On Two Notions of Imperfect Credibility in Optimal Monetary Policies *

Ippei Fujiwara† — *Keio University / ANU*

Timothy Kam‡ — *ANU*

Takeki Sunakawa§ — *Kobe University*

October 1, 2018

Abstract

We explore how outcomes of optimal monetary policy with *loose commitment* (Schaumburg and Tambalotti, 2007; Debortoli and Nunes, 2010) with the non-reoptimization probability of α can be interpretable as outcomes of deeper optimal policy under *sustainable plans* (Chari and Kehoe, 1990) with N-period punishment. In a standard monetary-policy framework, we show that, for any sufficiently high value of α, there exists an integer N such that impulse responses to the cost-push shock under each policy are similar to each other.

JEL Classification: E52; E58; E61

Keywords: imperfect credibility; monetary policy; sustainable policy

---

*Fujiwara gratefully acknowledges financial support from the Japanese Banker's Association Research Fund. First Draft: November 2015.

†Faculty of Economics, Keio University, Tokyo, Japan. Crawford School of Public Policy, Australian National University, Canberra, Australia. Email: ippei.fujiwara@keio.jp

‡Research School of Economics, Australian National University, Canberra, Australia. Email: tcy.kam@gmail.com

§Center for Social Systems Innovation, Kobe University, Japan. Email: takeki.sunakawa@gmail.com

1 Background and Problem Statement

As in the mainstream literature on optimal monetary policy, we define *imperfect credibility* generically as the imperfect ability of a monetary-policy plan to influence the private sector's belief about its continuation into the indefinite future. However, in the literature, this notion of imperfect credibility has taken on two alternative, and (structurally) very different, modelling interpretations. The first is the *limited commitment* approach to time-consistent policy design, and can be couched in terms of a *sustainable plans* problem (see, e.g., Chari and Kehoe, 1990; Kurozumi, 2008; Sunakawa, 2015). The second approach takes on the interpretation of a policy planner reoptimizing on its previously given policy plans (with exogenous probability): This is also known as *stochastic replanning* (Roberds, 1987), *quasi commitment* (Schaumburg and Tambalotti, 2007), or *loose commitment* (Debortoli and Nunes, 2010; Debortoli et al., 2014).[1]

Under quasi/loose commitment, a policy maker will renege on its commitment with a constant probability, say, $1 - \alpha$. When $\alpha = 1$ (or when $\alpha = 0$), such a policy collapses to the traditional commitment (or discretion) policy regime. Thus, it becomes possible to analyze "a continuum of monetary policy rules characterized by differing degrees of credibility" (see Schaumburg and Tambalotti, 2007).

In the quasi/loose commitment approach to imperfect credibility, the key friction to commitment, α, is a free parameter. Although computationally and quantitatively appealing, what could possibly be a theoretical justification for such a quasi/loose commitment? It has been conjectured that the sustainable plans approach to imperfect policy credibility may provide a deeper foundation for the more reduced-form quasi/loose commitment story. We make the quantitative connection between these two ideas. Specifically, we take up and explore the claims that the two approaches to imperfect policy credibility—quasi/loose commitment and sustainable plans—are "alternatives" (Schaumburg and Tambalotti, 2007) and "may share similarities" (Bodenstein et al., 2012), from a dynamic outcome or behavioral perspective. In particular, using the standard analytical framework for optimal monetary policy (Woodford, 2003), we compare impulse responses between quasi/loose commitment and the *optimal sustainable monetary policy* (Kurozumi, 2008; Sunakawa, 2015).

In the *sustainable plans* approach (Kurozumi, 2008; Sunakawa, 2015), a monetary policy plan (viz. central bank) is either sustainable (credible) or not. Whereas, in the language of quasi/loose commitment, a policy plan may be credible with some probability. To connect these two ideas, we consider tighter notions of the sustainability constraint (interpreted as simple penal codes), where there is some finite duration N of the punishment phase if a policy maker were to deviate from its original plan (Loisel, 2008; Nakata, 2018). We aim to understand how α, the degree of quasi/loose commitment, can or cannot be mapped back, and behaviorally equivalent to, the severity of the punishment threat, N.

We provide the following insights: First, even though quasi/loose commitment is based on

[1] See, for example, Debortoli and Nunes (2010) for fiscal policy, Debortoli and Nunes (2013) for the optimal level of debt, and Bodenstein et al. (2012) for the optimal monetary policy under the zero lower bound.

an *ad-hoc* assumption to rationalize a policy equilibrium between commitment and discretion, our study shows that it turns out to be a reasonably good approximation of a sustainable policy equilibrium behavior. In a standard calibration of the monetary-policy framework, we show that, for any sufficiently high value of α, there exists an integer N such that impulse responses to the cost-push shock under optimal sustainable policy with N-period punishment are similar to those under optimal policy with loose commitment with the non-reoptimization probability of α. In particular, we have that $\alpha = 0.82$ in the quasi/loose commitment world (the value estimated by Debortoli and Lakdawala, 2016) corresponds to $N = 12$ in the N-period punishment optimal sustainable monetary policy.

