mrs = 1.08 > 1$ in the steady state under RTM, as shown in Table 2, which implies that there is a positive inefficiency gap in the labor market (Galí, Gertler, and Lopez-Salido, 2007). The inefficiency gap also implies that the workers' effective bargaining power $\chi = \delta\eta/(1 - \eta + \delta\eta) = 0.28$ is less than the workers' actual bargaining power $\eta = 0.6$. When $\eta = \xi = 0.225$, $\eta = \xi = \chi$ holds and the steady state is efficient under both EB and RTM, as shown in Proposition 2, which is the case examined in Section 4.2.

[Table 2 is inserted here]

4 Results

In this section, the Ramsey-optimal monetary policies are numericaly investigated. First, the case of the inefficient steady state (Faia, 2009) without and with real wage rigidity is examined in Section 4.1. The case of the efficient steady state (Thomas, 2008) and a comparison between the Lagrange and LQ methods are shown in Section 4.2.

4.1 The inefficient steady state

In the case of the inefficient steady state, it is shown that price stability in response to technology shocks is *nearly* optimal in the model with RTM without real wage rigidity. This result starkly contrasts with the Faia's (2009) result in the model with EB, and is in line with the result in a model with the Walrasian labor market. With real wage rigidity, deviation from price stability becomes optimal. Under RTM, the optimal volatility of inflation is high when the workers' bargaining power is low, which also offers an interesting contrast with Faia's result. Similarly, a larger unemployment benefit to amplify the labor market volatility (Hagedorn and Manovskii, 2008) does not increase the optimal volatility of inflation under RTM, whereas it does under EB.

4.1.1 Without real wage rigidity

Figure 1 shows the impulse responses to a technology shock ($0.8 = 100\sigma_a\%$) without real wage rigidity under both EB and RTM. First of all, price stability is nearly optimal under RTM; namely, the optimal response of inflation to a positive technology shock is very small.

In response to a positive technology shock, under RTM, real wages respond immediately, and firms post more vacancies to produce more. A larger labor demand makes the aggregate labor market condition tighter, and employment gradually adjusts so that the labor supply is increased. Under EB, real wages are sluggish even without real wage rigidity, which makes the labor market condition even tighter. Inflation deviates from zero and responds negatively.

[Figure 1 is inserted here]

What is the intuition behind the different results under EB and RTM? Under EB, as in Faia (2009), endogenous price markups stemming from the net hiring cost and a related trade-off between inflation and unemployment undo price stability, even with both the intensive and extensive margins, compared to the case with the extensive margin only in Faia (2009). Under RTM, real wages and hours per worker replicate the allocation in the Walrasian labor market with variable wage markups. The wage channel to inflation leads to stable price markups, as they consists of only real marginal cost.

The left column of Figure 2 breaks down the inefficiency gap into the price and wage markups, as shown in Proposition 1. Under EB, the price markup responds positively, as the net hiring cost is higher with a tighter labor market condition. The wage markup responds negatively to offset the labor market inefficiency measured by the inefficiency gap. The gap responds positively, as the former effect dominates the latter. Under RTM, the wage markup responds negatively, but it stems from δ_t in Equation (19), as the marginal gain for workers (relative to the marginal loss for firms) by incremental real wages gets larger. Price markups are stable in response to the shock, and wage markups are the main driving force of the inefficiency gap.

The allocation in a model with the Walrasian labor market is also computed in Figure 3. Real wages and hours per worker are determined by $w_t = mrs_t = \varphi_t mpl_t$. The divine coincidence in Blanchard and Galí (2007) holds and fluctuations in the price and wage markups are exactly zero. The price stability result under RTM is a quantitative one, as opposed to the case with the Walrasian labor market.[12]

[Figure 2 is inserted here]

4.1.2 With real wage rigidity

Figure 3 shows the impulse responses with real wage rigidity. It is shown that deviation from price stability is optimal both under EB and RTM. Real wages are sluggish and cannot respond immediately to a positive technology shock. Firms post even more vacancies, and the aggregate labor market condition becomes even tighter to take advantage of lower real wages, especially under EB, because firms mainly adjust labor input at the extensive margin, as hours per worker is determined by bargaining.[13]

Under RTM, sluggish real wages make wage markups countercyclical and volatile because the marginal rate of substitution is related to real wages, as shown in the labor supply equation (19). Hours per worker increase, as firms can adjust labor input at the intensive margin. Under EB, firms adjust labor input mainly at the extensive margin and hours per worker decreases instead because there is no direct link between real wages and hours per worker. Price markups respond strongly, reflecting aggressive adjustments at the extensive margin, whereas wage markups are relatively stable compared to the ones under RTM.

[Figure 3 is inserted here]

The right column of Figure 2 also breaks down the inefficiency gap into price and wage markups in the case with real wage rigitity. As in the case without real wage rigidity, price markups are volatile and wage markups just offset the gap under EB, whereas wage markups mainly explain fluctuations in the gap under RTM. The responses under RTM closely resemble the ones in the model with the Warlasian labor market.

Under RTM, the allocational role of real wages is the key to explain the equilibrium dynamics. Firms can adjust labor input at the intensive margin and take advantage of low and sluggish real wages by increasing hours per worker via the wage channel. The adjustment at the intensive margin, however, yields wage markup fluctuations and makes the optimal inflation volatile, depending on the degree of real wage rigidity, as in the model with the Walrasian labor market. Under EB, the allocational role of real wages is limited, as there is no direct link between real wages and hours per worker. Firms mainly adjust labor input at the extensive margin, which generates fluctuations in the net hiring cost and price markups.

4.1.3 Robustness checks

The price stability result under RTM without real wage rigidity is a quantitative one. Some robustness checks are done for different shocks and parameter values. The analysis herein focuses on the case without real wage rigidity.

Government expenditure shocks Figure 4 shows the response to a positive government expenditure shock in the case without real wage rigidity. Faia (2009) showed that in response to the government expenditure shock, deviation from price stability arises, although the response is rather small. Faia's result depends on the assumption that firms adjust labor input at the extensive margin only. When $\phi = 10^4$ so that adjusting at the intensive margin is too costly, the model under EB in this paper replicates the case with the extensive margin only in Faia (2009). Employment decreases through a lower tightness of labor market. However, when $\phi = 2$ so that firms also adjust labor input at the intensive margin, employment increases as well as the labor market tightness and hours per worker, which leads to more price markup fluctuation and deviation from price stability. Under RTM, employment decreases insteadly, whereas hours per worker increase. The response of price markups is muted, and price stability is as nearly optimal as in the case of technology shocks.

