Sustainable Pricing in a Durable Goods Monopoly*

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Abstract

Ausubel and Deneckere [1989. Reputation in bargaining and durable goods monopoly. Econometrica 57, 511-531] showed that the monopoly profit is approximately sustainable when agents are patient enough and the length of time is small enough. This paper considers the case in which these conditions do not hold, and examines the short-run dynamics, which are, to my knowledge, neglected in previous studies. This paper finds that the sustainable pricing is intermediate between the monopolistic pricing and the Markov-perfect pricing, and the monopolist quotes the price between the monopolistic price and marginal costs for every period until the market is satiated. At a low discount factor, the sustainable pricing converges to marginal costs, but very slowly. At a sufficiently high discount factor, the speed of convergence becomes arbitrarily slow, which is consistent with Ausubel and Deneckere's result. This paper also considers the effect of introducing depreciation and stochastic marginal cost into the model.

Keywords: Durable goods monopoly; reputation; policy function iteration method

JEL codes: C63; D21; D42

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1 Introduction

Let's consider a monopolist which offers prices of infinitely durable goods to an infinite mass of consumers over time. Assume that in period 0, the monopolist quotes the monopolistic price with markup over marginal costs, and sell goods to consumers less than the amount at which goods are sold in perfect competition. Also assume that after period 1, no transaction occurs. This pricing strategy is *not* time-consistent; the monopolist has an incentive to offer lower prices than the precommitted price, and sell more goods after period 1. The monopolist continues to have such an incentive until the market is satiated. Also, consumers form expectations that the prices will be lower, and postpone their purchases in period 0.

Based on this consideration, Ronald Coase (1972) conjectured that the monopolist quotes the price equal to the competitive price (i.e., marginal costs) for every period. This is known as the Coase conjecture; the monopolist does not have monopolistic power. Economists have tried to find out how reasonable this conjecture is. Stokey (1981) showed that there is a unique time-consistent Markov-perfect equilibrium. As the length of time between each timing of price setting approaches zero, the monopolist immediately satiates the market at the competitive price. Stokey supported the Coase conjecture in the Markov-perfect equilibrium in a continuous-time setting. On the contrary, Ausubel and Deneckere (1989, hereafter AD) considered the subgame-perfect equilibrium (SPE), and found that the monopolistic price is "approximately" supported as the best SPE if the agents (both the monopolist and consumers) are patient enough and the length of time is small enough. If the monopolist deviates from the optimal strategy, the economy switches to the Markov-perfect equilibrium as defined in Stokey (1981) forever. The monopolist's reputation in a long-run relationship with consumers makes it possible to sell goods at higher prices. AD showed that the Coase conjecture does not necessarily hold even in a durable goods market; "a monopoly *is* a monopoly (AD, p. 512)."

However, in the case that the agents are not patient enough, and building a long-run relationship is difficult, it is not clear what kind of pricing strategy is optimal. Also, the short-run dynamics of prices and quantities in the best SPE, which are, to my knowledge, neglected in previous studies, are of natural interest because the nature of the problem is dynamic.

In this paper, I solve the monopolist's problem utilizing the idea of Chari and Kehoe's (1990) Sustainable Plan, which has been used in macroeconomics to solve time-inconsistency problems. I also examine the short-run dynamics by a version of policy function iteration methods with participation constraints (Kehoe and Perri, 2002).

There are three main findings. First, the sustainable pricing is intermediate between the monopolistic pricing and the Markov-perfect pricing. The monopolist quotes the price between the monopolistic price and marginal costs for every period until the market is satiated. Second, at a low discount factor β, the sustainable pricing converges to marginal costs, but very slowly; the monopolist offers lower prices over time and continues to sell until the market is satiated. Reputation has the effect like an adjustment cost. Third, at a sufficiently high discount factor β, the speed of convergence becomes arbitrarily slow, which is consistent with AD's result.

Chari and Kehoe's (1990) seminal paper proposed a concept of sustainable plans, which is optimal for the strategic player's dynamic decision making when the strategic player cannot commit to future plans. The equilibrium concept is very close to subgame perfect equilibria in the theory of repeated games between a strategic player and an infinite mass of small agents, and the authors characterized the sustainable equilibrium using game theory; they used Abreu's (1988) technique of using the worst sustainable equilibrium to characterize the entire set of the sustainable equilibrium. The equilibrium condition is summarized into an inequality, which is called the sustainability constraint.

In time-inconsistency problems, computations are often difficult because I have to deal with dynamic incentive constraints with which the Bellman's principle of optimality does not hold. Marcet and Marimon (1998; 2011) developed the recursive saddle point method to consider such dynamic incentive constraints. Kehoe and Perri (2002) applied the method for a two-country model with endogenous incomplete markets. They used a version of policy function iteration methods to solve for the policy function, which is also used in this paper. It is closely related to the method dealing with occasionally binding constraints (e.g., Christiano and Fisher, 2000), because the incentive constraints are occasionally binding.

The rest of paper proceeds as follows. In the next section, I explain the basic setup and each pricing strategy in turn. In Section 3, I show the numerical result. Section 4 includes some extensions: depreciation and shocks to marginal costs. Finally Section 5 concludes. Some proofs and the details of computational method are shown in the appendix.

2 Setup and Three Pricing Strategies

I consider three pricing strategies in turn; the monopolistic pricing, the Markov-perfect pricing (Stokey, 1981), and the sustainable pricing (Ausubel and Deneckere, 1989). The setup is common among these different pricing strategies.

Time is discrete and infinite; $t = 0, 1, 2, ..., \infty$. The monopolist produces infinitely durable goods and sell them to a continuum of identical consumers. Consumers can resell goods in the future, or a rental market is available. The dynamic demand equation is given by

$$p_t = f(q_t) + \beta p_{t+1}, \quad \forall t \geq 0. \tag{1}$$

where $f(0) > 0$ and $f'(q) < 0$. Durable goods are stock, and the price at period t is the discounted sum of future resale values. The monopolist chooses the sequence of $\{p_t\}_{t=0}^{\infty}$ (and hence $\{q_t\}_{t=0}^{\infty}$ given (1)) so as to maximize its discounted sum of future profits

$$R_0 \equiv \sum_{t=0}^{\infty} \beta^t (p_t - c)(q_t - q_{t-1}), \tag{2}$$

subject to (1).

2.1 Monopolistic Pricing

I begin with the situation that the monopolist can commit to future prices. I call it the monopolistic pricing, because the monopolist quotes the price associated with the quantity determined at the level with which the marginal cost is equal to the marginal revenue in every period. The monopolistic pricing is optimal in terms of the monopolist's profit in period 0. However, the monopolistic pricing is not time-consistent without such commitment technologies. The monopolist has an incentive to deviate from the original plan after period 1.

Let λ_t be the Lagrange multiplier associated with (1). If the monopolist can commit to future prices and quantities, the first-order necessary conditions (FONCs) of the maximization problem of (2) subject to (1) are

$$q_t - q_{t-1} = \lambda_t - \lambda_{t-1}, \tag{3}$$

$$p_t - c - \beta(p_{t+1} - c) + \lambda_t f'(q_t) = 0, \tag{4}$$

$$p_t - \beta p_{t+1} - f(q_t) = 0. \tag{5}$$

There are two state variables in the dynamic system. One is the amount of goods sold until the end of period $t-1$, q_{t-1}, and the other is the Lagrange multiplier in period $t-1$, λ_{t-1}, which shows the monopolist's commitment in the past. It is natural to assume that no goods are sold and no commitment is made before period 0. Given the initial conditions $q_{-1} = \lambda_{-1} = 0$, by solving equation (5) backward, I have $q_t = \lambda_t$ for all $t \geq 0$. A positive amount of goods sold also implies the monopolist's commitment in the past, because the monopolist has to commit to a particular level of prices to sell their goods to consumers who otherwise postpone their purchase. Equations (4) and (5) together with $q_t = \lambda_t$ become

$$(1-\beta)c = f(q_t) + q_t f'(q_t), \tag{6}$$

$$p_t = \sum_{i=0}^{\infty} \beta^i f(q_{t+i}), \tag{7}$$

where equation (6) shows that "average" marginal cost $\bar{c} = (1-\beta)c$ ($c = \bar{c} + \beta\bar{c} + ...$) is equal to the marginal revenue $f(q_t) + q_t f'(q_t)$ in periods $t \geq 0$. Equation (6) determines the amount of goods sold, and equation (7) sets prices. Because $q_t = q^m$ is constant by equation (6), the monopolist sets the monopolistic price $p_t = p^m$ constant over time by equation (7), and all transactions occur in period 0. The monopolist sells the amount of goods $q_0 = q^m$ and earn all profits $R_0 = R_0^m$ in period 0, and no profits after that, $R_t = 0$ for all $t \geq 1$.