2 Model

We employ the standard framework for optimal monetary policy by Woodford (2003). The state variable is u_t, a cost-push shock realized at the beginning of date $t \in \mathbb{N}$.[2] Let $\mathbb{E}_t \equiv \mathbb{E}\left\{\cdot \mid h^t\right\}$ denote the expectation operator conditional on information at date t, as summarized by some t-history h^t. Denote $\pi_t \equiv \pi_{\tilde{\sigma}}\left(h^t\right)$ and $x_t \equiv x_{\tilde{\sigma}}\left(h^t\right)$, respectively, as the inflation-rate and the output-gap selections from a policy plan $\tilde{\sigma} = \left\{h^t \mapsto \tilde{\sigma}_t\left(h^t\right) : t \in \mathbb{N}\right\}$, where $h^t := (u_0, x_{-1}, ..., u_t, x_{t-1})$. The cost-push (markup) shock u_t is assumed to follow an AR(1) process:[3]

$$u_t = \rho u_{t-1} + e_t; \qquad e_t \sim N\left(0, \sigma_e^2\right). \tag{1}$$

Given policy plan $\tilde{\sigma}$, the total expected value to the planner (and society) at the initial state is $W_t \equiv W_{\tilde{\sigma}}\left(h^t\right)$.[4] The central bank's objective function is given by

$$W_t = -\frac{1}{2}\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}L\left(\pi_{t+\tau}, x_{t+\tau}\right)\right\}; \qquad L\left(\pi_t, x_t\right) = \left(\pi_t^2 + \lambda x_t^2\right), \tag{2}$$

where $\lambda = \kappa/\varepsilon$. For any fixed policy plan $\tilde{\sigma}$, the competitive equilibrium is sufficiently characterized by the Phillips curve constraint:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + u_t; \qquad \kappa := \frac{(1-\theta)(1-\beta\theta)(\sigma+\eta)}{\theta(1+\eta\varepsilon)}, \tag{3}$$

where κ is a function on underlying microeconomic taste and technology parameters: β, σ, η, ε and θ, respectively, denote the subjective discount factor, the inverse of the intertemporal elasticity of substitution, the inverse of Frisch elasticity, the elasticity of substitution among differentiated products, and the Calvo (1983) parameter.

[2]This shock can be interpreted a wedge between the flexible-price output and the efficient output. Any "efficient" shocks that appear in the consumption Euler equation (such as technology shocks) can be completely offset by controlling the nominal interest rate one-by-one in the absence of the interest-rate lower bound.

[3]When computing the optimal sustainable monetary policy, the AR(1) process is approximated by a finite-state Markov chain following Tauchen (1986).

[4]Woodford (2003) demonstrates how the flow criterion function L can be derived from a second-order accurate approximations of the representative household preference function and the competitive equilibrium conditions.

Timing of events and actions. The following timeline holds for all policy regimes we consider below.[5] At the beginning of each date $t \in \mathbb{N}$: (1) The shock u_t is realized. (2) The central bank chooses a policy plan $\tilde{\sigma}'$ (or continues with a previously promised plan $\tilde{\sigma}$). (3) Simultaneously with (2), measure-zero and homogeneous agents form rational expectations $\mathbb{E}_t \pi_{t+s}$, for all $s \geq 0$, consistent with the central bank's plan. (4) Current outcomes (π_t, x_t) are realized consistent with competitive equilibrium condition (3).

2.1 Standard policy regimes

Commitment equilibrium. The policy maker chooses a plan $\{\pi_t, x_t\}_{t \in \mathbb{N}}$ to maximize the objective function (2) subject to the functional equation (3). The optimal plan satisfies the condition

$$\pi_t = -\frac{1}{\varepsilon}\left(x_t - x_{t-1}\right). \tag{4}$$

Given a process for u_t, the Phillips curve functional equation (3), together with the targeting rule in equation (4), characterize the equilibrium under monetary policy *commitment*. Denote the value of an optimal commitment plan to the policy maker as $V^C\left(u_t, x_{t-1}\right)$, which additionally depends on an auxiliary state variable x_{t-1} since the commitment plan ties the policy maker's hands to its past promise.

