Financial Frictions, Capital Misallocation and Structural Change[*]

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May 2019

Abstract

We develop a two-sector growth model with financial frictions to examine the effects of a decline in the working population ratio and change in the structure of household demand on sectoral TFP and structural change. Our findings are twofold. First, with financial frictions, a decline in labor input reduces the real interest rate and increases excess demand for borrowing, tightening collateral constraints at a given credit-to-value ratio and generating capital misallocation and lower sectoral TFP. Second, compared to the case with no financial frictions, such changes in sectoral TFP impede structural change driven by the change in the structure of household demand. We also estimate the model's parameters using the Japanese data and undertake a counter-factual simulation to demonstrate the role of financial frictions and capital misallocation in structural change.

Keywords: Financial frictions; heterogeneous firms; capital misallocation; total factor productivity; structural change

JEL codes: E23, E44, O41, O47

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[*] We appreciate the comments and discussions from an anonymous referee, Kosuke Aoki, Ryoichi Imai, Aubhik Khan, Kiminori Matsuyama, Kengo Nutahara, Satoshi Tanaka, Julia Thomas, Takayuki Tsuruga, Kozo Ueda, as well as seminar participants at Asian Meeting of the Econometric Society 2013 and Summer Workshop on Economic Theory. Any remaining errors are ours. The views expressed herein are ours and should not be interpreted as those of the Bank of Japan.

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1 Introduction

In recent years, many developed countries have experienced significant demographic change, especially in the form of rapidly aging populations. As the number of retirees increases, the value-added share of industries producing those goods and services necessary for the elderly, such as hygiene and medical services, has also increased. For example, as shown in Figure 1, since about the 1980s in Japan, the population ratio of persons aged 65 years and over to the total population has continued to increase. This ratio was around 10% at the beginning of the 1980s, but increased to 26.7% in 2015 and is projected to continue to increase to around 38% in 2060. During the same period, the nominal value–added share of service sectors related to aging has increased by more than 5%. In addition, firms in these industries in Japan are typically new and small, as shown in Table 1. They tend to have low capital adequacy ratios and face higher loan interest rates, which implies that they are financially constrained.[1] Also, regarding the household side, Table 2 shows that the consumption share of health of the elderly (age of 65 over) is much higher than that of young persons (ages 0-14 and 15-64).

Table 1: Financial conditions for firms by sector in Japan.

	All	Manu.	Service	Medical
Interest rates on loans	1.3	1.3	1.2	1.9
Capital/Assets	39.9	46.4	45.4	28.4
Small firms ratio	63.6	51.9	72.3	84.9
Liquidity/Short-term debt	140.3	149.5	126.9	133

Source: FY2015 Financial Statements Statistics of Corporations by Industry, Ministry of Finance of Japan.

Table 2: Consumption share of health in Japan.

age 0–14	age 15–64	age 65+
0.07	0.09	0.27

Source: Tung (2011). The data is for Japan in 2004 from National Transfer Accounts accessed at http://www.ntaccounts.org/web/nta/show/Browse%20database on April 2, 2019. The consumption of health includes the private and public expenditures of health.

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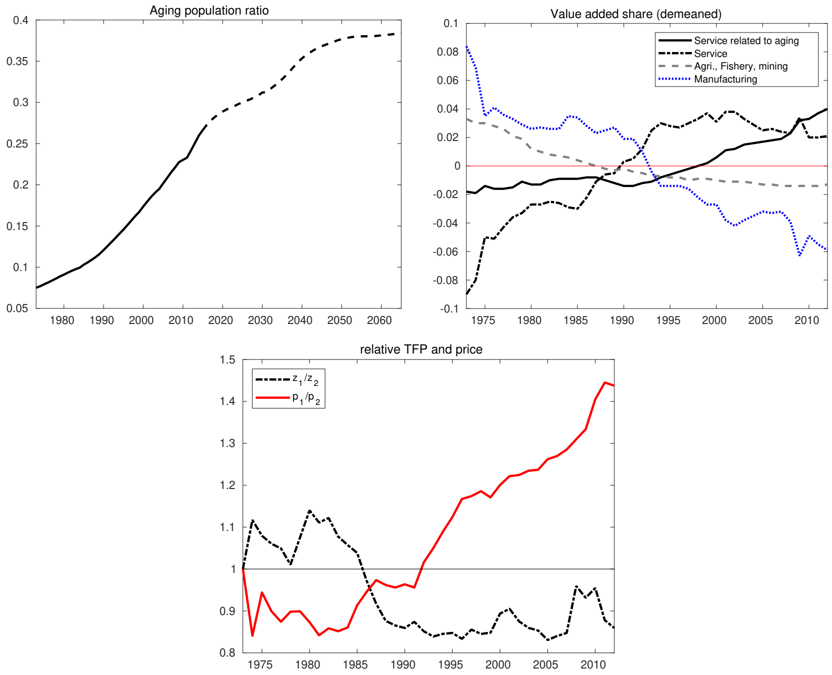
[1] Note that the service sector is more mature as a whole, and firms in the sector generally have higher capital adequacy ratios and lower interest rates than those in the medical sector on average.

Figure 1: Aging population, value-added share, and relative TFP and price in Japan.



Notes: The relative price is the ratio of the value-added deflator. See Appendix A for details of the data.