This pricing strategy is, however, *not* time-consistent. If the monopolist can reset prices in period 1, the monopolist also can reset commitment made in the past, by setting $\lambda_0 = 0$. Given $q_0 > 0$ and $\lambda_0 = 0$, I have $q_t - q_0 = \lambda_t$ for all $t \geq 1$. Then

$$(1-\beta)c = f(\tilde{q}_t) + \tilde{q}_t f'(\tilde{q}_t) - q_0 f'(\tilde{q}_t) \tag{8}$$

holds. Comparing equations (6) and (8), the additional term $-q_0 f'(\tilde{q}_t) > 0$ shifts the marginal revenue curve toward right. This yields $\tilde{p}_t < p_t$ and $\tilde{q}_t > q_t$ for all $t \geq 1$, and $\tilde{R}_1 = (\tilde{p}_1 - c)(\tilde{q}_1 -$

$q_0) > 0$; the monopolist has an incentive to quote lower prices than the prices committed in the past, sell more goods, and earn positive profits in period 1 by such a deviation. The new plan $\{\tilde{p}_t, \tilde{q}_t\}_{t=1}^{\infty}$ is not consistent with the original plan $\{p_t, q_t\}_{t=0}^{\infty}$.

Figure 1 also shows that the monopolist continues to have such an incentive until the market is satiated. An "irresponsible" monopolist, which abandons the price committed in the past in every period, quotes lower prices in every period to sell more goods. The curve of the marginal revenue in equation (8) continues to shift toward right, and given a constant marginal cost, the amount of goods sold until the end of period t gets higher, until the market is satiated at $q = q^c$ with which the price is equal to the marginal cost.

2.1.1 Analytical solutions in the LQ framework

As in the most of previous studies, I assume a linear demand function of $f(q) = a - bq$ and a quadratic profit function so that the problem is one in the linear-quadratic (LQ) framework. Then I can analytically solve for the monopolistic price, quantity and profit

$$\begin{aligned} p_t = p^m &= (a + \bar{c})/(2 - 2\beta), && \forall t \geq 0, \\ q_t = q^m &= (a - \bar{c})/(2b), && \forall t \geq 0, \\ R_0 = R_0^m &= (a - \bar{c})^2/[4b(1 - \beta)], && \\ R_t &= 0, && \forall t \geq 1. \end{aligned}$$

The monopolist quotes p^m which is higher than the competitive price $p^c = c$, and sell q^m which is lower than the competitive quantity $q^c = 2q^m = (a - \bar{c})/b$ in period 0.

2.2 Markov-Perfect Pricing

Stokey (1981) showed that there is a unique Markov-perfect equilibrium in the monopolist's problem.[1] The problem is defined as a recursive form:

$$W(q_{t-1}) = (p_t - c)(q_t - q_{t-1}) + \lambda_t[f(q_t) + \beta p(q_t) - p_t] + \beta W(q_t).$$

[1] Stokey originally solved the finite-horizon dynamic programming and set the initial period infinite past.

q_{t-1} is the only natural state variable, because the monopolist cannot commit to future prices and there is no commitment made in the past in every period.[2] The future price $p_{t+1} = p(q_t)$ is also function of q_t. Even though the monopolist cannot commit to future prices, the monopolist considers the effect of changing the amount of goods on future prices. The FONCs are

$$\partial p_t: \quad q_t - q_{t-1} = \lambda_t, \tag{9}$$

$$\partial q_t: \quad p_t - c - \beta W'(q_t) + \lambda_t \left[f'(q_t) + \beta p'(q_t) \right] = 0, \tag{10}$$

$$\partial \lambda_t: \quad p_t - \beta p(q_t) - f(q_t) = 0. \tag{11}$$

Also, $W'(q_t) = -(p_{t+1} - c)$ holds by the envelope theorem. Additional amount of goods sold decreases future profits, which the monopolist could earn otherwise in the next period. Equations (9)-(11) become

$$\bar{c} = f(q_t) + q_t f'(q_t) - q_{t-1} f'(q_t) + \beta p'(q_t)(q_t - q_{t-1}). \tag{12}$$

There are two additional terms to the marginal revenue as seen in equation (6). $-q_{t-1} f'(q_t) > 0$ appears as in the case of the monopolistic pricing seen in equation (8). The monopolist cannot control future prices by commitment, which shifts the marginal revenue curve toward right, decreases prices, and increases quantities. The Markov-perfect pricing is time-consistent, because the monopolist quotes lower prices than the monopolistic pricing to sell more amount of goods, so that the monopolist's strategy in the current period is consistent with the one in the next period. The monopolist also internalizes the effect of changing the amount of goods on future prices; $\beta p'(q_t)(q_t - q_{t-1}) < 0$ diminishes the marginal revenue and increases prices.

Note that, $p'(q_t)$ in equation (10) implies a difficulty to solve for the policy functions $p(q_{t-1})$ and $q(q_{t-1})$, because I need to know $p(q_{t-1})$ to obtain $p'(q_{t-1})$ and solve for the policy functions (Krusell et al, 2002). In the LQ framework, the guess and verify method is easily applied to avoid this problem.

[2] If I consider such a commitment, the monopolist's problem becomes non-recursive, because the constraint have a future variable p_{t+1}; Marcet and Marimon (2011) defines the saddle point problem to consider this type of the problem. I solve for the monopolistic pricing by solving the sequential problem, instead of solving the recursive problem, as I have done in the previous section. To solve for the sustainable pricing, I extensively use the saddle point problem. See section 2.3.

2.2.1 Analytical solutions in the LQ framework

I assume a linear function of $f(q) = a - bq$. Then I have

$$
\begin{aligned}
&q_t - q_{t-1} = \lambda_t, \\
&p_t - \beta p_{t+1} - a + bq_t = 0, \\
&p_t - c - \beta(p_{t+1} - c) + [-b + \beta(\partial p_{t+1}/\partial q_t)]\lambda_t = 0.
\end{aligned}
$$

The system has the steady state

$$
\begin{aligned}
&p^c = c, \\
&q^c = (a - \bar{c})/b.
\end{aligned}
$$

At the steady state, prices and quantities are at the associated competitive level. If the solution of the dynamic system is stable, prices and quantities converge to the steady state. I can analytically solve for the stable solution, by using the undetermined coefficient method to guess and verify the policy functions

$$
\begin{aligned}
&p_t = \tau_p + \tau_{pq} q_{t-1}, \\
&q_t = \tau_q + \tau_{qq} q_{t-1},
\end{aligned}
$$

where $\tau_{qq} = [1 - (1-\beta)^{.5}]/\beta \in (0,1)$, $\tau_{pq} = -b\tau_{qq}/(1 - \beta\tau_{qq}) < 0$, $\tau_q = (1 - \tau_{qq})q^c > 0$, and $\tau_p = p^c - \tau_{pq} q^c > 0$. $\tau_{qq} \in (0,1)$ is the stable root of the quadratic equation $\beta\tau_{qq}^2 - 2\tau_{qq} + 1 = 0$. The solution is the Markov-perfect equilibrium, because q_{t-1} is the only natural state variable. It is also unique, because the quadratic equation has only one stable root and the other root is not stable (Blanchard and Kahn, 1980).

2.2.2 Comparison of the profit functions in the LQ framework

In the LQ framework, I can also analytically solve for the profit of the Markov-perfect pricing, which is a function of the amount of goods sold, q_{t-1}

$$
W(q_{t-1}) \equiv R_t = \mu + \mu_q q_{t-1} + \mu_{qq} q_{t-1}^2. \tag{13}
$$

Then I can prove

Proposition 1. *$W(q_{t-1}) > 0$ and $W'(q_{t-1}) = -(p(q_{t-1}) - c) < 0$ for $q_{t-1} \in [0, q^c)$. Also, $W(q^c) = W'(q^c) = 0$.*

Proof. See Appendix A.1. □

Proposition 1 shows that the monopolist can always earn positive profit by the Markov-perfect pricing when the amount of goods already sold are between zero and the competitive level. The profit is decreasing and zero at the competitive level. This result suggests that, if there are no exogenous shocks, the amount of goods monotonically converges to the competitive level. I can also analytically solve for the profit of the monopolistic pricing in period t for any values (q_{t-1}, λ_{t-1}), by solving the FONCs of the monopolistic pricing (5)-(5) and the profit function (2)[3]

$$R^m(q_{t-1}, \lambda_{t-1}) = \mu^m + \mu_q^m q_{t-1} + \mu_\lambda^m \lambda_{t-1} + \mu_{qq}^m q_{t-1}^2 + \mu_{\lambda\lambda}^m \lambda_{t-1}^2 + \mu_{q\lambda}^m q_{t-1}\lambda_{t-1}, \quad (14)$$

Note that, $R^m(0,0) = \mu^m = R_0^m$ (see Appendix A.2), and as shown in the previous section, $q_t = \lambda_t$ for all t holds and there is no dynamics after period 0 in the monopolistic pricing. Still, the profit function is important to consider the monopolist's incentive to deviate from the current pricing in period t, given the amount of goods sold and the past commitment (q_{t-1}, λ_{t-1}). I can prove

Proposition 2. *$D_1 R^m(q_{t-1}, \lambda_{t-1}) < 0$ for $q_{t-1} \in [0, q^c]$ and $D_2 R^m(q_{t-1}, \lambda_{t-1}) < 0$ for $\lambda_{t-1} \in [0, q^c]$. Also, $R^m(q_{t-1}, \lambda_{t-1}) \leq 0$ for $\lambda_{t-1} \in [q^c - q_{t-1}, q^c]$.*

Proof. See Appendix A.2. □

Proposition 2 shows that as q_{t-1} or λ_{t-1} is higher, the monopolist more likely deviate from the current pricing, because the monopolist can earn positive profit by the Markov-perfect pricing whenever $q_{t-1} \in [0, q^c)$ (Proposition 1). When $\lambda_{t-1} > q^c - q_{t-1}$ (or equivalently, $q_{t-1} > q^c - \lambda_{t-1}$), the profit from the monopolistic pricing is always non-positive. Thus, considering propositions 1 and 2, I can show that

Corollary 3. *$R^m(q^m, q^m) < W(q^m)$, i.e., the monopolistic pricing is not sustainable for any $\beta \in (0,1)$.*

[3]Note that in the monopolistic pricing, $q_{t-1} = \lambda_{t-1} = 0$ holds without deviation.