Discretion equilibrium. The policy maker chooses (π_t, x_t) each period to minimize the per-period loss function $L\left(\pi_t, x_t\right)$ in (2), subject to the constraint (3). The optimal targeting rule under policy discretion can be derived as

$$\pi_t = -\frac{1}{\varepsilon} x_t. \tag{5}$$

Given a process for u_t, equation (3) together with the targeting rule in equation (5) characterize the (Markov perfect) equilibrium under monetary policy *discretion*. The total expected value to the policy maker at any state u_t can be calculated analytically as

$$V^D\left(u_t\right) = -\frac{1+\varepsilon\kappa}{\left(1-\beta\rho+\varepsilon\kappa\right)^2\left(1-\beta\rho^2\right)}\left(u_t^2 + \frac{\beta\sigma_e^2}{1-\beta}\right). \tag{6}$$

3 Imperfect credibility in monetary policy: Two stories

Now, we describe the two popular ways that have been taken to define environments with imperfect credibility in monetary-policy plans. Each has a completely different interpretation of the notion of imperfect credibility.

[5] This allows us to focus on just the "stabilization bias" problem due to time inconsistency later in the case of a sustainable plan equilibrium (see, e.g., Kurozumi, 2008, for more details).

3.1 Quasi/Loose Commitment

In this environment, we have a reduced-form notion of limited (or "loose") commitment indexed by a parameter, $\alpha \in [0,1]$. The parameter $1-\alpha$ measures the (common-knowledge) probability that the policy maker will re-optimize over its initial policy plan. Under quasi/loose commitment, the welfare maximization problem by the monetary authority is:

$$Q\left(u_t, x_{t-1}\right) = \max_{\pi_t, x_t} \left\{ -\frac{1}{2}\left(\pi_t^2 + \frac{\kappa}{\varepsilon}x_t^2\right), \right. \\ \left. +\beta\mathbb{E}_t\left[\alpha Q\left(u_{t+1}, x_t\right) + (1-\alpha) Q\left(u_{t+1}, 0\right)\right]\right\} \tag{7}$$

subject to:

$$\pi_t = \beta\mathbb{E}_t\left[(1-\alpha)\,\pi_{t+1} + \alpha\pi_{t+1}^R\right] + \kappa x_t + u_t, \tag{8}$$

where with probability $1-\alpha$, $\pi_{t+1}^R := \pi\left(u_{t+1}, 0\right)$ would be the new inflation selection supporting the continuation value $Q\left(u_{t+1}, 0\right)$. Observe in the functional equation (7), with probability $1-\alpha$, nature "resets" the optimal planning problem to begin from a zero output gap state, to deliver a continuation value of $Q\left(u_{t+1}, 0\right)$, regardless of history. This is interpreted in the literature as the planner having loose commitment.

3.2 Sustainable Plans Policy Regime

Consider now the *sustainable plans equilibrium* (see Chari and Kehoe, 1990; Kurozumi, 2008; Sunakawa, 2015). Suppose that a planner deviates from a given plan that induces total expected welfare W_t. We assume that a deviation from a precommitted plan is punished by the (Markov-perfect) discretion equilibrium, which delivers the total expected payoff $V^D\left(u_t\right)$.[6] In this setting, the constrained-efficient planner maximizes the objective (2) subject to the Phillips curve constraint (3), and the sustainability constraint:

$$V^S\left(u_t, x_{t-1}\right) \geq V^D\left(u_t\right), \tag{9}$$

for all u_t, where $V^S\left(u_t, x_{t-1}\right)$ is the value of the optimal sustainable plan at the given state x_{t-1}. Condition (9) encodes the requirement that social welfare under the optimal sustainable monetary policy to be at least as high as that under a deviation to the discretion equilibrium, $V^D\left(u_t\right)$.

So that we can compare between these two stories of imperfect credibility (in Section 4), we will consider tighter versions of the sustainability constraint (9). That is, instead of a threat of reversion to forever discretion, consider the case that in an event of deviation from an initial plan, the continuation to a discretion equilibrium will only last for N periods, and this is public information. In this setting, the RHS of the sustainability constraint (9) will be modified by a nonstationary value of discretion, $V_0^D\left(u_t\right)$, where the nonstationarity arises from the finite-

[6] Kurozumi (2008) suggests that the discretion equilibrium is the worst sustainable equilibrium in this model.

duration of punishment. We describe how this problem is characterized in our online appendix.

4 Comparison

We use a standard calibration of the NK model to discipline the comparison exercise. This is summarized in Table 1.

Table 1: Parameter Values

Parameters	Values	Explanation
β	.99	Subjective discount factor
σ	1	Inverse of intertemporal elasticity of substitution
η	1	Inverse of Frisch elasticity
ε	6	Elasticity of substitution, differentiated products
θ	.75	Calvo parameter
ρ	0	Shock persistence
σ_e	0.77	Standard deviation of shock

Consider the dynamics of inflation and output gap induced by the policy plan under an assumed commitment regime versus under a discretion regime. These regimes, respectively, are given by the solid green line and the dashed red line in both panels of Figure 1. Given a positive markup shock, we observe the well-known insight that the response of both variables under the commitment regime is less aggressive and is more gradual than that under the discretion equilibrium.[7]