[Figure 4 is inserted here]

Different values of η and b_w Faia (2009) showed that the optimal volatility of inflation is increasing in η because a higher bargaining power η results in more external congestion and unemployment fluctuations under EB. The left window of Figure 5 shows the optimal volatility of inflation for different values of η under both EB and RTM.[14] Under EB, it replicates Faia's result.[15] Under RTM, the optimal volatility of inflation is high when η is low. The external congestion is irrelevant to the optimal inflation because firms can smooth the net hiring cost at the extensive margin by adjusting labor input at the intensive margin. When η is low (i.e., when firms have more bargaining power), firms adjust hours per worker in the presence of the wage channel, and wage markups become more volatile.[16]

Hagedorn and Manovskii (2008, hereafter HM) showed that a larger value of unemployment benefit, $b = b_w wh$, also yields a larger employment fluctuation under EB.[17] The right window of Figure 5 shows the optimal volatility of inflation for different values of b_w. Under EB, the larger b_w, the more volatile inflation is. The labor market condition becomes more volatile, as pointed out by HM. Under RTM, however, the wage channel kills the labor market volatility. The optimal volatility of inflation is high when b_w is low because workers want to work more with a lower outside option, which makes hours per worker and wage markups volatile.

[Figure 5 is inserted here]

4.2 The efficient steady state

The case of the efficient steady state is examined to compare it with the price stability result under EB in Thomas (2008). If and only if $\varphi = 1$ and $\xi = \eta = \chi$, the steady state is efficient and common between EB and RTM, as shown in Proposition 2. The condition for efficiency in the steady state is satisfied when $\xi = \eta^* = 0.225$, with the calibration in Section 3.2. The correct LQ approximation is also common between EB and RTM, as shown in Proposition 3, and is used to compute the Ramsey-optimal monetary policies.

Figure 6 shows the impulse responses to a technology shock ($0.8 = 100\sigma_a\%$) without real wage rigidity under both EB and RTM. When the steady state is efficient, the optimal response of inflation is exactly zero as in Thomas (2008). The reason is understood by examining Figure 7, which shows the decomposition of the inefficiency gap. Without real wage rigidity, under EB, the price and wage markups cancel each other out, and the inefficiency gap is exactly equal to zero. In other words, there is no external congestion. Thus, the monetary policymaker can focus on stabilizing inflation. Under RTM, even though the (logged) inefficiency gap is zero in the efficient steady state, it temporally deviates from zero due to wage markup fluctuations. With real wage rigidity, deviation from price stability is optimal under both EB and RTM. The responses of price and wage markups under RTM resemble the ones in the model with the Walrasian labor market, as in the case of the inefficient steady state.

Figure 8 shows the optimal volatility of inflation for different values of η. It is shown that the optimal volatility of inflation is zero under EB when $\xi = \eta^*$, as in Thomas (2008). On the contrary, under RTM, even when $\xi = \eta^*$, the optimal volatility of inflation is not zero nor minimized over η. The optimal volatility of inflation computed by the LQ method are shown with gray lines in Figure 8. When $\xi = \eta^*$, the optimal volatilities computed by the Lagrange and LQ methods are exactly the same under both EB and RTM.

The results of the paper are consistent with the ones in Thomas (2008) [price stability when the steady state is efficient] and Faia (2009) [deviation from price stability when the steady state is inefficient]. It is found that, when the steady state is efficient, the price stability result holds under EB, and the LQ and Lagrangean methods yield exactly the same results up to first order. When the steady state is inefficient, deviation from price stability arises, and the further we deviate from the efficient steady state, the larger the numerical error between the Lagrange and LQ methods.

[Figure 6-8 is inserted here]

5 Concluding Remarks

This paper analyzed the Ramsey-optimal monetary policy in labor search models with sticky prices, focusing on the role of the wage channel to inflation (i.e., a relationship between real wages and real marginal cost), based on empirical evidence. It is found that the nature of the Ramsey-optimal monetary policy under RTM is totally different from the one under EB. Under RTM, even when the steady state is inefficient, price stability is nearly optimal, whereas deviation from price stability is optimal under EB. Real wage rigidity creates the case against price stability, as hours per worker are more volatile than real wages, and wage markups are countercyclical and volatile, as in models with the Walrasian labor market. This paper also studied both cases with the inefficient and the efficient steady state. Under EB, price stability is optimal with the efficient steady state, and the more we deviate from the efficient steady state, the more volatilie optimal inflation is. Under RTM, however, even when the steady state is efficient, the optimal volatility of inflation is not zero nor minimized.

In the present paper, it is found that wage markups are countercyclical and volatile with the wage channel and real wage rigidity. In the U.S. economy, the inefficiency gap between the marginal product of labor and the marginal rate of substitution is mainly explained by countercyclical wage markup fluctuations, as shown in Galí, Gertler, and Lopez-Salido (2007); therefore, one of the interesting topics for future research will be to investigate this relationship more. Within the search and matching framework, the wage channel and real wage rigidity are potentially able to explain the labor wedge (Chari, Kehoe, and McGrattan, 2007), which is one of the main drivers for business cycles (Cheremukhin and Restrepo-Echavarria, 2014; Pescatori and Tasci, 2011).

A Appendix

A.1 Steady State

$r = 1/\beta$ and $\varphi = (1+\tau)(\varepsilon_p - 1)/\varepsilon_p$ are immediately obtained. Given the steady state unemployment $\tilde{u} = 1 - n$ and job filling rate $q = m/v$, the other steady state values of the labor market are

$$n = 1 - \tilde{u}, \quad v = \rho n/q, \quad \bar{m} = \rho n/(u^{\xi} v^{1-\xi}),$$
$$u = 1 - n + \rho n, \quad \theta = v/u, \quad p = \rho n/u,$$

which are common between the two bargaining schemes.

A.1.1 Efficient Bargaining

We normalize $a = 1$. Steady state conditions of matching values S and J are given by

$$\begin{aligned}
&\varphi\alpha h^{\alpha-1} = \kappa h^{\phi} c^{\sigma},\\
&[1-\beta(1-\rho)(1-p)]S = (1-b_w)wh - \kappa h^{1+\phi}c^{\sigma}/(1+\phi),\\
&[1-\beta(1-\rho)]J = \varphi h^{\alpha} - wh,\\
&1 = [(1-\eta)/\eta]S/J,
\end{aligned}$$

These equations can be solved for

$$wh = \frac{\eta[1-\beta(1-\rho)(1-p)] + (1-\eta)[1-\beta(1-\rho)]\alpha/(1+\phi)}{\eta[1-\beta(1-\rho)(1-p)] + (1-\eta)[1-\beta(1-\rho)](1-b_w)} x h^{\alpha},$$

S, and J. Given q and J, vacancy cost is $c_v = qJ$ and steady state consumption $c = h^{\alpha}n - g - c_v v - (1-n)b$ gets pinned down. With the calibration given in Section 3.2, steady state vacancy cost is about 2.18% of output and steady state consumption is about 72.82% of output. Steady state reservation wage bills for workers and firms are given by $\underline{\omega} = wh - S$ and $\overline{\omega} = wh + J$. Using the calibrated parameters, $[\underline{\omega}, \overline{\omega}] = [0.5952, 1.2699]wh$ is obtained.