Proof. Note that $q^m = q^c/2$. Then $W(q^m) > 0$ holds from proposition 1 and $R^m(q^m, q^m) \leq 0$ holds from proposition 2. □

Corollary 3 is consistent with AD's result. AD showed that for every ε, there exists a discount factor β such that $[\varepsilon, \pi^* - \varepsilon]$ is in the set supported by subgame perfect equilibria (SPE), where π^* denotes the monopoly profit. As $\beta \to 1$, ε becomes arbitrary small to support the monopoly profit as the best SPE. π^* is only approximately supported as long as $\beta < 1$ up to $\varepsilon > 0$.[4] In the present analysis, Figure 2 shows $\max\{R^m(q_{t-1}, \lambda_{t-1}), -W(q_{t-1}), 0\}$ on the state space (q_{t-1}, λ_{t-1}). In the region where the value takes zero, the monopolistic pricing is not sustainable, because the monopolist can earn positive profit by deviating to the Markov-perfect pricing as $R^m(q_{t-1}, \lambda_{t-1}) \leq W(q_{t-1})$ holds. Corollary 3 shows that it is always true for any discount factor $\beta \in (0, 1)$ at $q_{t-1} = \lambda_{t-1} = q^m$ in the LQ framework. As $\beta \to 1$, the gain by deviation becomes smaller. This makes the monopolistic pricing "approximately" sustainable. The discount factor is important for the monopolist's incentive to deviate, because the long-term relationship is more important as $\beta \to 1$ as usual in a repeated environment.

Figure 2 also shows that even if $\lambda_{t-1} < q^c - q_{t-1}$, the monopolist may deviate from the monopolistic pricing to the Markov-perfect pricing. If the discount factor β is low, the promises made in the past is too much for the monopolist to keep. The region where both $\lambda_{t-1} < q^c - q_{t-1}$ and $R^m(q_{t-1}, \lambda_{t-1}) < W(q)$ hold shrinks as $\beta \to 1$, because the value of commitment is higher as $\beta \to 1$. When $R^m(q_{t-1}, \lambda_{t-1}) \geq W(q_{t-1})$ holds, a pair (q_{t-1}, λ_{t-1}) is credible in a reputation mechanism. Also, at $(q_{t-1}, \lambda_{t-1}) = (q^c, 0)$, the monopolist does not have incentive to deviate for any $\beta \in (0, 1)$, because $W(q^c) = R^m(q^c, 0) = 0$. The competitive level of prices and quantities are supported by SPE, but it is the worst SPE as the monopolist may be better off in other equilibria. In the next section, I consider the dynamics in such equilibria supported by the monopolist's reputation among consumers.

2.3 Sustainable Pricing

AD considered the subgame perfect equilibrium (SPE) in durable goods monopoly. AD formulated the dynamic game between the monopolist and an continuum of infinitely-lived consumers. As usual in repeated games, the trigger strategy takes a form of punishment to the monopo-

[4] AD originally considered discount rate r and the length of time z to have discount factor $\beta = (1 + rz)^{-1}$, and then made z arbitrary small.

list for deviating from the optimal strategy, and the trigger strategy gives the monopolist an incentive to keep its promises in a long-term relationship with consumers. The monopolist's reputation among consumers makes it possible to sell goods at higher prices. The monopolist's deviation from the current pricing damages its reputation, but such a deviation is off the equilibrium. The monopolist's strategy is summarized into the sustainability constraint (Abreu, 1988; Chari and Kehoe, 1990), in which the continuation value is greater than the one from the deviation.

2.3.1 Pricing Game

The monopolist quotes the price p_t at $t = 0, 1, 2, ...$[5] A continuum of identical infinitely-lived consumers purchases goods.[6] In each period, the monopolist quotes the price first, then consumers who have not purchased goods decide whether or not to buy. Both the monopolist and consumers discount the future by a common discount factor β.[7]

History in period t is defined as the prices and the quantities up to period $t-1$. $h_0 = \emptyset$ and $h_t = (h_{t-1}, p_{t-1}, q_{t-1})$ for all $t > 0$. The monopolist's strategy is given by $p_t = \sigma_t(h_t)$ and contingent plans for any future histories. Consumers are price takers, and their decision rule is given by $q_t = \tau_t(h_t, p_t)$ and contingent plans for any future histories. Then, I can define a sustainable equilibrium as follows.

Definition 4. A sustainable equilibrium of the model is a pair (σ, τ) of a monopolist's strategy and consumers' reaction to the strategy such that

(i) given the pricing strategy σ and current history (h_t, p_t), the continuation of consumers' reaction τ satisfies

$$p_t = \beta\sigma_{t+1}(h_{t+1}) + f[\tau_t(h_t, p_t)], \qquad t \geq 0$$

$$\sigma_s(h_s) = \beta\sigma_{s+1}(h_{s+1}) + f[\tau_s(h_s, \sigma_s(h_s))], \quad s \geq t+1$$

[5] AD originally used $t = 0, z, 2z, ..., nz, ...$ where z is the time interval. I use t instead of n as the index of time.

[6] Here I assume consumers are identical so that I can ignore consumers' distribution whose status of purchasing goods depends on the history as well; only q_{t-1} summarizes the information of those who have already purchased goods among identical consumers. AD originally imposed measurability restrictions on consumers who have different reservation prices, so that the the monopolist's and consumers' strategies are a function of consumers' distribution rather than a quantity.

[7] AD originally used $e^{-rt} \approx (1+r)^{-t}$ as the discount factor.

for all possible future histories induced by σ.

(ii) given consumers' reaction τ and and current history h_t, the continuation of the pricing strategy σ solves

$$\begin{array}{ll} \max_{(\tilde{\sigma}_s)_{s\geq t}} & \sum_{s=t}^{\infty} \beta^{s-t}[\tilde{\sigma}_s(h_s) - c][\tau_s(h_s, \tilde{\sigma}_s(h_s)) - \tau_{s-1}(h_{s-1}, \tilde{\sigma}_{s-1}(h_{s-1}))] \\ \text{s.t.} & \tilde{\sigma}_s(h_s) = \beta\tilde{\sigma}_{s+1}(h_{s+1}) + f[\tau_s(h_s, \tilde{\sigma}_s(h_s))], \text{ for all } s \geq t, \end{array}$$

for all $t \geq 0$ and for all possible future histories induced by $(\tilde{\sigma}_s)_{s\geq t}$.

2.3.2 Sustainability Constraint

In the sustainable pricing, the monopolist sets prices so that

$$\sum_{s=t}^{\infty} \beta^{s-t}(p_s - c)(q_s - q_{s-1}) \geq W(q_{t-1}), \tag{15}$$

holds for all $t \geq 0$.[8] (15) is called the sustainability constraint. As long as this inequality holds, the monopolist has no incentive to deviate from the current pricing. The sustainable pricing is defined as a strategy that specifies to continue the current pricing as long as it has been adapted in the past. Such a pricing is sustained by the monopolist's reputation among consumers who utilize the Markov-perfect pricing as a trigger strategy.

I use the following propositions (see Appendix for proofs):

Proposition 5. *The Markov-perfect equilibrium is the worst sustainable equilibrium.*

Proposition 5 shows that $W(q_{t-1})$ actually attains the lowest value among the outcome of sustainable equilibrium. The worst sustainable equilibrium value in the sustainability constraint ensures the best sustainable equilibrium to be included in the set of all sustainable equilibria.