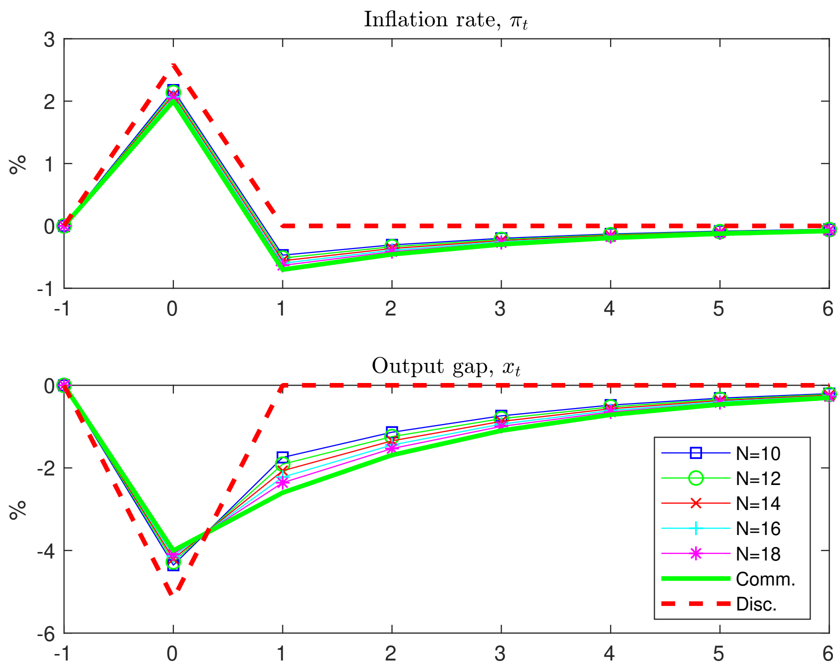

Figure 1: Sustainable plans (various N) vs. Commitment vs. Discretion equilibria.

Next consider the dynamics under sustainable plans equilibria for various lengths of the

[7]We feed shocks that make the sustainability constraint relevant. With small shocks, the sustainability constraint doesn't bind and the sustainable plan responses are the same as the ones under commitment.

punishment phase, N. What is interesting to observe is that for N large enough, the responses of inflation and output gap become close to those under the assumed commitment equilibrium. The intuition is as follows: For N sufficiently large, the value from deviating from a given sustainable plan becomes lower—i.e., deviating from commitment is not so attractive to the policy maker—since the threat of the punishment phase is prolonged.

Now consider the regime of loose commitment in Figure 2. As we increase (lower) the

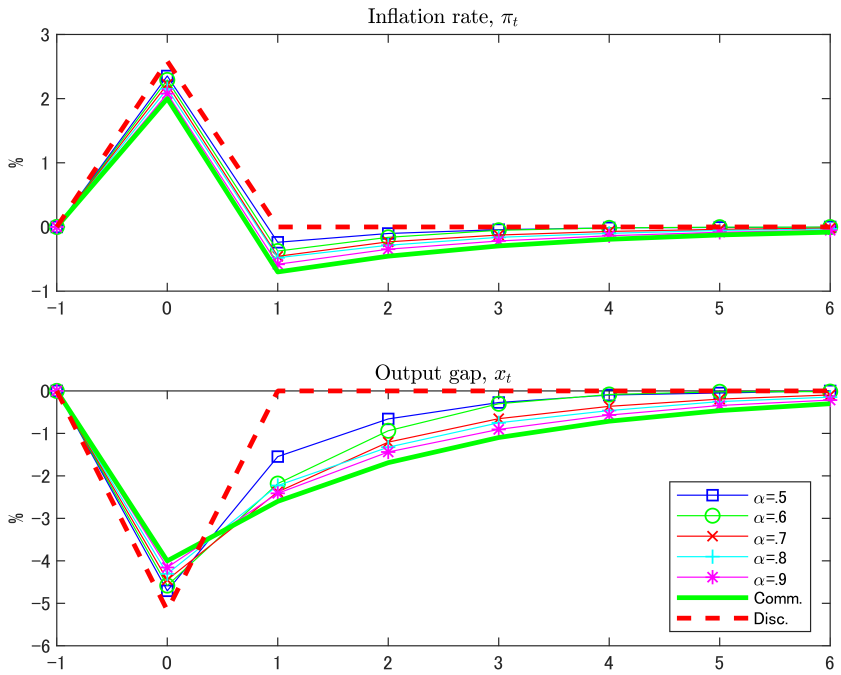

Figure 2: Loose Commitment (various α) vs. Commitment vs. Discretion equilibria.

parameter α toward unity (zero), the loose-commitment equilibrium outcomes for inflation and output gap approach that of the standard commitment (discretion) regime's. In fact, if we have either $\alpha = 1$ or $\alpha = 0$, the loose commitment solution is exactly that of the respective commitment and discretion regime's outcomes.