At the same time, the growth rate of total factor productivity (TFP) has declined over this period. The average growth rate of TFP from 1973 to 1990 was 1.5%, whereas that from 1991 to 2012 was only 0.3%. The relative TFP of the elderly-related sector (i.e., hygiene and medical services) to the other sectors has also been low, whereas its relative prices have increased during this period, especially after the 1980s.

Motivated by these facts, this paper provides a theory of how a reduction in the share of the working population can explain the slowdown of the economy through a decline in aggregate TFP caused by financial frictions and capital misallocation among firms. Our theory suggests that the reduced working population ratio may have led to a disproportionate decline in sectoral TFP, which affects the pace of structual change induced by demand shifts by the elderly.

This paper links capital misallocation, TFP, and structural change in the presence of financial frictions in a unified dynamic general equilibrium framework. We develop a two-sector neoclassical growth model with financial frictions and two household types: workers and retirees. We introduce two types of firms to analyze financial frictions: borrowers and savers. We assume that borrowers are newer and smaller firms dominant in industries such as hygiene and medical services, while savers are older and larger firms typically found in manufacturing and in services other than hygiene and medical services. For the most part, borrowers have large financial needs, but their borrowing is limited by collateral constraints (c.f., Kiyotaki and Moore, 1997). In contrast, savers have less need for financing, and their borrowing is unconstrained. Our model has two sectors: one new and the other old. The new sector is more credit constrained (i.e., the new sector has more borrowers than the old sector). As the number of retirees increases—that is, as the working population ratio declines—the demand for goods produced in the new sector increases. This is the driving force for the structural change explored in the paper.

Our findings are twofold. First, in the presence of financial frictions, a decline in the working population ratio distorts the allocation of capital, which lowers TFP in both the new and old sectors. The decline in the working population ratio also lowers real interest rates, thereby increasing borrowing demand. This increased demand for borrowing tightens the collateral constraint at a constant credit-to-value ratio because the value of collateral does not rise proportionally. This is because borrowers have too little capital, whereas savers have too much, and borrowers produce fewer goods than they would do in the absence of financial frictions. This capital misallocation then lowers sectoral TFP in both sectors.

Second, TFP in the new sector is more and disproportionately affected by financial frictions than that in the old sector, impeding the structural change driven by changes in the structure of household demand. The number of borrowers in the new sector exceeds that in the old sector, which means the new sector has a greater need for financial resources and is more vulnerable to financial frictions than the old sector. Thus, capital misallocation in the new sector is more severe than in the old sector. This difference in the effect of financial frictions between sectors ensures TFP in the new sector is lower than in the old sector, thereby impeding structural change. That is, it leads to too short a supply of goods produced in the new sector compared to that without financial frictions.

To examine whether these theoretical findings can be supported empirically, we estimate the model parameters by fitting the model to actual data. Our estimates are consistent with our assumptions. That is, there are more borrowers in the new sector than the old sector and retirees demand the new sector's goods more than workers. We simulate the model for the Japanese economy from 1973 to 2012 and undertake a counter-factual simulation in the absence of financial frictions. We find that financial frictions and capital misallocation indeed impede structural change as the price for the new sector's goods becomes higher.

Related litarature Some recent studies argue that capital misallocation among firms is one of the main reasons behind a decline in aggregate or sectoral TFP.[2] For example, Moll (2014) emphasizes the role of financial frictions as a cause of capital misallocation. Financial frictions may be affected by changes in the working population ratio. Poterba (2001) and Ikeda and Saito (2014) discusses the effects of demographic change on financial markets and asset prices.

Furthermore, a difference in TFP among sectors may affect inter-sectoral capital misallocation (i.e., structural change), a shift in sectoral output or employment share taking place over a long period of time.[3] Changes in the working population ratio, on the one hand, would affect the household demand structure. On the other hand, asymmetric effects of such demographic changes on sectoral TFPs would also be important for structural change. These economic consequences associated with a decline in the working population ratio is significant to consider since many countries have experienced or will experience such a decline.

Buera et al. (2011) develop a quantitative framework to explain the relationship between TFP, structural change measured by the ratio of employment or output in the service sector to that in the manufacturing sector, and financial development across countries. They analyze the effects of exogenous change in financial frictions on sectoral TFP (for manufacturing and services) to explain cross-country differences in TFP and structural change.[4] In contrast, the present paper focuses on tightening collateral constraints caused by a decline in labor input. We also focus on changes in

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[2] For example, see Hsieh and Klenow (2009); Restuccia and Rogerson (2008). There is also a special issue in *Review of Economic Dynamics* on "Misallocation and Productivity" (Restuccia and Rogerson, 2013).

[3] For an excellent and extensive survey, see Matsuyama (2008).

[4] Arellano et al. (2012); Khan and Thomas (2013) studies the effect of exogenous changes of financial frictions on the measured TFP in heterogeneous agent models.

the structure of household demand as a driving force for structural change, rather than traditional sector-biased technological progress or non-homothetic preferences (Kongsamut et al., 2001; Ngai and Pissarides, 2007).