Proposition 6. *Any pair (p, q) of contingent sequences of prices and quantities is the outcome of the sustainable equilibrium if and only if (i) the pair (p, q) satisfies (1) in every period $t \geq 0$ and (ii) the inequality (15) holds in every period $t \geq 0$.*

Proposition 6 also shows that any arbitrary sequence is an outcome of a sustainable equilibrium if and only if Equations (1) and (15) are satisfied for $t \geq 0$. The entire set of outcome

[8] There is no temporary gain from deviation because the monopolist moves first.

induced by the sustainable equilibrium is characterized by this proposition. The sustainable pricing is characterized by the outcome of the best sustainable equilibrium; the monopolist chooses $\{p_t, q_t\}_{t=0}^{\infty}$ so as to maximize (2) subject to Equations (1) and (15) for $t \geq 0$.[9] The sustainable pricing is obtained by solving for the constrained efficient allocation. Note that, the sustainable pricing is a strategy which assumes the absence of commitment technologies, whereas, the outcome of the sustainable pricing assumes the presence of commitment technologies.

Let $\varphi_t \geq 0$ be Lagrange multiplier associated with the sustainability constraint (15), and let the sum of the Lagrange multiplier $\Psi_t = \Psi_{t-1} + \varphi_t \geq 1$, given $q_{-1} = \lambda_{-1} = 0$ and $\Psi_{-1} = 1$. Then the recursive form of the saddle point problem as in Marcet and Marimon (2011) is written as

$$\begin{aligned} V(q_{t-1}, x_{t-1}) &= (p_t - c)(q_t - q_{t-1}) - (x_t - z_t x_{t-1})p_t + x_t f(q_t) \\ &\quad -(1 - z_t)W(q_{t-1}) + (\beta/z_{t+1})V(q_t, x_t), \end{aligned}$$

where $x_t = \lambda_t/\Psi_t \geq 0$, $z_t = \Psi_{t-1}/\Psi_t \in (0, 1]$ and $V_t = R_t/\Psi_t$. x_t is normalized Lagrange multiplier λ_t, and $x_t > 0$ shows the monopolist's commitment in the past. z_t is the ratio of the sum of Lagrange multiplier φ_t, and $z_t < 1$, which is equivalent to $\varphi_t > 0$, shows that the sustainability constraint is binding in the current period. The FONCs are

$$\begin{aligned} \partial p_t : &\quad (q_t - q_{t-1}) - (x_t - z_t x_{t-1}) = 0, \\ \partial q_t : &\quad (p_t - c) + x_t f'(q_t) + (\beta/z_{t+1}) D_1 V(q_t, x_t) = 0, \\ \partial x_t : &\quad -p_t + f(q_t) + (\beta/z_{t+1}) D_2 V(q_t, x_t) = 0. \end{aligned}$$

Also, the envelope theorem yields

$$\begin{aligned} D_1 V(q_{t-1}, x_{t-1}) &= -(p_t - c) - (1 - z_t)W'(q_{t-1}) \leq 0, \\ D_2 V(q_{t-1}, x_{t-1}) &= z_t p_t > 0. \end{aligned}$$

[9] One might wonder if other fallback equilibria were chosen with a somewhat less harsh punishment. As no theory seems to be available on how fallback equilibria are chosen, my discussion here assumes that both the monopolist and consumers choose the Markov-perfect pricing for sure if, at all, the monopolist abandons the commitment.

The first condition shows that by increasing marginal amount of goods sold, the monopolist loses future profit $-(p_t - c) \leq 0$, but outside option $-(1 - z_t)W'(q_{t-1}) \geq 0$. Note that $W'(q_{t-1}) \geq -(p_t - c)$ holds in the sustainable pricing, because the monopolist quotes higher prices in the sustainable pricing than the Markov-perfect pricing, in which $W'(q_{t-1}) = -(p_t - c)$ holds; therefore, I have $D_1V(q_{t-1}, x_{t-1}) = -(p_t - c) - W'(q_{t-1}) + z_tW'(q_{t-1}) \leq 0$. The monopolist's profit is decreasing in the amount of goods as in the Markov-perfect equilibrium. However, the monopolist can utilize the outside option to earn more profit than in the Markov-perfect pricing, when the sustainability constraint is binding, $(1 - z_t) > 0$.

The second condition shows that, by increasing marginal amount of commitment, the monopolist can sell marginal amount of goods and earn p_t (Note that $V_t = R_t/\Psi_t$; $z_tp_t = \Psi_tp_t/\Psi_{t-1}$ is a normalized value in period $t-1$). The commitment to future prices yields more profits for the monopolist, if such a commitment is feasible.

Combining the FONCs and the envelope theorem, I have

$$
\begin{aligned}
& q_t - q_{t-1} = x_t - z_tx_{t-1}, \\
& (p_t - c) + x_tf'(q_t) - (\beta/z_{t+1})\left[(p_{t+1} - c) + (1 - z_{t+1})W'(q_t)\right] = 0, \\
& -p_t + f(q_t) + \beta p_{t+1} = 0, \\
& R_t \geq W(q_{t-1}) \perp z_t \leq 1.
\end{aligned}
$$

The last condition is the complementary slackness condition, as the sustainability constraint is occasionally binding. Summarizing FONCs, I have

$$
\begin{aligned}
& (1 - \beta)c = f(q_t) + q_tf'(q_t) - (q_{t-1} - z_tx_{t-1})f'(q_t) \\
& -\beta(1 - z_{t+1})z_{t+1}^{-1}[p_{t+1} - c + W'(q_t)].
\end{aligned} \tag{16}
$$

where $p_{t+1} - c + W'(q_t) = z_{t+1}W'(q_t) - D_1V(q_t, x_t) > 0$ is the difference of marginal value between the current pricing and the Markov-perfect pricing.

The discretion and outside-option effect in marginal revenue Let me consider the following three different optimality conditions in each pricing strategies:

$$(\text{Monopolistic}) \quad (1-\beta)c = f(q_t) + q_t f'(q_t) \underbrace{-q_0 f'(q_t)}_{+},$$

$$(\text{Markov-perfect}) \quad (1-\beta)c = f(q_t) + q_t f'(q_t) \underbrace{-q_{t-1} f'(q_t)}_{+} \underbrace{+\beta p'(q_t)\lambda_t}_{-},$$

$$(\text{Sustainable}) \quad (1-\beta)c = f(q_t) + q_t f'(q_t) \underbrace{-(q_{t-1} - z_t x_{t-1}) f'(q_t)}_{+}$$

$$\underbrace{-\beta(1-z_{t+1}) z_{t+1}^{-1}[p_{t+1} - c + W'(q_t)]}_{-},$$

where $f'(q_t) < 0$, $p'(q_t) < 0$, and $p_{t+1} - c + W'(q_t) > 0$. The first equation is for the monopolistic pricing, the second equation is for the Markov-perfect pricing, the third equation is for the sustainable pricing each. Note that the marginal revenue is $\partial(pq)/\partial q = p + qp'(q)$ in the static case. In the dynamic case I consider here, with the monopolist's commitment to future prices, the marginal revenue is $f(q_t) + q_t f'(q_t)$. However, if such commitment technologies are not available, there are two changes in the marginal revenue. One is "the discretion effect" which lowers prices, and the other is "the outside-option effect" which highers prices.

The discretion effect stems from the fact that the monopolist quotes lower prices without commitment as consumers purchase more goods. In the monopolistic pricing, if the monopolist deviates and quotes lower prices, the monopolist obtains additional marginal revenue $-q_0 f'(q_t)$. In the Markov-perfect pricing, such a commitment is not feasible, and the marginal revenue increases in every period by $-q_{t-1} f'(q_t)$, as the monopolist quotes lower prices and sells more goods. In the sustainable pricing, the past commitment $z_t x_{t-1}$ weakens such an effect, and the monopolist can quotes higher prices than in the Markov-perfect pricing, as long as $x_{t-1} > 0$. In other words, the marginal revenue decreases by $z_t x_{t-1} f'(q_t) < 0$ compared to the Markov-perfect pricing.

The outside option effect is related to the future prices in the Markov-perfect pricing, or the outside option in the sustainable pricing, which shifts the marginal revenue curve toward left and the monopolist can quote higher prices. In the Markov-perfect pricing, marginal changes in quantity sold in period t decreases the future profit by $\beta p'(q_t)(q_t - q_{t-1}) < 0$. The monopolist internalizes the effect and quotes higher prices to sell less goods today. Also, in the sustainable pricing, marginal changes in quantity decreases the future value by $\beta(1 - z_{t+1}) z_{t+1}^{-1}[p_{t+1} - c + W'(q_t)] < 0$. Note that, as long as the sustainable constraint in the next period is not binding, $z_{t+1} = 1$, the monopolistic pricing is sustainable, and this term does not come in effect. However, once the sustainability constraint is binding, the monopolist internalize the effect of marginal changes in quantity in the outside option, and quotes higher prices as well as in the Markov-perfect pricing.