Debortoli and Lakdawala (2016) found that the posterior mean (mode) of their estimate of the equivalent of α is 0.82 (0.81). We find that $N = 12$ for the sustainable plans economy suffices to replicate the dynamics of inflation and output gap of the equilibrium under loose commiment with $\alpha = 0.82$.[8] We plot these in Figure 3.

[8]By searching across different values of $\alpha \in [0, 1]$, we solve each corresponding loose commitment equilibrium, and obtain each loose-commitment equilibrium's impulse response functions (for inflation and output gap) as a function of α. Denote these impulse response functions at the peak (i.e., Period 0) as $IRF_0(\alpha)$. For a fixed N in the optimal sustainable plan economy, we also have its corresponding impulse response functions $IRF_0(N)$. Then $N = 12$ minimizes the simple average of the ℓ_1-norm, $\sum_{j \in \{\pi, x\}} \left| IRF_{0,j}(\alpha) - IRF_{0,j}(N) \right|$, where $\alpha = 0.82$.

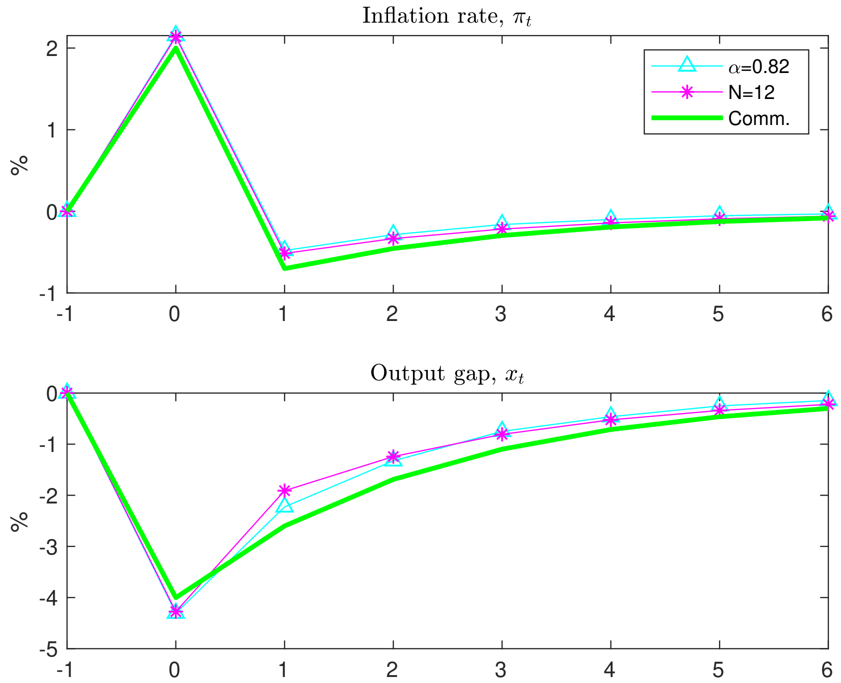

Figure 3: Loose Commitment ($\alpha = 0.82$) vs. Commitment vs. Sustainable Plan ($N = 12$).

More generally, we can consider the set of loose-commitment regimes, indexed by α, that most closely match with a corresponding N-period-punishent sustainable-plan equilibrium (given the lowest feasible setting of $N = 10$ in terms of existence of a sustainable-plan equilibrium), in terms of their impulse response statistics. While we cannot make a direct claim to the empirical validity of our result here, we can still argue that, informally, the set of observational equivalence between the two notions of imperfect credibility, as indexed by the (N, α) locus in Figure 4, involves quantitatively plausible values of α when shocks are large.