A.1.2 Right-to-Manage Bargaining

Steady state conditions of matching values are given by

$$\begin{aligned}
&wh = \varphi\alpha h^{\alpha},\\
&[1-\beta(1-\rho)(1-p)]S = (1-b_w)wh - \kappa h^{1+\phi}c^{\sigma}/(1+\phi),\\
&[1-\beta(1-\rho)]J = \varphi h^{\alpha} - wh,\\
&\delta = [(1-\eta)/\eta]S/J,
\end{aligned}$$

These equations can be solved for

$$\delta = \frac{(1-\eta)[(1+\phi)(1-b_w) - \alpha][1-\beta(1-\rho)]\alpha}{\eta[1-\beta(1-\rho)(1-p)](1+\phi)(1-\alpha) + (1-\eta)[1-\beta(1-\rho)]\alpha(1-\alpha)}, \qquad (22)$$

S, and J. Steady state vacancy cost is about 7.65% of output and steady state consumption is about 67.35% of output. $[\underline{\omega}, \overline{\omega}] = [0.5952, 2.0194]wh$ is obtained.

A.2 Proofs

A.2.1 Proposition 1

For price markups, under both EB and RTM, the hiring condition (9) at the extensive margin becomes

$$\begin{aligned} \varphi_t &= \left(1+\frac{x_t}{w_t h_t}\right)\frac{w_t h_t n_t}{y_t}, \\ &= \left(1+\frac{x_t}{w_t h_t}\right)\frac{\alpha w_t}{mpl_t}, \end{aligned}$$

where $mpl_t \equiv a_t f'(h_t) = \alpha y_t/(h_t n_t)$; therefore, $\mu_{pt}^{\mathrm{EB}} = \varphi_t^{-1}\alpha\left(1+\frac{x_t}{w_t h_t}\right)$. Under RTM, in addition to the hiring condition, the wage channel (15) becomes

$$\begin{aligned} w_t &= \varphi_t a_t f'(h_t), \\ &= \varphi_t mpl_t. \end{aligned}$$

Price markups are equal to the inverse of real marginal cost, i.e., $\mu_{pt}^{\mathrm{RTM}} = \varphi_t^{-1}$, which also implies $\alpha(1+x_t/(w_t h_t)) = 1$; the net hiring cost is smoothed.

For wage markups, under EB, the bargaining between firms and workers over hours per worker yields the efficiency condition (11) at the intensive margin;

$$mrs_t = \varphi_t mpl_t,$$

which implies that the inefficiency gap is equal to the inverse of real marginal cost. Wage markups are given by $\mu_{wt}^{\mathrm{EB}} = w_t/(\varphi_t mpl_t) = 1/(\varphi_t \mu_{pt}^{\mathrm{EB}}) = \alpha^{-1}\left(1+\frac{x_t}{w_t h_t}\right)^{-1}$. Under RTM, equation (11) does not hold, and the wage equation (16) becomes

$$w_t = \mu_{wt}^{\mathrm{RTM}} mrs_t,$$

where $\mu_{wt}^{\mathrm{RTM}} = \frac{\varepsilon_w}{\varepsilon_w - 1 + \delta_t}$. The efficiency gap is given by $\mu_{pt}^{\mathrm{RTM}}\mu_{wt}^{\mathrm{RTM}} = \varphi_t^{-1}\frac{\varepsilon_w}{\varepsilon_w - 1 + \delta_t}$.

Figure 9 graphically shows how real wages and hours per worker (w, h) are determined.[18] Under EB, by equation (11), hours per worker are determined at $h = h^{\mathrm{EB}}$ regardless of wage contracts. Real wages are determined independently at $w = w^{\mathrm{EB}}$ by equation (13). $(w^{\mathrm{EB}}, h^{\mathrm{EB}})$ is determined at the point E_1 in Figure 9. If the Hosios (1990) condition is satisfied and there is a subsidy to offset the distortion stemming from monopolistic competition, real wages and hours per worker are at the efficient level at the point E_0 in Figure 9, (w^*, h^*). Under RTM, the wage channel yields the labor demand schedule equation (14), whereas the bargaining over real wages between workers and firms yields the labor supply schedule equation (19), which shifts with variable wage markups μ_{wt}^{RTM}. The equilibrium is the intersection of the demand and supply at the point E_2 in Figure 9, $(w^{\mathrm{RTM}}, h^{\mathrm{RTM}})$.

[Figure 9 is inserted here]

A.2.2 Proposition 2

The efficient equilibrium is computed by solving the social planner's problem, in absence of price stickiness and monopolistic competition. The social planner chooses $\{c_t, h_t, n_t, v_t\}$ so as to maximize

$$
\begin{aligned}
& V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - g(h_t)n_t + \beta E_t V_{t+1}, \\
& \text{subject to} \\
& c_t + g_t + c_v v_t = a_t f(h_t) n_t + b(1-n_t), \\
& n_t = (1-\rho)n_{t-1} + m u_t^{\xi} v_t^{1-\xi},
\end{aligned}
$$

where $u_t = 1 - n_{t-1} + \rho n_{t-1}$ and $\theta_t = v_t/u_t$. From the FONCs, the equilibrium conditions for the efficient equilibrium are obtained as

$$
\begin{aligned}
& \frac{c_v}{q(\theta_t)} = (1-\xi)\left(a_t f(h_t) - b - g(h_t)c_t^{\sigma}\right) \\
& + \beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{c_v}{q(\theta_{t+1})} - \xi c_v \theta_{t+1}\right)\right\}, \\
& a_t f'(h_t) = g'(h_t)c_t^{\sigma}.
\end{aligned}
$$

In the steady state,

$$
[1-\beta(1-\rho)(1-\xi p)] = \frac{(1-\xi)q}{c_v}\left(af(h) - g(h)c^{\sigma} - b\right), \tag{23}
$$

$$
af'(h) = g'(h)c^{\sigma}. \tag{24}
$$

Under EB, the usual Hosios (1990) condition applies; when $\varphi = 1$ and $\xi = \eta$, the allocation in the decentralized economy is efficient. Under RTM, from equations (9), (19), and (16), the equilibrium conditions in the decentralized economy are obtained as

$$
\begin{aligned}
&\frac{c_v}{q(\theta_t)} = (1-\chi_t)\left(\varphi_t a_t f(h_t) - b - g(h_t)c_t^{\sigma}\right) \\
&+\beta(1-\rho)E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(1-\chi_t+\chi_t(\delta_t/\delta_{t-1})[1-p(\theta_{t+1})])\frac{c_v}{q(\theta_{t+1})}\right\}, \\
&\varphi_t a_t f'(h_t) = \mu_{wt} g'(h_t)c_t^{\sigma},
\end{aligned}
$$

where $\chi_t = \eta\delta_t/(1-\eta(1-\delta_t))$ and $\mu_{wt} = \varepsilon_w/(\varepsilon_w+1-\delta_t)$. In steady state,

$$
[1-\beta(1-\rho)(1-\chi p)]\frac{c_v}{q} = \frac{(1-\chi)q}{c_v}\left(\varphi f(h) - b - g(h)c^{\sigma}\right), \tag{25}
$$

$$
\varphi f'(h) = \mu_w g'(h)c^{\sigma}, \tag{26}
$$

By comparing equations (23)-(24) and (25)-(26), when $\varphi = 1$ and $\xi = \eta = \chi$, the equations coincide, and the allocation in the decentralized economy is efficient. Note that the efficient steady state is common between EB and RTM, as it satisfies (23)-(24) under both EB and RTM.