2.3.3 Solving for the Policy Functions Numerically

The solution can be obtained only numerically. By using some auxiliary variables, I have

$$\begin{aligned}
&R_t = (p_t - c)(q_t - q_{t-1}) + \beta R_{t+1}, \\
&p_t = f(q_t) + \beta p_{t+1}, \\
&p_t - c + x_t f'(q_t) = \beta \theta_{t+1}, \\
&z_t \theta_t = p_t - c + (1 - z_t) W'(q_{t-1}), \\
&q_t - q_{t-1} = x_t - x_{t-1}, \\
&R_t \geq W(q_{t-1}),
\end{aligned}$$

where R_t is the profit in period t and θ_t summarizes forward terms. Let the state variables $s = (q_{-1}, x_{-1})$, then I have a state-space representation

$$\begin{aligned}
&R(s) = (p(s) - c)(q(s) - q) + \beta R[q(s), x(s)], \\
&p(s) = f(q(s)) + \beta p[q(s), x(s)], \\
&p(s) - c + x(s) f'(q(s)) = \beta \theta[q(s), x(s)], \\
&z(s)\theta(s) = p(s) - c + (1 - z(s)) W'(q_{-1}), \\
&q(s) - q_{-1} = x(s) - z(s) x_{-1}, \\
&R(s) \geq W(q_{-1}).
\end{aligned}$$

I can solve this system by policy function iteration method with occasionally binding constraints as in Kehoe and Perri (2002).

The dynamic Lagrangean used here corresponds to the recursive saddle point method considered in Marcet and Marimon (1998; 2011). Messner and Pavoni (2004) pointed out that Marcet and Marimon's method may yield non-optimal and non-feasible solutions when the problem is not strongly concave. This criticism also may apply to the analysis here because the Markov-perfect equilibrium has an endogenous state variable and the sustainable constraints considered here may not be strongly concave. To check the optimality, I update the policy function slowly by randomly choosing updating grids. I also check the feasibility after the algorithm converges.

3 Numerical Results

3.1 Parametrization

In the LQ framework, $a = b = c = 1$ are arbitrarily chosen for just demonstration purposes. $\beta = \{0.6, 0.95\}$ are chosen for the case of impatient agents and the case of patient agents each. Grids of (q_{-1}, x_{-1}) are divided into $51 \times 51 = 2601$ points and linear interpolation is used to approximate values between grids.

3.2 Short-run Dynamics

Figure 3 shows the short-run dynamics in the model. First, in the case of impatient agents with $\beta = 0.6$, the sustainable pricing is intermediate between the monopolistic pricing and the Markov-perfect pricing. In the monopolistic pricing, price and quantity immediately jump to the monopolistic level after period 1, and maintain the level after that, even though the monopolist has incentive to deviate. In the Markov-perfect pricing, price and quantity quickly converge to the steady state until the market is satiated. This is consistent with Stokey's result when the length of period is small enough (note that I fixed the length of time to one). The sustainable pricing also converges to the steady state, but very slowly; the monopolist offers lower prices over time and continues to sell until the market is satiated. The monopolistic price is not sustainable for any $\beta \in (0, 1)$ (Corollary 3), so the sustainable price is lower than the monopolistic price for every period. In the case of patient agents with $\beta = 0.95$, the sustainable pricing and the monopolistic pricing are almost equivalent, but the sustainable price is slightly lower than the monopolistic price.

The binding pattern of the sustainability constraint makes the short-run dynamics of the sustainable pricing distinct from the others. Figure 4 shows the policy function of $z(q, x)$, the ratio of the sum of Lagrange multiplier on the sustainability constraint. Note that $z(q, x) < 1$ shows that the sustainability constraint is binding. The triangle formed by hyperplanes $q_{-1} = q^c$, $x_{-1} = q^c$ and $x_{-1} = q^c - q_{-1}$ is the area of state space (q, x) where $z(q, x) < 1$, which corresponds to Proposition 2. In the case of impatient agents with $\beta = 0.6$, the state of the economy (q, x) jumps to near (q^m, q^m) in period 0, where q^m is the quantity sold in the monopolistic pricing. After period 1, $z(q, x) < 1$ holds until the market is satiated; the monopolistic pricing is not sustainable. To keep the sustainability constraint with equality, the monopolist lowers prices and sells more goods. In the case of patient agents with $\beta = 0.95$, the state of the economy (q, x) jumps to near (q^m, q^m), but much closer than in the case with $\beta = 0.6$, as the area where $z(q, x) < 1$ shrinks as $\beta \to 1$. After period 1, $z(q, x) < 1$ holds, but the speed of convergence is much slower than in the case with $\beta = 0.6$, because $z(q, x)$ is close to one. As $\beta \to 1$, the speed of convergence becomes arbitrarily slow, which is consistent with the AD's result.

What is the mechanism behind the result? The spread between q and x and the Lagrange multiplier z are the key to understand the mechanism, which are shown in Figure 4. Let the spread between q and x be denoted by $\Delta_t \equiv q_t - x_t = q_{t-1} - z_t x_{t-1}$. Then I have

$$\Delta_t = \begin{cases} \Delta_{t-1} & \text{if } z_t = 1, \\ \Delta_{t-1} + (1 - z_t)x_{t-1} & \text{if } z_t \in (0, 1). \end{cases}$$

The more often the sustainability constraint is binding, the larger the spread is. By substituting $x_t = q_t - \Delta_t \geq 0$ into the FONCs, I have

$$\begin{aligned} (1 - \beta)c &= f(q_t) + q_t f'(q_t) - \Delta_t f'(q_t) \\ &-\beta(1 - z_{t+1})z_{t+1}^{-1}[(p_{t+1} - c) + W'(q_t)]. \end{aligned} \tag{17}$$

As the sustainability constraint is binding, the spread becomes larger, and the marginal revenue shifts toward right as in Figure 1. This is an analogy to the case when the monopolist can reset price in every period in the monopolistic pricing. However, in this case, the sustainable pricing is time consistent. Note that if $q_t = q^c = \Delta_t$ and $x_t = 0$, then $(1 - \beta)c = f(q_t)$ holds in (17); The sustainable pricing converges to the marginal cost. The speed of convergence is governed by the law of motion of Δ_t and the binding pattern of the sustainability constraint.

Figure 4 shows that in the case of impatient agents with $\beta = 0.6$, the spread $q - x$ becomes larger as time goes by after period 1. The sustainability constraint is binding, i.e., $z_t < 1$, until the market is satiated. In the case of patient agents with $\beta = 0.95$, the spread remains small, it becomes larger very slowly though.

4 Extensions

In this section, I consider two extensions to the basic setup: depreciation and stochastic marginal cost. Now suppose that the purchased goods are separated at rate δ and the marginal cost is stochastic. Then equation (1) becomes

$$p_t = f(q_t) + \beta(1-\delta)E_t p_{t+1}. \tag{18}$$

The future price is stochastic, and discounted by not only the discount factor β, but also one minus the depreciation rate $(1-\delta)$. The marginal cost c_t follows an exogenous stochastic process:

$$\log c_{t+1} = (1-\rho)c + \rho \log c_t + \varepsilon_{t+1}, \tag{19}$$

where $\varepsilon_{t+1} \sim N(0, \sigma^2)$ is the disturbance to the marginal cost, which follows a normal distribution with the standard deviation σ. The monopolist chooses the sequence of $\{p_t\}$ (and hence $\{q_t\}$ given (18) and (19)) so as to maximize its discounted sum of expected future profits

$$R_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t (p_t - c_t)(q_t - (1-\delta)q_{t-1}),$$

subject to (18) and (19) for all $t \geq 0$, given $q_{-1} = 0$ and $c_{-1} = c$. Note that the amount of newly purchased goods in period t is $q_t - q_{t-1} + \delta q_{t-1} > 0$.

4.1 Depreciation

Karp (1996) showed that, in a continuous-time setting, depreciation of durable goods yields multiple Markov-perfect equilibria and the Coase conjecture does not necessarily hold. In a discrete-time setting considered in the present paper, the equilibrium is unique, but the Markov-perfect pricing no longer converges to the marginal cost. Depreciation erodes the Coase conjecture, because the monopolist can make a positive profit, and the price is higher than the marginal cost in the steady state. The sustainable pricing is intermediate between the monopolistic pricing and the Markov-perfect pricing, and the sustainable pricing does not converge to the Markov-perfect pricing in the steady state. As depreciation rate δ is higher, the monopolistic pricing is more likely sustainable; for any discount factor $\beta \in (0,1)$, there is a positive depreciation rate $\delta \in (0,1]$ which makes the monopolistic pricing sustainable.

4.2 Stochastic Marginal Cost

Fabinger, Gopinath, and Itskhoki (2011) studied price markup dynamics with stochastic marginal cost, and found an incomplete pass-through in the Markov-perfect pricing. Although the shock to marginal cost itself may generate an interesting dynamics, when I consider stochastic marginal cost in the sustainable pricing, there are other interesting issues through the working of the sustainability constraint. For example, when the shock to marginal cost occurs, the outside option is worse, and the monopolist can quotes higher prices based on reputational power. Even without the shock realization, the possibility of the shock in the future diminishes the monopolist's expected future profit more intensely in the Markov-perfect pricing than in the monopolistic pricing; the possibility of the future shock also makes the monopolist easier to maintain his reputation.