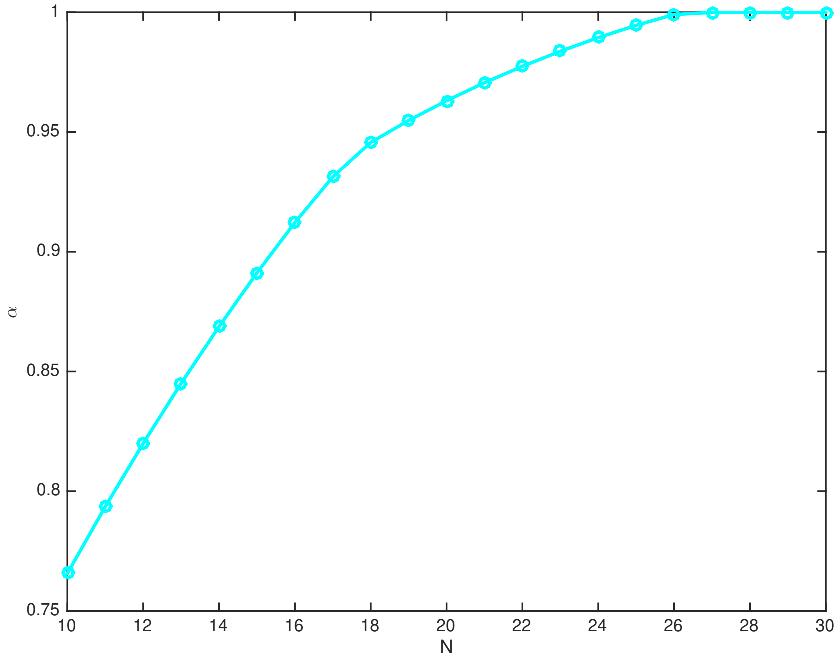

Figure 4: Loose Commitment (α) vs. Sustainable Plan (N): Matching impulse response dynamics.

5 Discussion

In a well-known monetary policy framework, we showed that two notions of *imperfect credibility* in monetary policy design—*loose commitment* versus *sustainable plans*—have similar impulse responses of inflation and output gap to the cost-push shock.

However, the caution is needed to interpret our results as the two notions of imperfect credibility have very distinctive economic interpretations. Moreover, they imply different welfare conclusions since the policymaker's objectives are clearly very different. This type of comparison is also model-specific, but we expect the similarity result is likely to hold in other models, for example, models with the zero lower bound on nominal interest rates.[9]

References

Bodenstein, Martin, James Hebden, and Ricardo Nunes, "Imperfect credibility and the zero lower bound," *Journal of Monetary Economics*, 2012, *59* (2), 135–149.

Calvo, Guillermo A., "Staggered Prices in A Utility-Maximizing Framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

Chari, V V and Patrick J Kehoe, "Sustainable Plans," *Journal of Political Economy*, August 1990, *98* (4), 783–802.

Debortoli, Davide and Aeimit Lakdawala, "How Credible Is the Federal Reserve? A Structural Estimation of Policy Re-optimizations," *American Economic Journal: Macroeconomics*, July 2016, *8* (3), 42–76.

[9]The discretion equilibrium may not be the worst sustainable equilibrium, e.g., see Dong (2018). In such a case, we may consider a revert-to-discretion sustainable plan as in Nakata (2018), which is presumably not the best sustainable equilibrium.

— **and Ricardo Nunes**, "Fiscal policy under loose commitment," *Journal of Economic Theory*, May 2010, *145* (3), 1005–1032.

— **and** — , "Lack Of Commitment And The Level Of Debt," *Journal of the European Economic Association*, October 2013, *11* (5), 1053–1078.

— **, Junior Maih, and Ricardo Nunes**, "Loose Commitment In Medium-Scale Macroeconomic Models: Theory And Applications," *Macroeconomic Dynamics*, January 2014, *18* (01), 175–198.

Dong, Bingbing, "Forward Guidance and Credible Monetary Policy," 2018. SSRN Working Paper.

Fujiwara, Ippei, Timothy Kam, and Takeki Sunakawa, "Sustainable international monetary policy cooperation," CAMA Working Paper 14/2016, Crawford School of Public Policy, Australian National University 2016.

Kehoe, Patrick J. and Fabrizio Perri, "International Business Cycles with Endogenous Incomplete Markets," *Econometrica*, May 2002, *70* (3), 907–928.

Kurozumi, Takushi, "Optimal sustainable monetary policy," *Journal of Monetary Economics*, October 2008, *55* (7), 1277–1289.

Loisel, Olivier, "Central bank reputation in a forward-looking model," *Journal of Economic Dynamics and Control*, November 2008, *32* (11), 3718–3742.

Nakata, Taisuke, "Reputation and Liquidity Traps," *Review of Economic Dynamics*, April 2018, *28*, 252–268.

Roberds, William, "Models of Policy under Stochastic Replanning," *International Economic Review*, October 1987, *28* (3), 731–55.

Schaumburg, Ernst and Andrea Tambalotti, "An investigation of the gains from commitment in monetary policy," *Journal of Monetary Economics*, March 2007, *54* (2), 302–324.

Sunakawa, Takeki, "A quantitative analysis of optimal sustainable monetary policies," *Journal of Economic Dynamics and Control*, 2015, *52* (C), 119–135.

Tauchen, George, "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.

Supplementary (Online) Appendix

A Sustainable plan with N-period punishment

We explain how the value function $u \mapsto V_0^D(u)$, discussed in section 3.2, is determined here. Note that this function does not depend on a current date t per se, but on the stage $n \in \{0, 1, ..., N-1\}$ *relative to any date* t where the policy deviation occured. Thus V_0^D, means the total expected value of reversion to an N-period punishment phase, at the beginning of that phase ($n = 0$). Once we have this function, we have the outside option value for the constrained-efficient planner in the description of an N-period punishment sustainable plan.