Unlike Buera et al. (2011), we build a prototype two-sector economy with wedges whose allocations and prices are equivalent to those in the original two sector economy introduced above in order to analytically investigate the relationship among financial frictions, capital misallocation and structural change.[5] This method was originally proposed by Chari et al. (2007). Although our approach employs a simplified modeling of heterogeneity across firms, we obtain analytical results as well as numerical ones.[6]

For estimating the model's parameters, we follow the strategy used in Herrendorf et al. (2013); Moro et al. (2017). Taking the sequence of the relative price as given, the intratemporal problem is separated from the intertemporal problem as shown in Herrendorf et al. (2014). The equilibrium conditions in the intratemporal problem are used for nonlinear estimations. Then, having estimates at hand, we simulate the model and obtain a dynamic perfect–foresight path as in Buera and Kaboski (2009). We also do a counter-factual simulation to evaluate the role of financial frictions and capital misallocation for sectoral TFP and structural change in a general equilibrium.

The reminder of the paper proceeds as follows. We present the original two-sector economy with financial frictions and households' demand structure in the following section. Section 3 analyzes certain correspondences from financial frictions and demand structure to the wedges in the prototype economy. Section 4 estimates the key parameters of financial frictions and household preferences using the data of working population ratio, value-added share, sectoral TFP and relative price from 1973–2012 in Japan. Then we presents numerical exercises using the prototype economy. Section 5 presents a conclusion. The details of data, the prototype economy with wedges, and the proofs of propositions are presented in online Appendix.

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[5] The prototype two-sector economy with wedges is used in other papers, such as Hayashi and Prescott (2008), Esteban-Pretel and Sawada (2014), and Cheremukhin et al. (2017).

[6] See also Buera et al. (2011); Midrigan and Xu (2014); Moll (2014).

2 Two-sector model with financial frictions and demand structure

We build a two-sector neoclassical growth model with financial frictions. On the firm side, there are two sectors, the old and the new. In each sector, there are a final-good producing firm and multiple intermediate-good producing firms. The final-good producing firm uses intermediate goods as inputs to produce final goods, and has a constant elasticity of substitution (CES) production function. We assume that financial needs are different among intermediate-good producing firms; a fraction of them have a lower discount factor. They are *borrowers*, who are bound by collateral constraints (Kiyotaki and Moore, 1997). The other firms with a higher discount factor are *savers*, who are unconstrained. Even though borrowers and savers have the same Cobb-Douglas production function, their levels of holding capital are different depending on their financial frictions. Therefore, the final-good producing firm with the CES function in each sector has lower sectoral TFP and hence output. We also assume that the number of borrowers is larger in the new sector, which leads to differences in sectoral TFP.

On the household side, there are workers and retirees. Workers supply labor inelastically and consume final goods in each sector, whereas retirees only consume final goods in each sector. Workers and retirees have different preferences over final goods produced. When the number of workers declines, there are two effects; one is a decline in labor input, and the other is a change in the structure of household demand.

2.1 Firms

There are two sectors, the old and the new. Let 1 denote the new sector, and 2 denote the old. The final-good producing firm in each sector $i = \{1, 2\}$ minimizes its expenditure:

$$p_t^i y_t^i = \int_0^1 p_{jt}^i y_{jt}^i dj, \tag{1}$$

subject to

$$y_t^i = \left[\int_0^1 (y_{jt}^i)^{1-\varepsilon^{-1}} dj \right]^{1/(1-\varepsilon^{-1})}, \tag{2}$$

The final-good producing firm in sector i purchases $p^i_{jt}y^i_{jt}$ of the intermediate good $j \in [0,1]$ and sells $p^i_t y^i_t$ of the final good to households, where p^i_{jt} and y^i_{jt} are the price and output of intermediate goods, and p^i_t and y^i_t are the price and output of final goods. The firm produces final goods by a CES production function (2), where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The first-order condition (FOC) is given by

$$y^i_{jt} = (p^i_{jt}/p^i_t)^{-\varepsilon} y^i_t. \tag{3}$$

This is the demand function of each intermediate good. The intermediate-good producing firm $j \in [0,1]$ maximizes its discounted sum of future profits:

$$\sum_{t=0}^{\infty} \beta^t_j \left(\frac{\lambda_t}{\lambda_0}\right)^{-1} \left\{(1-\tau^i_t)(p^i_{jt}y^i_{jt} - w_t n^i_{jt}) - p^2_t(k^i_{jt+1} - (1-\delta)k^i_{jt}) - q_t b^i_{jt+1} + b^i_{jt}\right\},$$

subject to

$$y^i_{jt} = \zeta^i_t (k^i_{jt})^{\alpha}(n^i_{jt})^{1-\alpha}, \tag{4}$$

$$y^i_{jt} = (p^i_{jt}/p^i_t)^{-\varepsilon} y^i_t,$$

$$-b^i_{jt+1} \leq \theta p^2_t k^i_{jt}, \tag{5}$$

Each intermediate-good producing firm j in sector i sells $p^i_{jt}y^i_{jt}$ of intermediate goods to the final-good producing firm in the same sector, pays wage bill $w_t n^i_{jt}$ to workers, and purchases capital goods produced in sector 2. w_t is the real wage, and we assume that labor is freely mobile between firms and sectors. $\beta^t_j(\lambda_t/\lambda_0)^{-1} = \prod_{s=1}^{t} \beta(\lambda_s/\lambda_{s-1})^{-1}$ is the cumulative stochastic discount factor, where $\beta_j \in (0,1)$ and λ^{-1}_t is the Lagrange multiplier on the household's budget constraint. Each firm produces using a Cobb-Douglas production function (4) with capital share $\alpha \in (0,1)$ subject to the demand function (3). ζ^i_t is a sector-specific exogenous productivity and $1-\tau^i_t$ is a sector-specific exogenous tax.[7] Each firm also borrows or lends b^i_{jt+1} bonds priced at the risk-free bond price q_t, and faces a collateral constraint (5) with a parameter for the credit-to-value ratio $\theta \geq 0$. Let ϕ^i_{jt}