5 Concluding Remarks

In this paper, the short-run dynamics of the sustainable pricing in a durable goods monopoly is studied. By taking advantage of reputation among consumers, the monopolist can offer higher prices in the sustainable pricing than in the Markov-perfect pricing. The sustainable pricing adjusts toward the Markov-perfect pricing, but the speed of adjustment is slow. Reputation has the effect like an adjustment cost. Reputation also makes the monopolist more profitable in the steady state, if durable goods depreciates over time. In the future research, I will examine the effect of marginal cost shocks more carefully, especially focusing on the incomplete pass-through.

A Appendix

A.1 Proof of Proposition 1

First, I solve for the coefficients of the policy functions by using undetermined coefficient method[10]

$$p_t = \tau_p + \tau_{pq} q_{t-1},$$

$$q_t = \tau_q + \tau_{qq} q_{t-1},$$

[10] The detail of calculation for the coefficients are available upon request.

where

$$\tau_{qq} = [1 - (1-\beta)^{.5}]/\beta \in (0,1), \quad \tau_{pq} = -b\tau_{qq}/(1-\beta\tau_{qq}) < 0,$$
$$\tau_q = (1-\tau_{qq})q^c > 0, \qquad \tau_p = p^c - \tau_{pq}q^c > 0,$$

and $\tau_{qq} \in (0,1)$ is the stable root of the quadratic equation $\beta\tau_{qq}^2 - 2\tau_{qq} + 1 = 0$. Given the policy functions, I can solve for the quadratic profit function analytically

$$W(q_{t-1}) = \mu + \mu_q q_{t-1} + \mu_{qq} q_{t-1}^2,$$

where

$$\mu = -\tau_{pq}(q^c)^2/2 > 0, \quad \mu_q = \tau_{pq}q^c < 0, \quad \mu_{qq} = -\tau_{pq}/2 > 0.$$

Then $W'(0) = \mu_q < 0$ and $W''(q) = 2\mu_{qq} > 0$ for all q, $W'(q^c) = \mu_q + 2\mu_{qq}q^c = 0$, and $W(q^c) = \mu + \mu_q q^c + \mu_{qq}(q^c)^2 = 0$ hold; therefore, $W'(q_{t-1}) < 0$ for $q_{t-1} \in [0, q^c)$ also holds.

A.2 Proof of Proposition 2

As in the case of the Markov-perfect pricing, I can solve for the coefficients of the policy functions

$$p_t = \tau_p^m + \tau_{pq}^m q_{t-1} + \tau_{px}^m \lambda_{t-1},$$
$$q_t = \tau_q^m + \tau_{qq}^m q_{t-1} + \tau_{qx}^m \lambda_{t-1},$$
$$\lambda_t = \tau_x^m + \tau_{xq}^m q_{t-1} + \tau_{xx}^m \lambda_{t-1},$$

where

$$\tau_p^m = p^m, \quad \tau_{pq}^m = -[b/(1-\beta)]/2, \quad \tau_{px}^m = [b/(1-\beta)]/2,$$
$$\tau_q^m = q^m, \qquad \tau_{qq}^m = 1/2, \qquad \tau_{qx}^m = -1/2,$$
$$\tau_x^m = q^m, \qquad \tau_{xq}^m = -1/2, \qquad \tau_{xx}^m = 1/2.$$

Given the policy functions, I can solve for the quadratic profit function analytically

$$R_t = \mu^m + \mu^m_x \lambda_{t-1} + \mu^m_q q_{t-1} + \mu^m_{xx} \lambda^2_{t-1} + \mu^m_{qq} q^2_{t-1} + \mu^m_{xq} \lambda_{t-1} q_{t-1},$$

where

$$\mu^m = [b/(1-\beta)](q^m)^2 > 0, \qquad \mu^m_x = 0, \qquad \mu^m_q = -2(p^m - c)\tau^m_{qq} < 0,$$
$$\mu^m_{xx} = \tau^m_{pq}\tau^m_{qq} < 0, \quad \mu^m_{qq} = -\tau^m_{pq}\tau^m_{qq} > 0, \quad \mu^m_{xq} = 0.$$

Then $D_1 R^m(q_{t-1}, \lambda_{t-1}) = \mu^m_q + 2\mu^m_{qq} q_{t-1} = -2\tau^m_{qq}[(p^m - c) + \tau^m_{pq} q_{t-1}] \leq 0$ for $q_{t-1} \in [0, q^c]$ and $D_2 R^m(q_{t-1}, \lambda_{t-1}) = 2\mu^m_{xx}\lambda_{t-1} \leq 0$ for $\lambda_{t-1} \in [0, q^c]$ hold. Then,

$$\begin{aligned}
R^m(q, q^c - q) &= \mu^m + \mu^m_q q + \mu^m_{qq} q^2 + \mu^m_{xx}(q^c - q)^2, \\
&= \mu^m + \mu^m_q q - \mu^m_{qq}(-2q^c q + (q^c)^2), \\
&= [b/(1-\beta)](q^m)^2 - \mu^m_{qq}(q^c)^2 + (\mu^m_q + 2\mu^m_{qq} q^c)q, \\
&= [b/(1-\beta)](q^m)^2 + \tau^m_{pq}\tau^m_{qq}(q^c)^2 - 2\tau^m_{qq}[(p^m - c) + \tau^m_{pq} q^c]q.
\end{aligned}$$

Note that $p^m = (a + (1-\beta)c)/(2 - 2\beta)$, $q^m = (a - (1-\beta)c)/(2b)$ and $q^c = 2q^m$. Then,

$$\begin{aligned}
(p^m - c) + \tau^m_{pq} q^c &= p^m - c - [b/(1-\beta)]q^c/2, \\
&= 0, \\
[b/(1-\beta)](q^m)^2 + \tau^m_{pq}\tau^m_{qq}(q^c)^2 &= [b/(1-\beta)](q^m)^2 - [b/(1-\beta)](q^c)^2/4, \\
&= 0.
\end{aligned}$$

Therefore, $R^m(q, q^c - q) = 0$ for all q holds.

A.3 Proof of Proposition 5

See Ausubel and Deneckere (1989).

A.4 Proof of Proposition 6

See Ausubel and Deneckere (1989).

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Figure 1: Demonstrating time inconsistency.

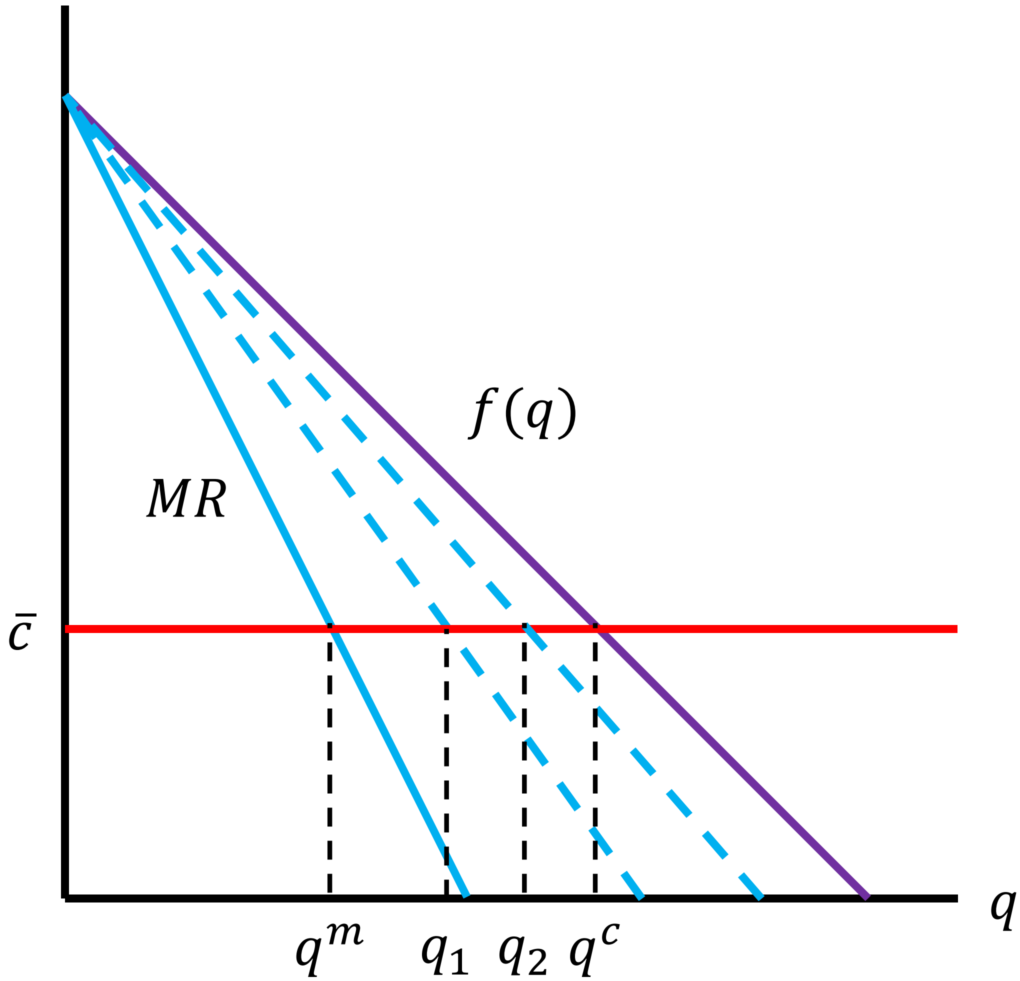


Figure 2: Difference of value functions.