As δ can be viewed as a function of η and $\chi = \delta(\eta)\eta/(1-\eta+\delta(\eta)\eta)$ is a monotone function of η, there is the unique $\eta = \eta^*$ satisfying $\eta = \chi(\eta)$ such that $\delta(\eta^*) = 1$ holds and the steady state is efficient (Sunakawa, 2012).[19] Equation (22) can be solved for such η^*.

A.2.3 Proposition 3

Available upon request from the author.

References

Andolfatto, D. (1996): "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112–132.

Barro, R. J. (1977): "Long-Term Contracting, Sticky Prices, and Monetary Policy," *Journal of Monetary Economics*, 3(3), 305–316.

Benigno, P., and M. Woodford (2012): "Linear-quadratic Approximation of Optimal Policy Problems," *Journal of Economic Theory*, 147(1), 1–42.

Blanchard, O., and J. Galí (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39(s1), 35–65.

Blanchard, O., and J. Galí (2010): "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2(2), 1–30.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

Cheremukhin, A. A., and P. Restrepo-Echavarria (2014): "The Labor Wedge as a Matching Friction," *European Economic Review*, 68, 71–92.

Christoffel, K., and K. Kuester (2008): "Resuscitating the Wage Channel in Models with Unemployment Fluctuations," *Journal of Monetary Economics*, 55(5), 865–887.

Christoffel, K., and T. Linzert (2010): "The Role of Real Wage Rigidity and Labor Market Frictions for Inflation Persistence," *Journal of Money, Credit and Banking*, 42(7), 1435–1446.

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297–308.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), 281–313.

Faia, E. (2009): "Ramsey Monetary Policy with Labor Market Frictions," *Journal of Monetary Economics*, 56(4), 570–581.

Galí, J., M. Gertler, and D. Lopez-Salido (2007): "Markups, Gaps, and the Welfare Costs of Business Fluctuations," *Review of Economics and Statistics*, 89(1), 44–59.

Gertler, M., and A. Trigari (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117(1), 38–86.

Hagedorn, M., and I. Manovskii (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98(4), 1692–1706.

Hall, R. E. (2005): "Employment Fluctuations with Equlibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

Hall, R. E., and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," 98(4), 1653–1674.

Hosios, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *The Review of Economic Studies*, 57(2), 279–298.

Krause, M. U., and T. A. Lubik (2007): "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions," *Journal of Monetary Economics*, 54(3), 706–727.

Leontief, W. (1946): "The Pure Theory of the Guaranteed Annual Wage Contract," *Journal of Political Economy*, 54(1), 76–79.

Merz, M. (1995): "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics*, 36(2), 269–300.

Nickell, S. J., and M. Andrews (1983): "Unions, Real Wages and Employment in Britain 1951-79," *Oxford Economic Papers*, 35(0), 183–206.

Pescatori, A., and M. Tasci (2011): "Search Frictions and the Labor Wedge," Working Paper 1111, Federal Reserve Bank of Cleveland.

Ravenna, F., and C. E. Walsh (2011): "Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework," *American Economic Journal: Macroeconomics*, 3(2), 130–62.

Rotemberg, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49(4), 517–31.

Schmitt-Grohé, S., and M. Uribe (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28(4), 755–775.

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

SHIMER, R. (2010): *Labor Markets and Business Cycles.* Princeton University Press, Princeton, NJ.

SUNAKAWA, T. (2012): "Efficiency in a Search and Matching Model with Right-to-Manage Bargaining," *Economics Letters*, 117(3), 679–682.

THOMAS, C. (2008): "Search and Matching Frictions and Optimal Monetary Policy," *Journal of Monetary Economics*, 55(5), 936–956.

TRIGARI, A. (2006): "The Role of Search Frictions and Bargaining for Inflation," IGIER Working Paper n 304, Bocconi University.

WOODFORD, M. (2010): "Optimal Monetary Stabilization Policy," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3, chap. 14, pp. 723–828. Elsevier.

Notes

[1]In the present paper, Nash bargaining over both real wages and hours per worker is called efficient bargaining (Christoffel and Linzert, 2010). Under efficient bargaining (and perfect competition), hours per worker is determined at the efficient level such that the marginal rate of substitution is equal to the marginal product of labor.

[2]However, RTM yields a larger inefficiency than EB does. Firms with some monopolistic power, instead, may offer a take-it-or-leave contract to workers that specifies both real wages and hours per worker (Leontief, 1946).

[3]Price stability to technology shocks is one of the central results of previous studies (Woodford, 2010). Erceg, Henderson, and Levin (2000) showed that deviation from price stability is optimal due to wage markup fluctuations in a model with sticky prices and wages and the Walrasian labor market.

[4]Shimer (2005; 2010) introduced real wage rigidity as a real wage norm à la Hall (2005) to amplify labor market volatility to bring the model to the data. Christoffel and Linzert (2010) argued that real wage rigidity under RTM, in contrast with under EB (Krause and Lubik, 2007), is important to explain inflation persistence.

[5]Benigno and Woodford (2012) showed that, if the correct LQ approximation is used, the Lagrange and LQ methods yield exactly the same result up to first order. They also derived the correct LQ approximation to the Ramsey-optimal policies when the steady state is inefficient.

[6]This timing assumption is used in Faia (2009), and different from Trigari (2006) and Christoffel and Linzert (2010).

[7]If prices are flexible in equation (7), $\psi = 0$, then $(1+\tau)P_t = [\varepsilon_p/(\varepsilon_p - 1)]\tilde{\varphi}_t$ holds, where $\tilde{\varphi}_t = P_t\varphi_t$ is nominal marginal cost and $\varepsilon_p/(\varepsilon_p - 1)$ is a steady-state price markup. When $1+\tau = \varepsilon_p/(\varepsilon_p - 1)$, $\varphi_t = 1$ holds, namely, there is no markup and hence distortion.

[8]Trigari (2006) split firms' price setting and hiring processes into two steps. Here, the two steps are merged into one step by following Faia (2009).