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[7] These variables may have trends and are needed for only fitting the model to the data of the Japanese economy in Section 4.3.

be the Lagrange multiplier on the collateral constraint. The FOCs are

$$\begin{aligned}
\partial k^i_{jt+1}: &\quad p^2_t\lambda_{t+1} = \beta_j\lambda_t\left\{(1-\tau^i_t)(1-\varepsilon^{-1})\alpha p^i_{jt+1}y^i_{jt+1}/k^i_{jt+1} + p^2_{t+1}(1-\delta+\theta\phi^i_{jt+1})\right\}, && (6)\\
\partial n^i_{jt}: &\quad w_t = (1-\varepsilon^{-1})(1-\alpha)p^i_{jt}y^i_{jt}/n^i_{jt}, && (7)\\
\partial b^i_{jt+1}: &\quad (q_t-\phi^i_{jt})\lambda_{t+1} = \beta_j\lambda_t. && (8)
\end{aligned}$$

The complementary slackness conditions are

$$\begin{aligned}
&\phi^i_{jt}(b^i_{jt+1}+\theta p^2_t k^i_{jt}) = 0,\\
&\phi^i_{jt} \geq 0.
\end{aligned}$$

Note that firms have different discount factors β_j, which determine their financial needs and bond positions. We assume that there are only two types of intermediate-good firms, *borrowers* and *savers*, denoted by $j = \{b, s\}$. We further assume that

Assumption 1. *$\beta_b < \beta_s = \beta$ so that only the borrowers' collateral constraint binds.*

From Assumption 1, we immediately obtain $b^i_{bt+1} = -\theta p^2_t k^i_{bt} < 0 < b^i_{st+1}$, $\phi^i_{bt} > 0$ and $\phi^i_{st} = 0$. Note that different financial needs and discount factors are the only source of heterogeneity among firms, and as long as the borrowers' discount factor β_b is common among sectors, $\phi_t \equiv \phi^i_{bt} > 0$ does not depend on i.

Sectoral output y^i_t, capital k^i_t and labor n^i_t are given by

$$\begin{aligned}
y^i_t &= \left[\chi^i(y^i_{bt})^{1-\varepsilon^{-1}} + (1-\chi^i)(y^i_{st})^{1-\varepsilon^{-1}}\right]^{1/(1-\varepsilon^{-1})}, && (9)\\
k^i_t &= \chi^i k^i_{bt} + (1-\chi^i)k^i_{st}, && (10)\\
n^i_t &= \chi^i n^i_{bt} + (1-\chi^i)n^i_{st}, && (11)
\end{aligned}$$

where $\chi^i \in (0,1)$ is the ratio of borrowers in each sector. We assume that

Assumption 2. *$\chi^1 > \chi^2$ so that sector 1 is the more constrained sector.*

This assumption corresponds to the observation that the new sector such as medical is more likely financially constrained. In Section 4.1, we estimate these parameters and show that the assumption is indeed satisfied. The TFP in each sector is defined as

$$z_t^i \equiv \frac{y_t^i}{(k_t^i)^\alpha (n_t^i)^{1-\alpha}}. \tag{12}$$

From Assumption 2, the more intermediate-good producing firms that are constrained, the more allocation between the two types of firms is distorted; therefore, the sectoral TFP in the more constrained sector is lower; $z_t^1 < z_t^2$ holds. See also Section 3.

2.2 Households

There are two types of households, workers and retirees, denoted by $k = \{w, r\}$. They have different preferences over different final goods produced in each sector. Each household k minimizes its expenditure:

$$p_t^k c_t^k = p_t^1 c_t^{1k} + p_t^2 c_t^{2k},$$

subject to

$$c_t^k = \left[(\mu^k)^{-\epsilon^{-1}} (c_t^{1k})^{1-\epsilon^{-1}} + (1-\mu^k)^{-\epsilon^{-1}} (c_t^{2k})^{1-\epsilon^{-1}} \right]^{1/(1-\epsilon^{-1})}, \tag{13}$$

where p_t^k is the price of composition goods c_t^k consumed by household k. Composition goods are produced by a CES function (13) of c_t^{1k} and c_t^{2k}, which are the consumption of each sector's goods by household k. The function has parameters for the elasticity of substitution $\epsilon \geq 0$ and the sector-bias effect $\mu^k \in (0, 1)$ for each type of household k. The FOCs are

$$c_t^{1k} = \mu^k (p_t^1 / p_t^k)^{-\epsilon} c_t^k, \tag{14}$$

$$c_t^{2k} = (1-\mu^k)(p_t^2 / p_t^k)^{-\epsilon} c_t^k. \tag{15}$$

The sector-bias parameters μ^w and μ^r determine each household's demand for goods produced in sector 1. We assume that

Assumption 3. *$\mu^w < \mu^r$ so that retirees demand for goods produced in sector 1 more than workers do.*

From Assumption 3, the decline in labor input implies a shift in the structure of household demand from the old sector to the new sector. An increase in the number of retirees leads to an increase in demand for goods produced in the new and more constrained sector (sector 1). As in the previous case with Assumption 2, we show that this assumption holds with the estimated parameters.