A.1 Determining V_0^D

Below, we denote a nonstationary value function at punishment stage n as $V_n^D(\cdot)$, where $n \in \{0, 1, ..., N\}$. The timing of events is as follows:

- Suppose the policy maker deviates from the current policy plan at date t.
- Private agents observe it and punish the policymaker for $t, ..., t+N-1$.
- The policymaker is allowed to commit to a new plan in $t+N$; i.e., $x_{t+N-1} = 0$ in Period $t+N$.

A.1.1 Backward induction

Given $u \equiv u_t$, $V_N^D(u) = V^S(u, 0)$ and $\pi_N^D(u) = \pi(u, 0)$, the following backward induction is used to obtain $V_0^D(u)$.

1. In period $t+N-1$, at the penultimate punishment state $n = N-1$, solve

$$
\begin{aligned}
V_{N-1}^D(u) &= \max_{\pi, x} \left\{ -\pi^2 - \lambda x^2 + \beta \sum_{u'} p(u'|u) V_N^D(u') \right\}, \\
\text{s.t.} \quad \pi &= \kappa x + u + \beta \sum_{u'} p(u'|u) \pi_N^D(u).
\end{aligned}
$$

   for $\pi_{N-1}^D(u)$ and $V_{N-1}^D(u)$, where p is the Markov matrix for the stochastic process of u.

2. Repeat this for $n = N-2, ..., 0$ to obtain $V_{N-2}^D(u), ..., V_0^D(u)$.

A.1.2 Iterative procedure

$V_0^D(u)$ depends on $V^S(u, x_{-1})$, which in turn, depends on $V_0^D(u)$. We will denote $V^{S,(i)}$ as a *candidate guess of a (sustainable-equilibrium) value function after exiting an N-period punishment phase*. Note that in a sustainable equilibrium, it would be that $\lim_{i\to\infty} V^{S,(i)} = V^S$. The idea is that, if we have found V^S, then we would also know V_0^D.

To find the fixed point in terms of the functions V^S, we iterate on the following steps:

1. Set an initial guess for the value function $V^{S,(0)}$ (after the final punishment stage) as the value under a commitment equilibrium:

$$V^{S,(0)}\left(u, x_{-1}\right) \leftarrow V^C\left(u, x_{-1}\right);$$

   and, get its corresponding policy as that under a commitment equilibrium:

$$\pi^{(0)}\left(u, x_{-1}\right) \leftarrow \pi^C\left(u, x_{-1}\right),$$

   for all (u, x_{-1}). This is a good, but arbitrary, initial guess.

2. For each iteration $i \geq 0$, set

$$V_N^{D,(i)}(u) \leftarrow V^{S,(i)}\left(u, 0\right)$$

   and set policy

$$\pi_N^{D,(i)}(u) \leftarrow \pi^{(i)}\left(u, 0\right)$$

   for all u.

3. Given function $V_N^{D,(i)}$, solve by backward induction (see section (A.1.1) above), for $V_0^{D,(i)}$ (the candidate approximant for the equilibrium V_0^D).

4. Given function $V_0^{D,(i)}$, solve for a candidate pair of sustainable equilibrium value and policy functions using the recursions defined in section (A.2) below. We get updated guesses: $V^{S,(i+1)}(u, x_{-1})$ and $\pi^{(i+1)}(u, x_{-1})$ for all (u, x_{-1}).

5. Repeat Steps 2-4 until the sequence of function approximants converge: $V^{S,(i)} \rightarrow V^S$ and $\pi^{(i)} \rightarrow \pi$.

A.2 Characterizing the optimal sustainable plan

The relevant state variables are $s := (u, x_{-1})$. Define a record-keeping function $z := \Psi_{t-1}/(\Psi_{t-1} + \psi_t)$, where $\Psi_t = \sum_{s=0}^{t} \psi_t$ and $\psi_t \equiv \psi(u_t, x_{t-1})$ is a Lagrange multiplier or gradient function on the sustainability constraints. This sufficiently encodes history dependence in the constrained-efficient optimal sustainable plan, and in a recursive way.[10] A recursive characterization of the

[10]Intuitively, if $z(s) = 1$ almost everywhere, and the sustainability constraints are never binding, then we have the traditional commitment regime. If $N \rightarrow 1$, $z(s) = 0$, and the sustainability constraint is always binding, then we have the literature's notion of a discretion (Markov perfect) equilibrium regime.

optimal sustainable policy plan is a list of policy functions $s \mapsto (\pi, x, z)(s)$, a value function V^S, such that:

$$
\begin{aligned}
V^S(s) &= -[\pi(s)]^2 - \lambda [x(s)]^2 + \beta \sum_{u'} p(u'|u) V^S [u', x(s)]\,; \\
\pi(s) &= -\frac{\lambda}{\kappa} [x(s) - z(s) x_{-1}]\,; \\
\pi(s) &= \kappa x(s) + u + \beta \sum_{u'} p(u'|u) \pi([u', x(s)]\,; \\
V^S(s) &\geq V^D(u)\,;
\end{aligned}
$$

This system defines a recursive operator that allows us to solve for $s \mapsto (\pi, x, z)(s)$.