[9]Galí, Gertler, and Lopez-Salido (2007) originally defined the gap as the log difference between the marginal rate of substitution and the marginal product of labor; their gap is $gap_t \equiv \log mrs_t - \log mpl_t = -(\log \mu_{pt} + \log \mu_{wt})$, opposed to mpl_t/mrs_t in the present paper. When $\mu_{pt} > 1$ and $\mu_{wt} > 1$, their gap is negative, whereas this gap (after logged) is positive.

[10]All values except (α, ϕ) are the same as in Faia (2009); (α, ϕ) are missing in Faia because there is only the extensive margin in the model. Details of the steady state calculation are found in Appendix A.1.

[11]Steady state unemployment seems high, but this value may include discouraged and occasionally participating workers. Andolfatto (1996) used $n = 0.57$ (before the match), which implies $u = 0.43$ by including a non-labor force population.

[12]The impulse response in Figure 1 shows an initial rise of inflation, although it is admittedly small. Also, Figure 5 shows that the optimal volatility of inflation under RTM is positive.

[13]This result is in line with Shimer (2005), who argued that real wage rigidity has an amplifying effect on labor market variables.

[14]For displayed values of η and b_w in Figure 5, the actual wage bill $w_t h_t$ lies in the bargaining set $[\underline{\omega}_t, \bar{\omega}_t]$.

[15]Note that, even when the Hosios condition is met at $\eta = \xi$ under EB, the steady state is not efficient because of the distortion stemming from monopolistic competition. See also Figure 3 in Faia (2009).

[16]It is analytically shown that as $\eta \to 0$, the marginal gain for workers by incremental real wages $\delta_t = -(\partial S_t/\partial w_t)/(\partial J_t/\partial w_t)$ increases, and wage markups μ_{wt} are more elastic to changes in δ_t.

[17] With both intensive and extensive margins, there is an upper bound of $b_w \leq \bar{b}_w = 1 - \frac{1}{\mu_w(1+\phi)} \in (0,1)$ with which $S \geq 0$ holds; i.e., workers want to continue the matching with firms. The upper bound is $\bar{b}_w = 0.6912$, a much lower value than the one used in HM, with the calibration in the present paper. As $\phi \to \infty$, the model replicates the case with extensive margin only (Faia, 2009), and $\bar{b}_w$ approaches one.

[18] All variables are logged. A similar figure is found in Galí, Gertler, and Lopez-Salido (2007).

[19] $\delta(\eta)$ has the following property: If $(1+\phi)(1-b_w) > 1$, $\delta(0) = [(1+\phi)(1-b_w) - \alpha]/(1-\alpha) > 1$, $\delta(1) = 0$, and $\delta'(\eta) < 0$ hold.

Table 1: Parameter values.

Parameter	Value	Description
β	0.99	Discount factor
σ	2	Risk aversion
ε_p	6	Elasticity of demand
ψ	20	Price adjustment cost
ξ	0.4	Elasticity of matches to unemp.
ρ	0.1	Job separation rate
η	0.6	Workers' actual bargaining power
b_w	0.6	Unemployment benefits
α	0.9	Elasticity of production to hours per worker
ϕ	2	Inverse of Frisch elasticity
$1-n$	0.4	Steady state unemployment after matching
q	0.7	Steady state job filling rate for firms
g_y	0.25	Steady state govt. expenditure
ρ_a	0.95	Auto corr. of technology
σ_a	0.008	Std. dev. of technology

Table 2: Steady state values.

Variable	Value		Description
	RTM	EB	
n	0.6000	0.6000	Employment after matching
v	0.0857	0.0857	Vacancies
θ	0.1863	0.1863	Labor market tightness
$\bar{m}$	0.3575	0.3575	Scaling parameter in matching function
p	0.1304	0.1304	Job finding rate for workers
w	0.7500	0.8095	Real wages
b	0.4500	0.4857	Unemployment benefits
c/y	0.6735	0.7282	Consumption-output ratio
$c_v v/y$	0.0765	0.0218	Vacancy cost-output ratio
$\underline{\omega}/(wh)$	0.5952	0.5952	Reservation wage bill for workers
$\overline{\omega}/(wh)$	2.0194	1.2699	Reservation wage bill for firms
κ	4.2546	3.9293	Scaling parameter in labor disutility
δ	0.2648	-	Marginal gain for workers
μ_w	1.0794	-	Wage markup
χ	0.2843	-	Workers' effective bargaining power

Notes: Steady state values are computed by the model's steady state conditions, which are shown in Appendix A.1.

Figure 1: Impulse responses to a positive technology shock without real wage rigidity.

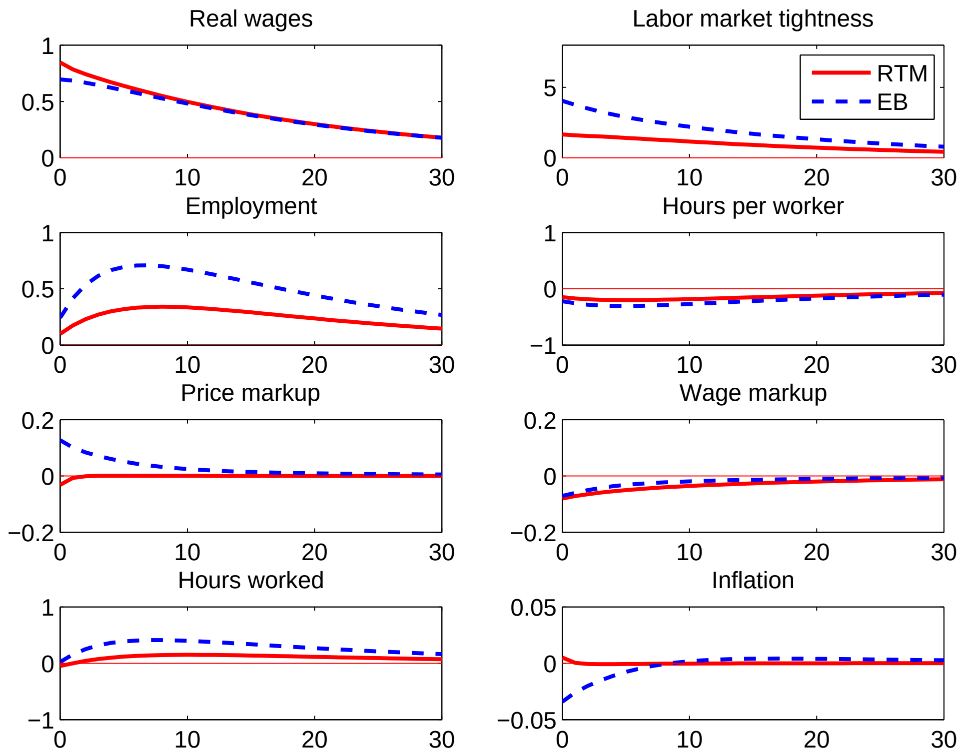


Figure 2: Composition of the inefficiency gap.