Utilitarian economy We assume that there is a utilitarian who insures the consumption risks that each household faces. Te consumption profile (c_t^w, c_t^r) then depends only on the household demand structure.[8] We ignore the lifecycle effect of demographic changes on consumption and saving and hence the real interest rate (e.g., Carvalho et al., 2016), as this may also affect any misallocation between firms and endogenous TFP. This effect usually exerts downward pressure on the real interest rate and endogenous TFP may decline even more.[9]

The utilitarian chooses the allocation of composition goods to consume c_t^w and c_t^r, and the amount of the risk-free bond B_t. Given the prices of the composition goods p_t^w and p_t^r, and the risk-free bond price q_t, the utilitarian maximizes the joint life-time utility of workers and retirees

$$\sum_{t=0}^{\infty} \beta^t \left\{ n_t^w \log c_t^w + (1 - n_t^w) \log c_t^r \right\},$$

subject to

$$n_t^w p_t^w c_t^w + (1 - n_t^w) p_t^r c_t^r + q_t B_{t+1} \leq w_t n_t^w + B_t + D_t,$$

where $n_t^w = n_t^1 + n_t^2$ is the total number of workers and D_t is the sum of firm transfers. Let λ_t^{-1} be

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[8] For example, if their demand preferences are identical, i.e., $\mu^w = \mu^r$, workers and non-workers consume the same amount of goods.

[9] We ignore household lifecycle because we wish to obtain an analytical mapping from the model with financial frictions and the structure of household demand to the prototype model via the wedges.

the Lagrange multiplier. The FOCs are

$$\partial c_t^w : \quad \lambda_t = p_t^w c_t^w, \tag{16}$$

$$\partial c_t^r : \quad \lambda_t = p_t^r c_t^r, \tag{17}$$

$$\partial B_{t+1} : \quad q_t \lambda_t^{-1} = \beta \lambda_{t+1}^{-1}. \tag{18}$$

Note that a decline in the number of workers n_t^w involves two effects; (i) a decline in the labor input and the corresponding wage bill, and (ii) an increase in the number of retirees and the total demand for goods produced in sector 1.[10]

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[10] We can decentralize the utilitarian economy as follows. Each household type maximizes their life-time utility

$$\sum_{t=0}^{\infty} \beta^t \log c_t^k$$

subject to

$$p_t^k c_t^k + q_t B_{t+1}^k \leq h_t^k + B_t^k + D_t^k,$$

where h_t^k is the labor income and the transfers among households. The FOCs are

$$\lambda_t^k = p_t^k c_t^k,$$

$$q_t / \lambda_t^k = \beta / \lambda_{t+1}^k.$$

We need to show $\lambda_t^w = \lambda_t^r$. Note that $\lambda_{t+1}^k / \lambda_t^k = \beta / q$. Therefore, under the assumption of perfect foresight, it suffices to show $\lambda_0^w = \lambda_0^r$. We assume that there is a transfer between households in period 0 so that the marginal utilities are equalized between workers and retirees and $\lambda_0^w = \lambda_0^r$ holds.

2.3 Market-clearing conditions

The good market in each sector, the capital market, labor market and bond market all clear:

$$c_t^1 = n_t^w c_t^{1w} + (1 - n_t^w) c_t^{1r}, \tag{19}$$

$$c_t^2 = n_t^w c_t^{2w} + (1 - n_t^w) c_t^{2r}, \tag{20}$$

$$c_t^1 = y_t^1, \tag{21}$$

$$c_t^2 = (1 - \psi_t) y_t^2 + (1 - \delta) k_t - k_{t+1}, \tag{22}$$

$$k_t = k_t^1 + k_t^2, \tag{23}$$

$$n_t^w = n_t^1 + n_t^2, \tag{24}$$

$$\sum_{i \in \{1,2\}} \left[\chi^i b_{b,t+1}^i + (1 - \chi^i) b_{s,t+1}^i \right] + B_{t+1} = 0. \tag{25}$$

c_t^1 and c_t^2 are the total amounts of consumption of each sectoral good. Goods produced in sector 1 are only consumed, whereas goods produced in sector 2 are also used for each firm's investment and the government expenditure.[11] $\psi_t = g_t / y_t^2$ is the ratio of government expenditure to output in sector 2. There is an integrated bond market to which all firms and households have access, with a unique market-clearing bond price q_t.

We denote the price of goods produced in sector 1 $p_t^1 = p_t$ and normalize $p_t^2 = 1$ hereafter. p_t is the relative price, i.e., the ratio of the price of goods produced in sector 1 to those in sector 2. We detrend the model as the exogenous component of the sectoral TFP ζ_t^i for each sector may have a trend in time. A competitive equilibrium is defined as the set of prices and allocations satisfying the relevant equations. See Appendix **??** for details.

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[11] These assumptions may have influence on the measurement of the value-added output in the old sector. However, the value-added share of the old sector is more than 90% and hygiene and medical services are usually not included in investment.