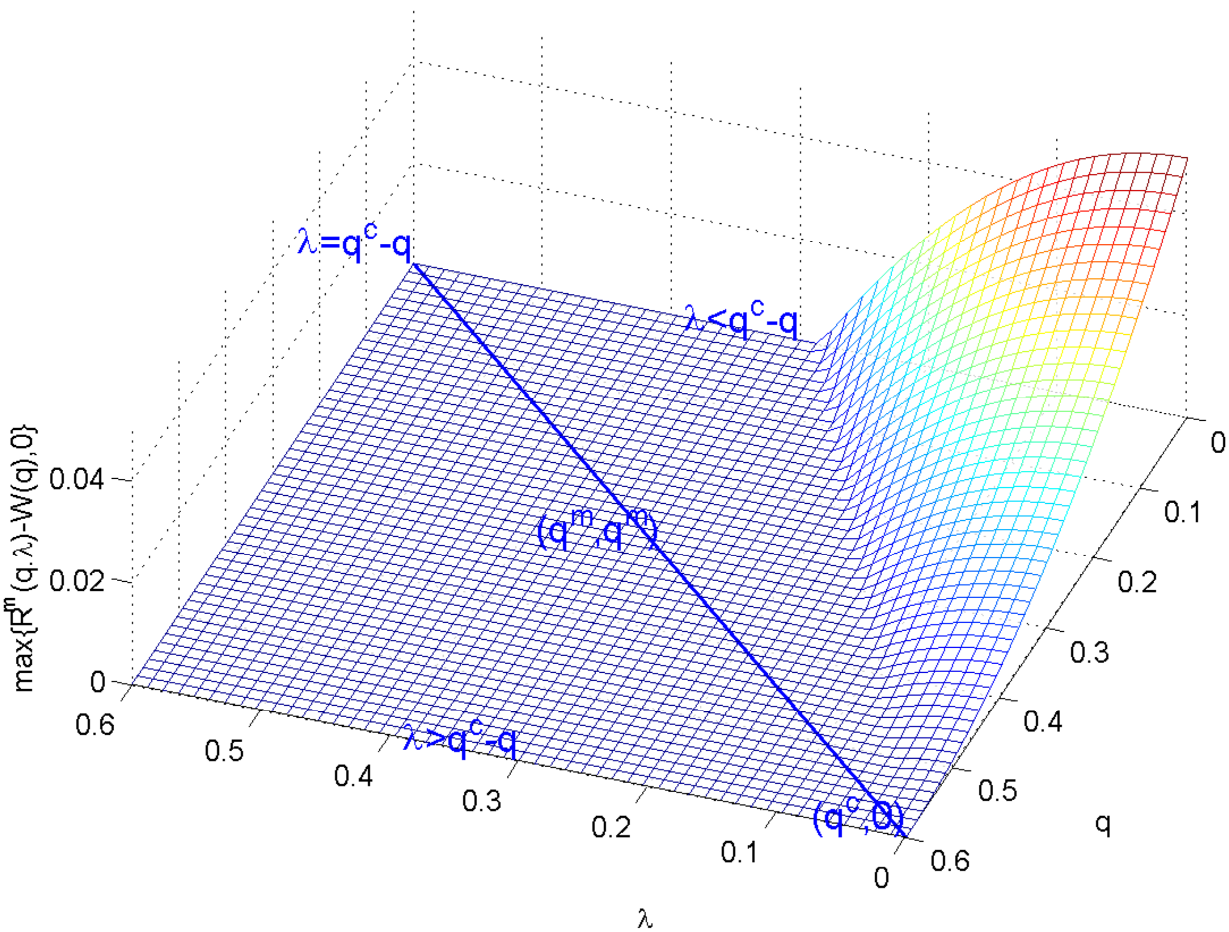
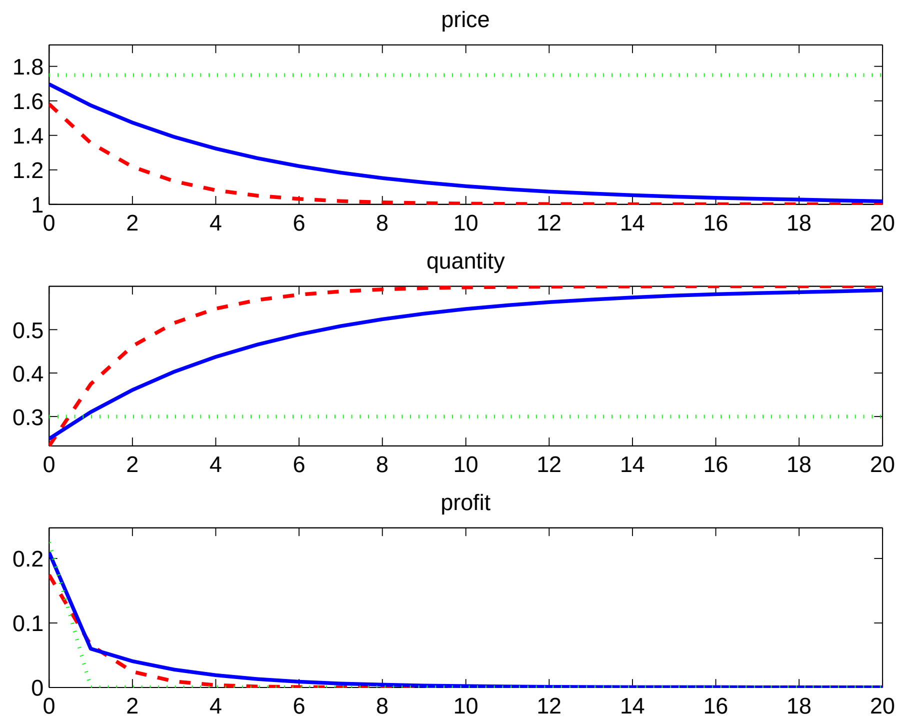


Figure 3: Short-run dynamics.