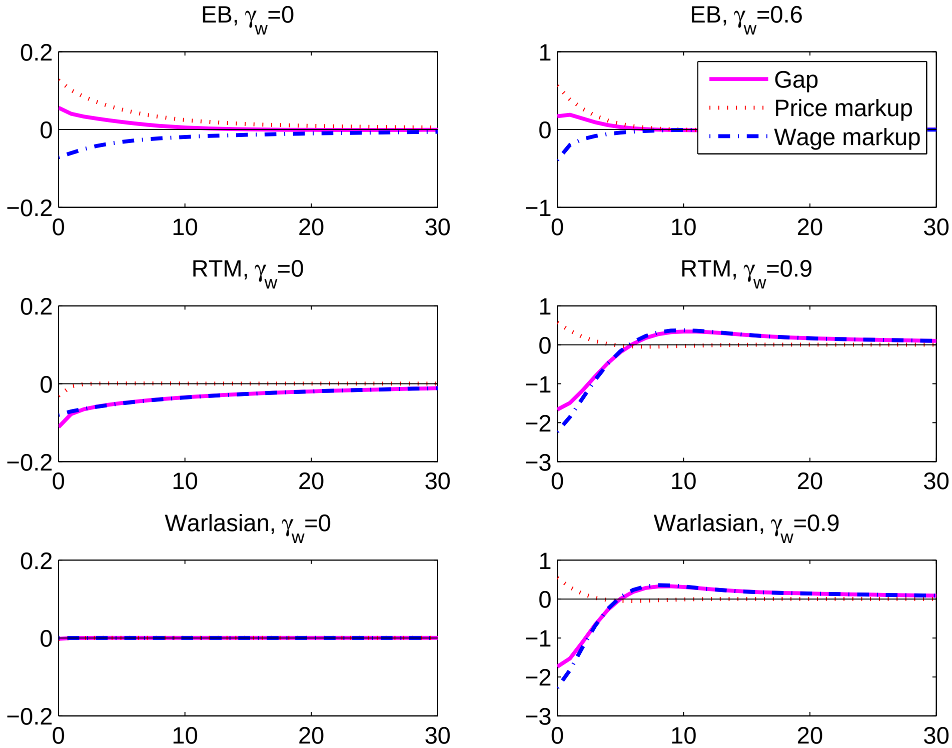
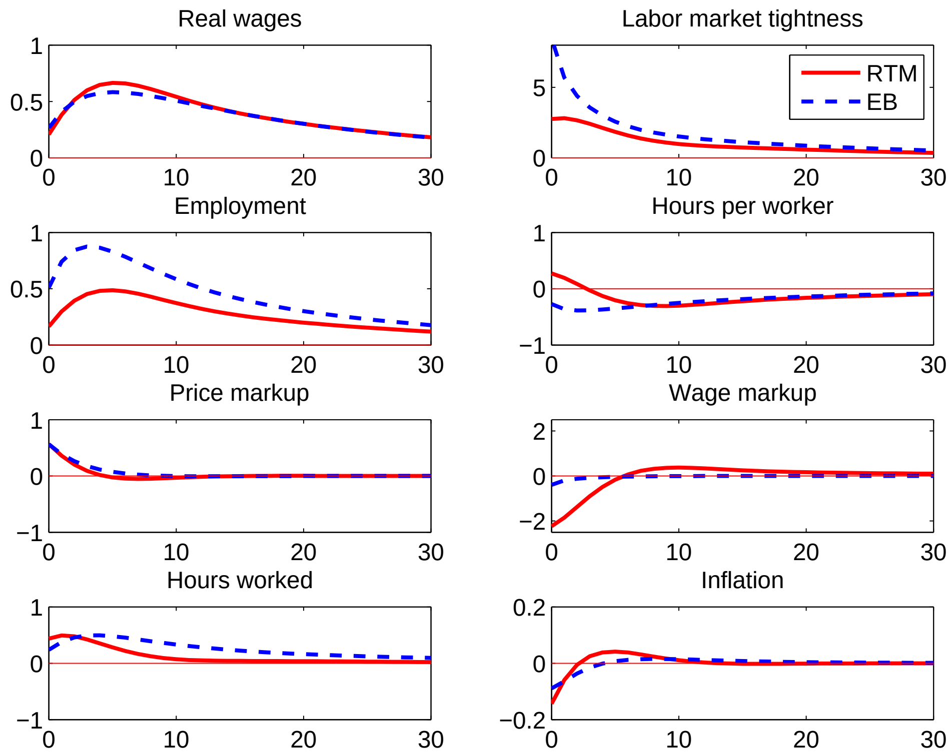
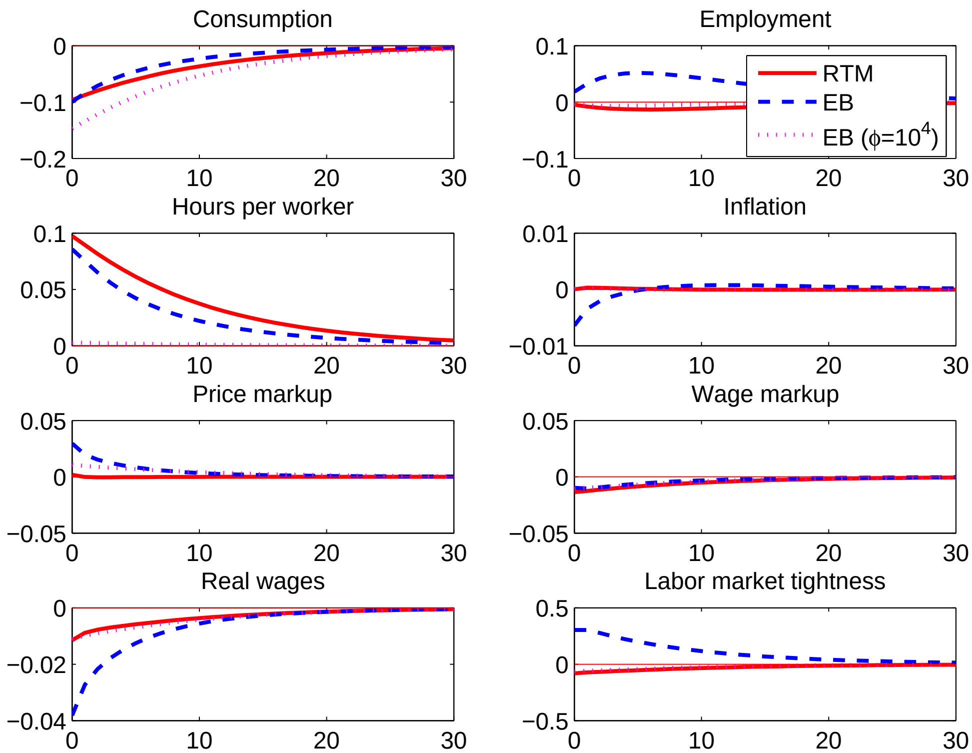


Figure 3: Impulse responses to a positive technology shock with real wage rigidity.