3 Analysis

In this section, we present an analysis using the detailed model presented in the previous section.[12] In neoclassical growth models, a decline in labor input leads to temporally lower real interest rates, as the capital-labor ratio rises. We analytically show that such a drop in real interest rates, i.e., higher risk-free bond prices, in turn leads to tighter collateral constraints and capital misallocation among firms, which is also linked to sectoral TFP. Regarding structural change, we present a prototype two-sector model with wedges (Chari et al., 2007) whose allocations and prices are equivalent to the detailed two-sector model with financial frictions and structure of households demand. By using this equivalence, we derive the demand and supply curves in terms of the relative price and output.

3.1 Capital misallocation and sectoral TFPs

There is a relationship between the tightness of collateral constraints and capital misallocation among firms. In Proposition 1, we analytically show that, given the relative price, a tighter collateral constraint leads to capital misallocation. Also, capital misallocation leads to lower sectoral TFP.

Note that the FOC of the risk-free bond held by the utilitarian (18) determines the risk-free bond price q_t. Combining it with the FOC of borrowers (8), we have

$$\phi_t = (1 - \beta_b/\beta) q_t. \tag{26}$$

From Assumption 1, $\beta_b < \beta$ and $\phi_t > 0$ holds; only borrowers' collateral constraint binds. There is a one-to-one relationship between the risk-free bond price and the tightness of collateral constraints; that is, the higher risk-free bond price is, the tighter collateral constraint is. As the risk-free bond price increases, investment returns are relatively higher than bond returns, and borrowers have more incentive to borrow, but the collateral constraint prevents them from doing so; therefore, the collateral constraint becomes tighter.[13]

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[12] In this section, we use the model without detrending, but the discussion extends to the model with detrending in a straight-forward way. Specifically, only the discussion on the sectoral TFP and relative price will be affected by incorporating trends.

[13] Note that borrowers want to borrow infinite amount as their discount factor is less than savers' discount factor,

The risk-free bond price, or the tightness of collateral constraints is linked to misallocation among borrowers and savers. We show that

Lemma 1. *(i) The ratio of marginal revenue product of capital (MRPK) among borrowers and savers is given by*

$$\begin{aligned} x_{t+1} &\equiv \frac{p^i_{bt+1} y^i_{bt+1} / k^i_{bt+1}}{p^i_{st+1} y^i_{st+1} / k^i_{st+1}}, \\ &= \frac{\beta}{\beta_b} \frac{1 - (q_t - \phi_t)(1 - \delta + \theta\phi_{t+1})}{1 - q_t(1 - \delta)}. \end{aligned} \tag{27}$$

(ii) If $\beta_b < \beta$ and the credit-to-value ratio is smaller than the threshold,

$$\theta < \bar{\theta}_{t+1} = \frac{\beta}{\beta_b} \frac{1}{q_t q_{t+1}},$$

then $x_{t+1} > 1$ holds.

Proof. See Appendix C.1. □

If $x_{t+1} = 1$, MRPKs of borrowers and savers are equalized, and capital is efficiently allocated among borrowers and savers. If $x_{t+1} > 1$, MRPK of borrowers is greater than that of savers, which implies capital misallocation among borrowers and savers. We assume θ is far enough below $\bar{\theta}_{t+1}$ so that $x_{t+1} > 1$ holds. As we assume that the borrowers' discount factor β_b is common among sectors, and it is the only source of heterogeneity among firms, the ratio of MRPK is also common among sectors.

Also, capital misallocation among firms is linked to sectoral TFP. We show the following proposition

Proposition 1. *(i) The sectoral TFP z^i_{t+1} can be decomposed into its exogenous component ζ^i_{t+1} and its endogenous component η^i_{t+1}. η^i_{t+1} is a function of the ratio of MRPK x_{t+1} and the ratio of*

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the risk-free bond price in steady state, i.e., $\beta_b < \beta = q$.

borrowers χ^i*,*

$$\begin{aligned} z^i_{t+1} &= \zeta^i_{t+1}\eta^i_{t+1}, \\ &= \zeta^i_{t+1}\left[\frac{\left(1-\chi^i+\chi^i x^{\gamma}_{t+1}\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x^{\gamma-1}_{t+1}\right)^{\tilde{\alpha}}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned} \tag{28}$$

where $\tilde{\alpha} = (1-\varepsilon^{-1})\alpha$*,* $\nu = (1-\varepsilon^{-1})(1-\alpha)$ *and* $\gamma = \tilde{\alpha}/(\tilde{\alpha}+\nu-1) < 1$*.*