A.3 Details on computing the optimal sustainable plan

To compute the policy function in the sustainable plans, a version of the policy function iteration method with occasionally binding constraints is used, as in Kehoe and Perri (2002); Sunakawa (2015); Fujiwara et al. (2016). Let $s = (u, x_{-1}) \in U \times X$ where U and X are closed sets. We discretize the elements in U and X into grid points $(u_j, x_{-1,k})$ indexed by a pair of integers (j, k) for $j = 1, ..., n_u$ and $k = 1, ..., n_x$ where n_u and n_x are the number of grid points for each. We denote V^S as V by omitting the superscript. The algorithm is as follows:

1. Set an initial guess for the functions $V^{(0)}(s)$ and $\pi^{(0)}(s)$.
2. Taking the functions $V^{(i-1)}(s')$ and $\pi^{(i-1)}(s')$ and the values of $V^D(u_j)$ as given, solve the relevant equations for the values $(V_{jk}, \pi_{jk}, x_{jk}, z_{jk})$ at each grid point.
3. Update the functions by setting functional values $V^{(i)}(s) = V_{jk}$ and $\pi^{(i)}(s) = \pi_{jk}$ at each grid point.
4. Iterate 2-3 until $\left\| V^{(i)}(s) - V^{(i-1)}(s) \right\|$ and $\left\| \pi^{(i)}(s) - \pi^{(i-1)}(s) \right\|$ are small enough.

In Step 2, there are two possible binding patterns of the sustainable constraint: (i) the constraint is slack or (ii) the constraint binds.

(i) $z_{jk} = 1$. Then, we can solve

$$
\begin{aligned}
V_{jk} &= -([\pi_{jk}]^2 + \lambda [x_{jk}]^2) + \beta \sum_{u'} p(u'|u_j) V^{(i-1)}(u', x_{jk}), \\
x_{jk} &= -\frac{\kappa}{\lambda} \pi_{jk} + x_{-1,k}, \\
\pi_{jk} &= \kappa x_{jk} + u_j + \beta \sum_{u'} p(u'|u_j) \pi^{(i-1)}(u', x_{jk}),
\end{aligned}
$$

for the values of $(x_{jk}, \pi_{jk}, V_{jk})$.

(ii) $z_{jk} \in (0,1)$ and $V_{jk} = V^D(u_j)$. We can solve

$$\begin{aligned}
V^D(u_j) &= -([\pi_{jk}]^2 + \lambda[x_{jk}]^2) + \beta \sum_{u'} p(u'|u_j) V^{(i-1)}(u', x_{jk}), \\
x_{jk} &= -\frac{\kappa}{\lambda}\pi_{jk} + z_{jk} x_{-1,k}, \\
\pi_{jk} &= \kappa x_{jk} + u_j + \beta \sum_{u'} p(u'|u_j) \pi^{(i-1)}(u', x_{jk}),
\end{aligned}$$

for the values of $(x_{jk}, \pi_{jk}, z_{jk})$.

x_{jk} may not be on the grid points, therefore, the functions need to be approximated. Spline interpolation is used between the grid points and linear outerpolation is used outside of X.[11] More specifically, the conditional expectations are approximated as $\hat{V}^e(x; u_j, \boldsymbol{\theta}_V) \approx V^e(x; u_j) \equiv \sum_{u'} p(u'|u_j) V(u', x)$ and $\hat{\pi}^e(x; u_j, \boldsymbol{\theta}_\pi) \approx \pi^e(x; u_j) \equiv \sum_{u'} p(u'|u_j)\pi(u', x)$ for each $j = 1, ..., n_u$ by piecewise cubic splines, where $\boldsymbol{\theta}_V$ and $\boldsymbol{\theta}_\pi$ are vectors of the spline coefficients.

The upper (lower) bound of the disturbance is $m\sigma_\varepsilon$ (-$m\sigma_\varepsilon$) and $m = 6$ is set.[12] Maximum and minimum values of X are chosen to be consistent with the optimal volatility of output gap under the optimal commitment policy. The number of grids is $n_u = 31$ for U and $n_x = 15$ for X. It is assumed that ε_t follows a normal distribution, and Tauchen's (1986) method is used to approximate AR(1) process equation (1) with parameters $(\rho, \sigma_\varepsilon, m)$ by the set of grid points U and a Markov chain $p(u'|u)$.

[11] It may be justified by observing that the policy function in the optimal commitment policy is linear, which is obtained by solving the problem without the sustainability constraint.

[12] This is larger than $m = 3$, the value commonly used in the literature.