a. the case of impatient agents: $\beta = 0.6$



b. the case of patient agents: $\beta = 0.95$

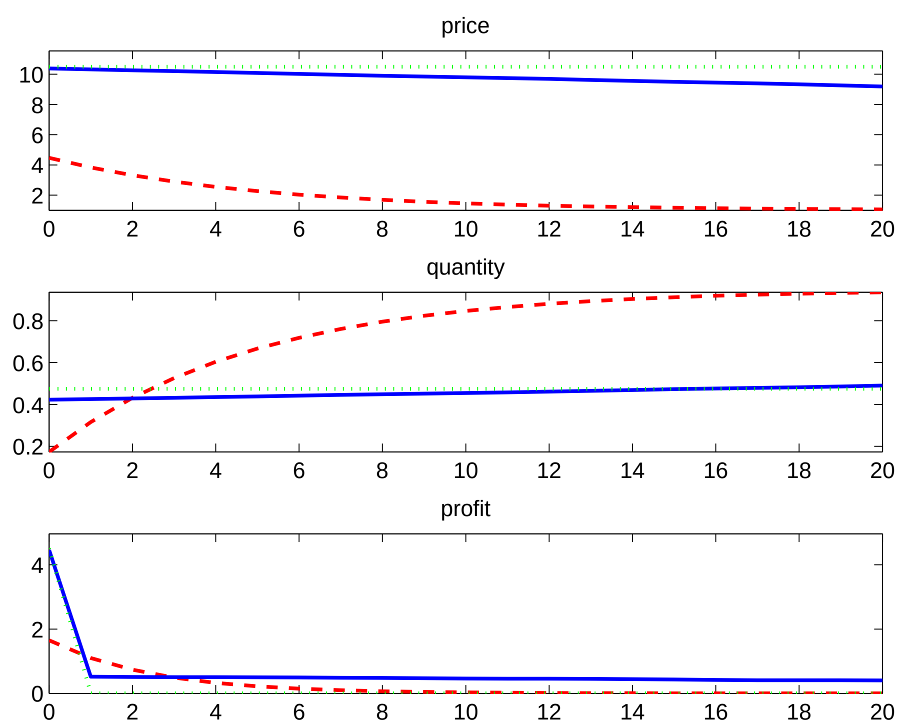
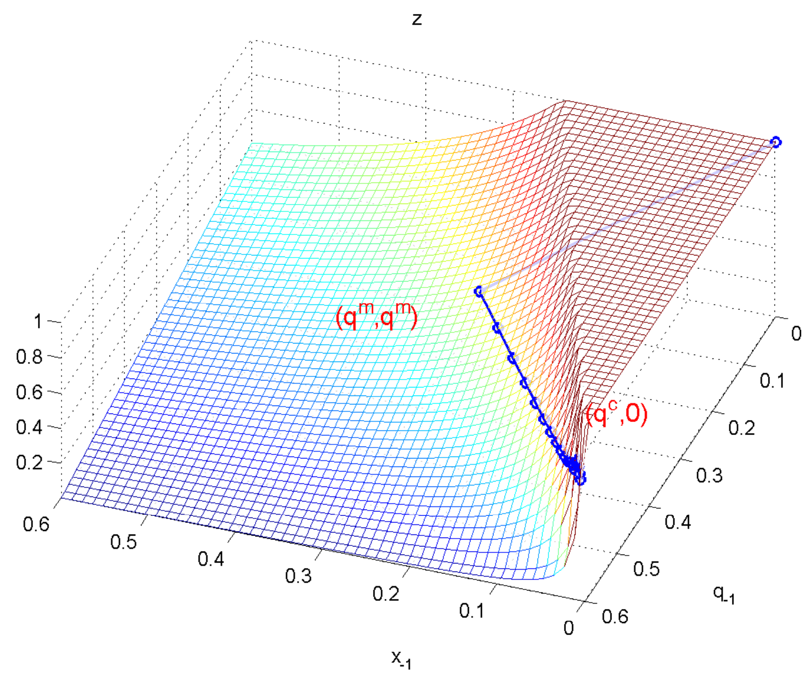
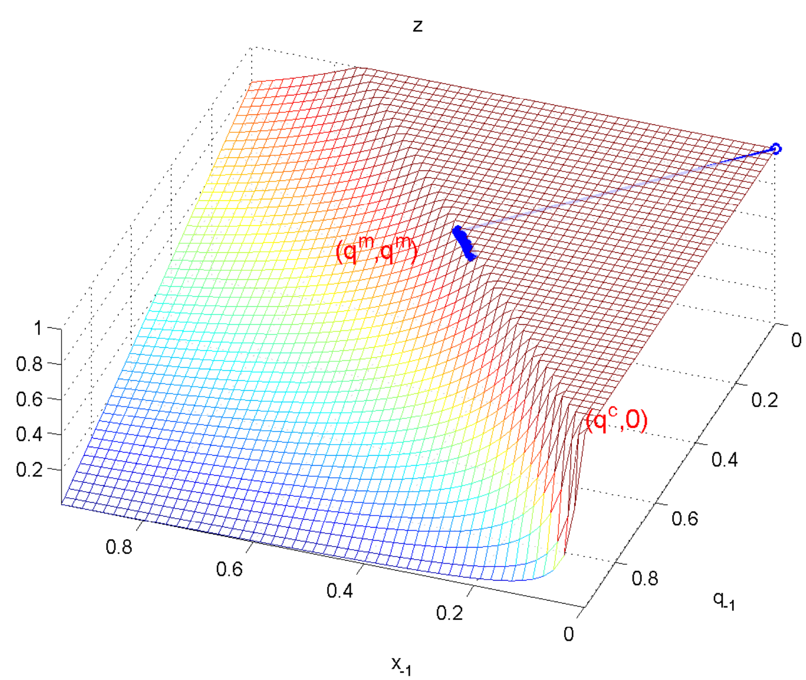


Figure 4: Policy function of Lagrange multipliers.

a. the case of impatient agents: $\beta = 0.6$



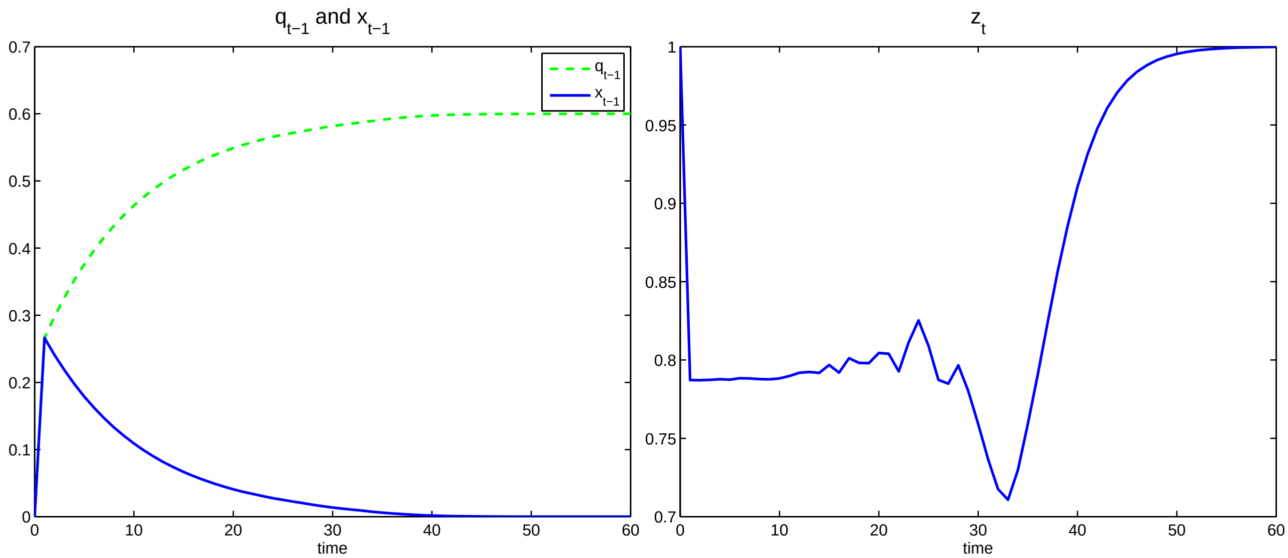
b. the case of patient agents: $\beta = 0.95$



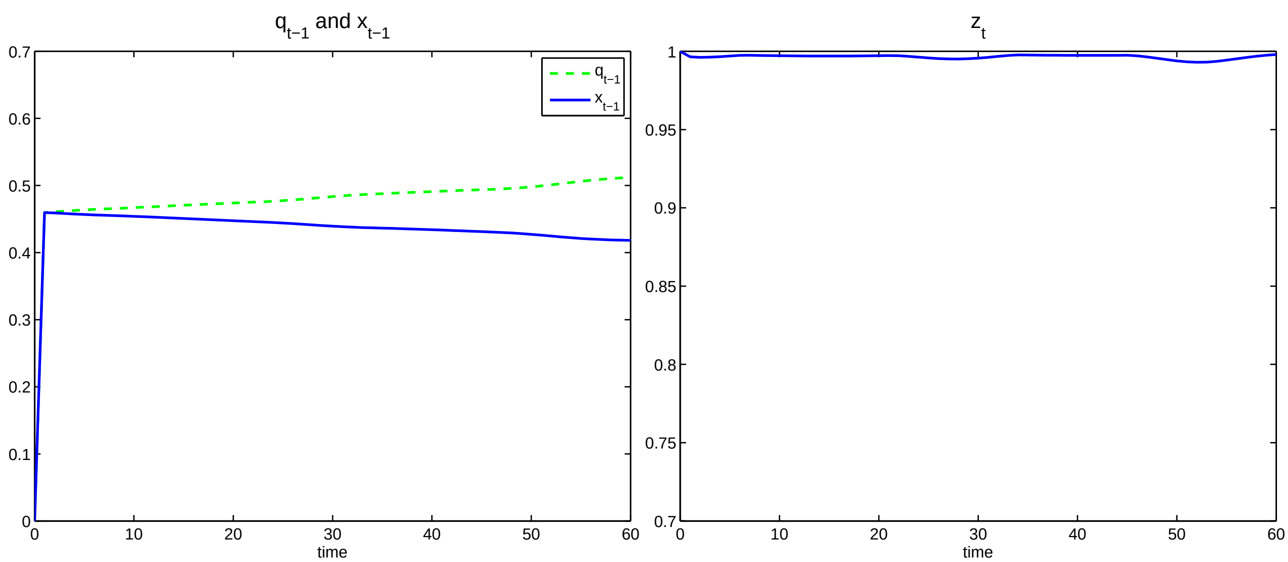
Notes: x_{t-1} is normalized Lagrange multiplier, and $x_{t-1} > 0$ shows the monopolist's promise in the past. z_t is the ratio of the sum of Lagrange multiplier, and $z_t < 1$ shows the sustainability constraint is binding.

Figure 5: Short-run dynamics of Lagrange multipliers.

a. the case of impatient agents: $\beta = 0.6$



b. the case of patient agents: $\beta = 0.95$



Notes: x_{t-1} is normalized Lagrange multiplier, and $x_{t-1} > 0$ shows the monopolist's promise in the past. z_t is the ratio of the sum of Lagrange multiplier, and $z_t < 1$ shows the sustainability constraint is binding.