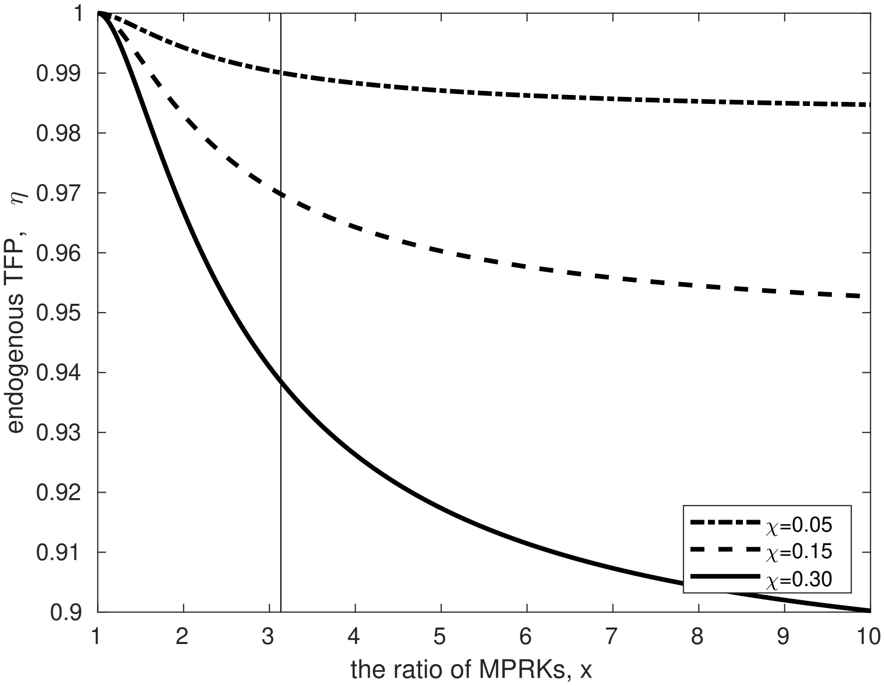
(ii) $\frac{\partial x_{t+1}}{\partial \phi_t}\frac{\phi_t}{x_{t+1}} > 0$ and $\frac{\partial \eta^i_{t+1}}{\partial x_{t+1}}\frac{x_{t+1}}{\eta^i_{t+1}} < 0$ for $i = \{1,2\}$ hold.

(iii) If $\chi^1 > \chi^2$, then $\left|\frac{\partial \eta^1_{t+1}}{\partial x_{t+1}}\frac{x_{t+1}}{z^1_{t+1}}\right| > \left|\frac{\partial \eta^2_{t+1}}{\partial x_{t+1}}\frac{x_{t+1}}{\eta^2_{t+1}}\right|$ holds.

Proof. See Appendix C.2. □

Figure 2 shows the relationship between misallocation (the ratio of MRPK between borrowers and savers) x_t and endogenous TFP η^i_t. Note that if $x_t = 1$, then $\eta^i_t = 1$ holds; if resources are efficiently allocated among firms, the sectoral TFP is only exogenously given. Otherwise, the sectoral TFP endogenously drops due to capital misallocation. The larger x_t, the more η^i_t drops. An increase in the bond price leads to a tighter collateral constraint. As the collateral constraint is tighter, borrowers cannot produce a sufficient amount of their intermediate goods and sell them to the final-good producing firm; borrowers have too little capital, whereas savers have too much capital. Such a capital misallocation hurts the efficiency of final goods production and lowers sectoral TFP. Also, the sectoral TFP in the more constrained sector (i.e., there are more borrowers) is dampened more by capital misallocation among firms. The larger χ^i, the more η^i_t drops, given the degree of misallocation between borrowers and savers x_t.

Figure 2: Misallocation and Productivity.



Notes: The vertical line corresponds to the value of x_t in the steady state calibrated in Section 4.1.

3.2 Structural change

In the previous subsection and Proposition 1, we showed that the sectoral TFP (we also call it the efficiency wedge hereafter, and these two terms are used interchangeably) $z_t^i = \zeta_t^i \eta_t^i$ is a function of the MRPK ratio, x_t, which measures the degree of capital misallocation. In this subsection, we will show that the relative price p_t, which is defined by the relative efficiency of each sector, is also a function of x_t. The relative price of goods produced in the new sector increases as the new sector is more constrained than the old sector. Such a change impedes structural change.

Structural change is measured by the relative price and output. To derive the relative demand and supply curves of the relative price and output, we write down the prototype two-sector economy with wedges, which is described in details in Appendix B. We show the following "equivalence result" (Chari et al., 2007):

Lemma 2. *In the prototype model with wedges, allocations and prices are equivalent to those in* *the detailed model, if and only if the wedges* $\{z^i_{t+1}, (1+\tau^i_{kt+1})^{-1}, \varphi_t\}$ *satisfy*

$$\begin{aligned} z^i_{t+1} &= \zeta^i_{t+1}\left[\frac{\left(1-\chi^i+\chi^i x^{\gamma}_{t+1}\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x^{\gamma-1}_{t+1}\right)^{\tilde{\alpha}}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ (1+\tau^i_{kt+1})^{-1} &= (1-\tau^i_{t+1})(1-\varepsilon^{-1})\left[\frac{1-\chi^i+\chi^i x^{\gamma-1}_{t+1}}{1-\chi^i+\chi^i x^{\gamma}_{t+1}}\right], \\ \varphi_t &= n^w_t\left[1+(1/\mu^w-1)(p_t)^{\epsilon-1}\right]^{-1} \\ &\quad +(1-n^w_t)\left[1+(1/\mu^r-1)(p_t)^{\epsilon-1}\right]^{-1}. \end{aligned} \tag{29}$$

Proof. See Appendix C.3. □

Two things are worth noting here. First, the efficiency and capital wedges z^i_{t+1} and $(1+\tau^i_{kt+1})^{-1}$ depend on the MRPK ratio x_{t+1} and the ratio of borrowers χ^i; these wedges represent financial frictions and capital misallocation. If allocation among the sectors is efficient, $x_t = 1$, $z^i_t = \zeta^i_t$ and $(1+\tau^i_{kt})^{-1} = (1-\tau^i_t)(1-\varepsilon^{-1})$ hold, which includes the inverse of a gross markup stemming from monopolistic competition of intermediate-good producing firms in the original detailed economy. Second, the preference wedge φ_t is a utility-based weight on consumption in sector 1 in the prototype economy (see also Equation (32)), and depends on the demand structure of households, i.e., the number of workers n^w_t, the sector-bias parameters μ^k, and the relative price p_t. As the number of workers declines and $\mu^w < \mu^r$, φ_t increases and a change in demand structure from the old sector (sector 2) to the new sector (sector 1) occurs.