Notes: The degree of real wage rigidity is $\gamma_w = 0.6$ for EB and $\gamma_w = 0.9$ for RTM.

Figure 4: Impulse responses to a positive government expenditure shock without real wage rigidity.



Notes: the government spending shock follows $\ln(g_t/g) = \rho_g \ln(g_{t-1}/g) + \varepsilon_{gt}$, where $\varepsilon_{gt} \sim N(0, \sigma_g^2)$, $\rho_g = 0.9$, and $\sigma_g = 0.0074$, following to Faia (2009).

Figure 5: Optimal volatility of inflation.

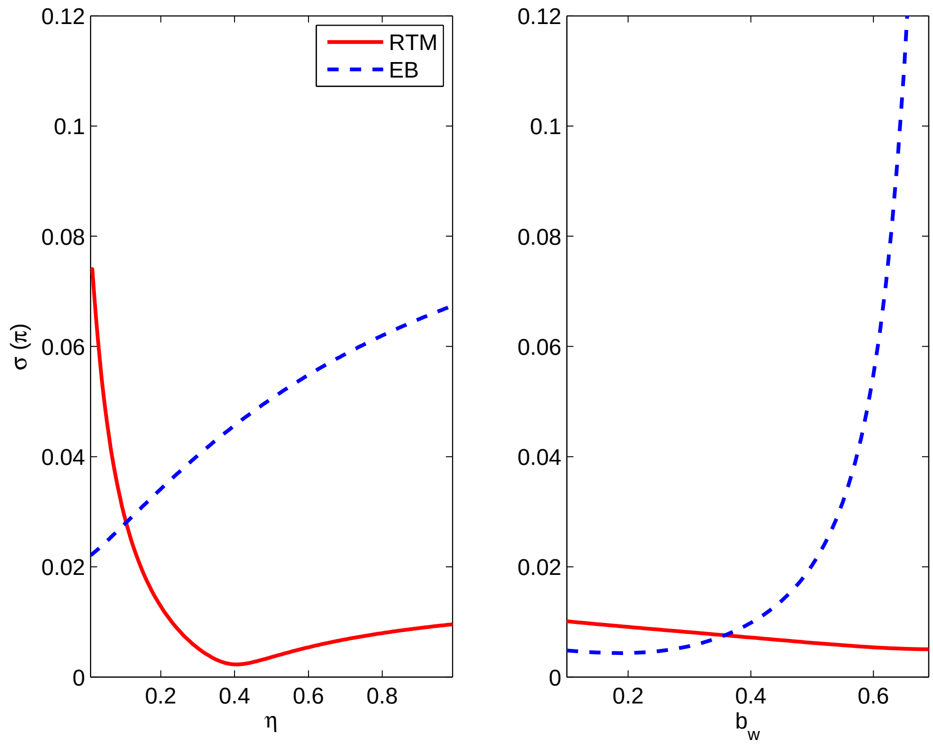


Figure 6: Impulse responses to a positive technology shock without real wage rigidity (with the efficient steady state).

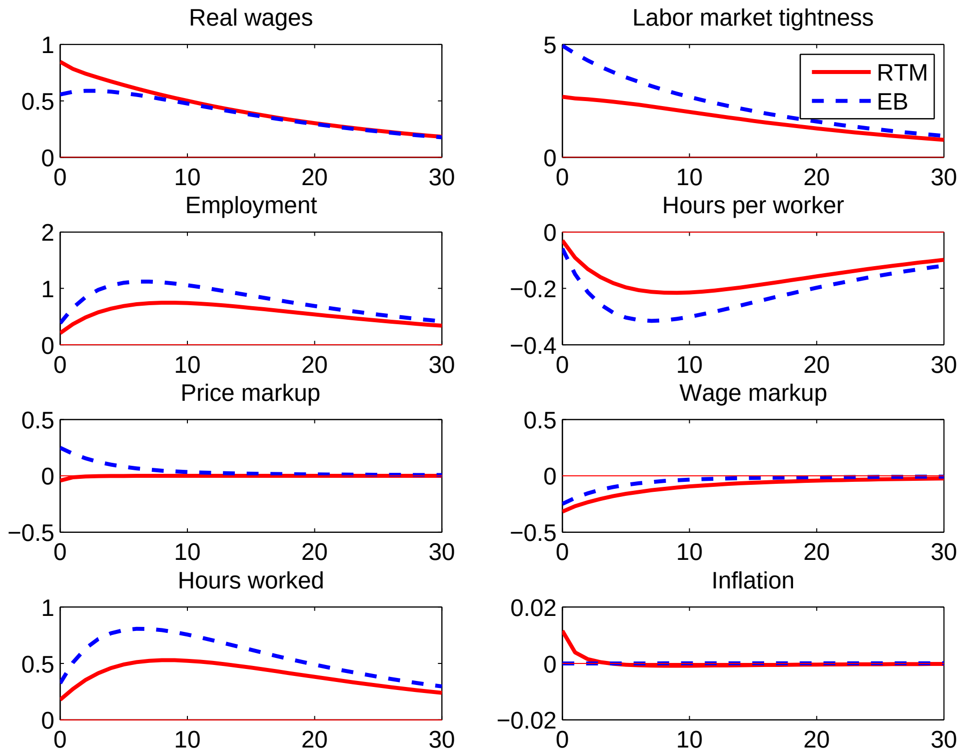


Figure 7: Composition of the inefficiency gap (with the efficient steady state).

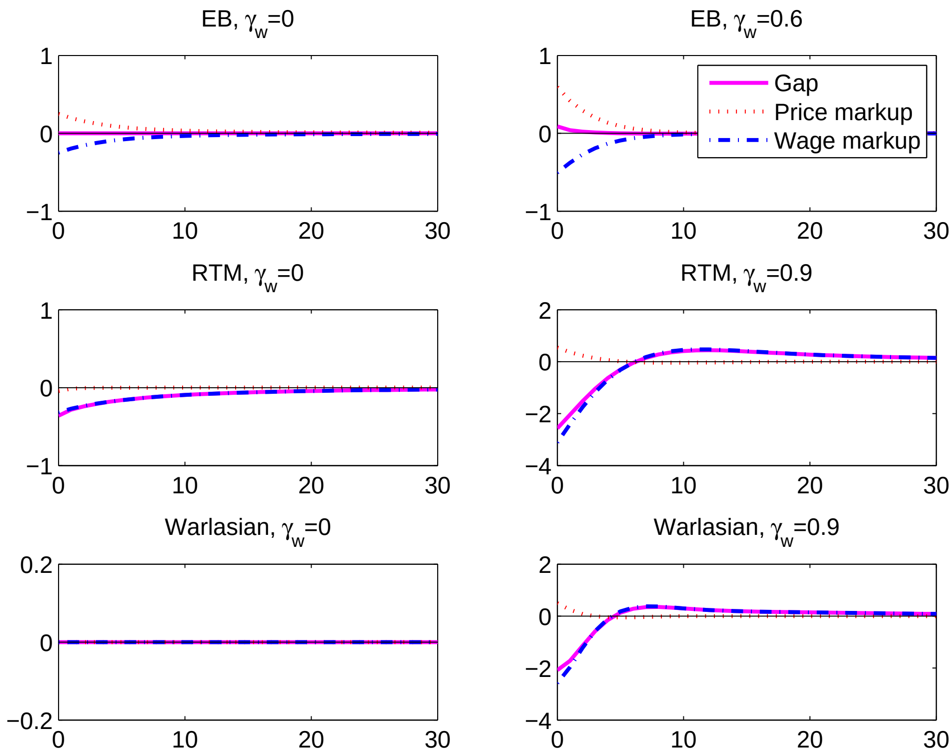
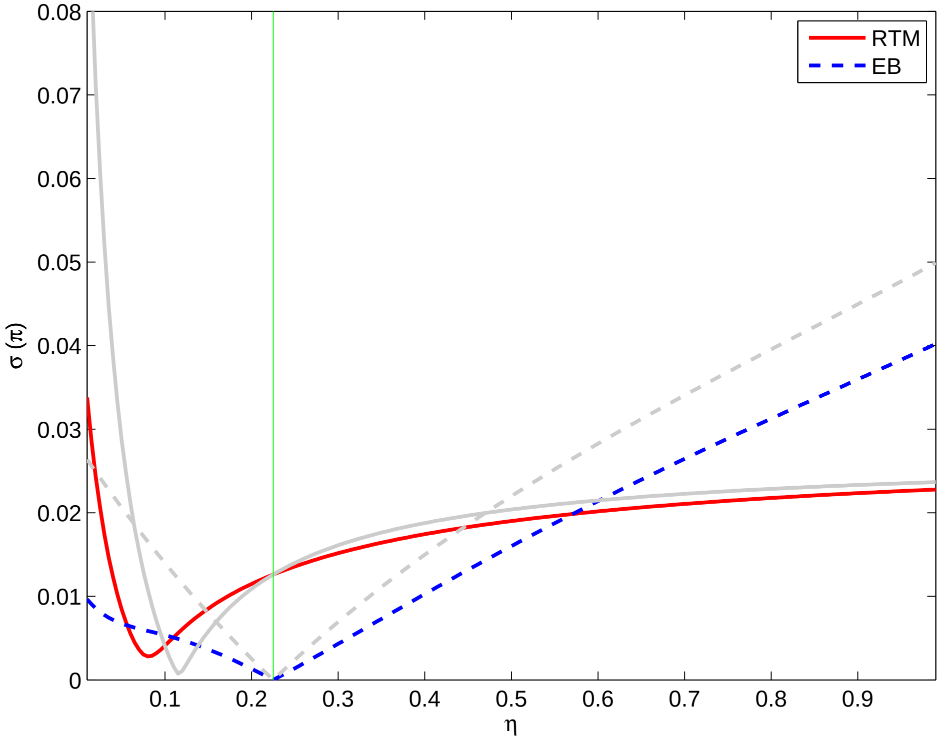
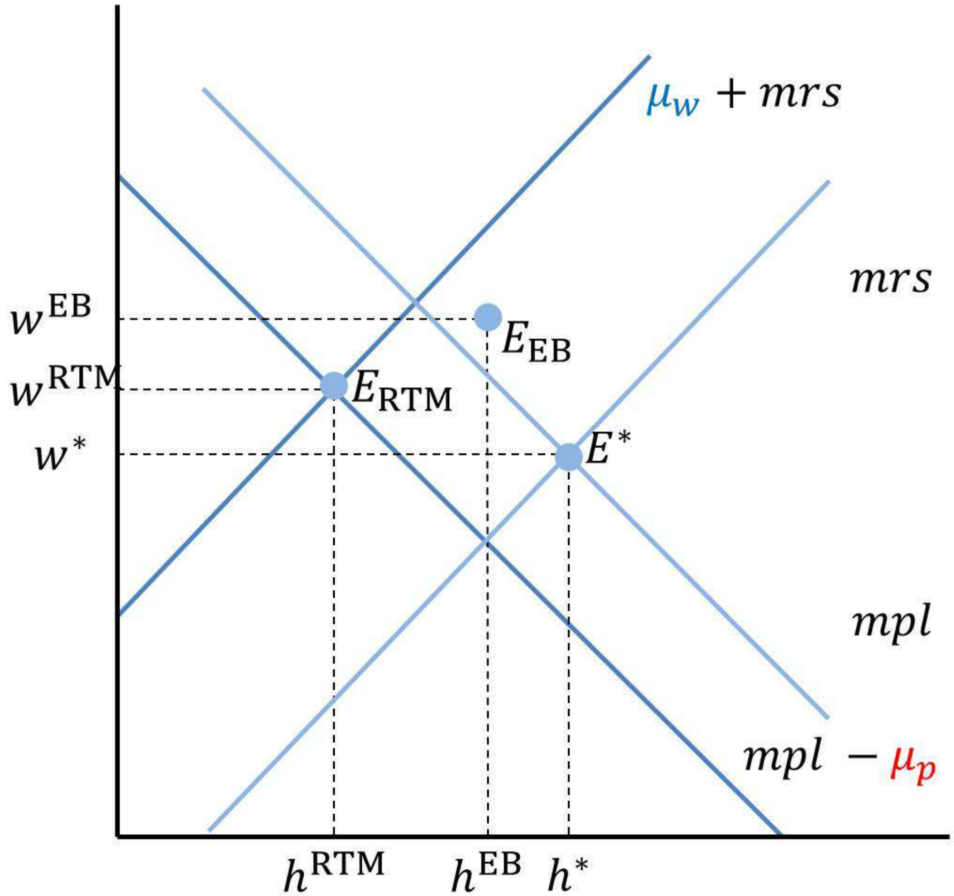


Figure 8: Optimal volatility of inflation (with the efficient steady state).



Notes: the vertical line shows $\xi = \eta^* = 0.225$ at which the steady state is efficient under both EB and RTM. Dark lines are the ones computed by the Lagrange method, whereas light lines are the ones computed by the LQ method.

Figure 9: Labor market allocation.



Notes: EB denotes allocation in efficient bargaining. RTM denotes allocation in right-to-manage bargaining. * denotes allocation at the social optimum.