In the prototype economy with wedges, we can easily derive the relative supply curve (Buera and Kaboski, 2009). From firms' profit maximization,

$$\begin{aligned} p_t &= \frac{[(1+\tau^1_{kt})r_t]^{\alpha}\omega_t^{1-\alpha}}{z^1_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \\ 1 &= \frac{[(1+\tau^2_{kt})r_t]^{\alpha}\omega_t^{1-\alpha}}{z^2_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \end{aligned}$$

Then we have

$$p_t = \frac{z_t^2}{z_t^1}\left(\frac{1+\tau_{kt}^1}{1+\tau_{kt}^2}\right)^{\alpha}. \tag{30}$$

Note that the supply curve is horizontal, as the production function has constant returns to scale. The relative price is a function of the wedges z_t^i and $(1+\tau_{kt}^i)^{-1}$, which in turn depend on the MRPK ratio x_t and the ratio of borrowers χ^i. Next, we show the following proposition:

Proposition 2. *(i) The relative price p_t is a function of the ratio of MRPK x_t and the ratio of borrowers χ^i,*

$$p_t = \frac{\zeta_t^2}{\zeta_t^1}\left(\frac{1-\tau_t^2}{1-\tau_t^1}\right)^{\alpha}\left[\frac{1-\chi^1+\chi^1 x_t^{\gamma}}{1-\chi^2+\chi^2 x_t^{\gamma}}\right]^{\frac{1}{1-\varepsilon}}. \tag{31}$$

(ii) If $\chi_1 > \chi_2$, $\zeta_t^2(1-\tau_t^2)^{\alpha} > \zeta_t^1(1-\tau_t^1)^{\alpha}$, and $x_t > 1$, then $p_t > 1$ holds.

(iii) If $\chi_1 > \chi_2$, then $\frac{\partial p_t}{\partial x_t}\frac{x_t}{p_t} > 0$ holds.

Proof. See Appendix C.4. ☐

The price of goods in the new sector (sector 1), which is more constrained, becomes higher than that in the old sector (sector 2), as its sectoral TFP is more dampened and there is too little supply in the new sector.

We can also derive the relative demand curve. From the utilitarian's utility maximization,

$$\begin{aligned} p_t c_t^1/\lambda_t &= \varphi_t, \\ c_t^2/\lambda_t &= (1-\varphi_t). \end{aligned}$$

Then we have

$$p_t = \frac{1-\varphi_t}{\varphi_t}\left(\frac{c_t^1}{c_t^2}\right)^{-1}. \tag{32}$$

Note that the demand curve has a 45-degree downward slope, as the elasticity of substitution in the utilitarian's utility function is one.[14]

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[14] For the detrended economy, Equation (31) becomes $\tilde{p}_t = \left(\frac{1-\tau_t^2}{1-\tau_t^1}\right)^{\alpha}\left[\frac{1-\chi^1+\chi^1 x_t^{\gamma}}{1-\chi^2+\chi^2 x_t^{\gamma}}\right]^{\frac{1}{1-\varepsilon}}$ and Equation (32) becomes

Figure 3 graphically shows the relative demand and supply curves, (30) and (32). Let us suppose there is a decline in labor input. The demand curve shifts upward. As the number of workers declines and the number of retirees increases, there is a change in households' demand structure from the old sector (sector 2) to the new sector (sector 1). Also, with financial frictions, the decline in labor input leads to endogenously strengthened collateral constraints and capital misallocation, which lowers sectoral TFP (efficiency wedges), as shown in Proposition 1. In the more constrained sector, the efficiency and capital wedges decrease more, the relative price increases and the supply curve shifts upward, as shown in Proposition 2. In sum, after a decline in labor input, the equilibrium shifts from A to B, with financial frictions. On the other hand, in the case with no financial frictions, the supply curve is unchanged and only the demand curve shifts upward. Equilibrium shifts from A to C. Comparing the two equilibria, equilibrium B with financial frictions and equilibrium C with no financial frictions, the relative price is higher and the relative output is lower in B than C. That is, financial frictions and capital misallocation impede the structural change driven by the change in households' demand structure.

4 Numerical results

In this section, we numerically demonstrate the analytical findings shown in the previous section using calibrated parameters. We first estimate the parameters for household preferences and firm technologies. Then, having the parameters calibrated and estimated at hand, we simulate the model for the Japanese economy during the period 1973–2012.

4.1 Calibration and estimation

The following set of parameters $(\epsilon, \mu^w, \mu^r, \varepsilon, \chi^1, \chi^2)$ are estimated to fit the model to the data. To obtain the parameters for household preferences (ϵ, μ^w, μ^r), we estimate the consumption share for

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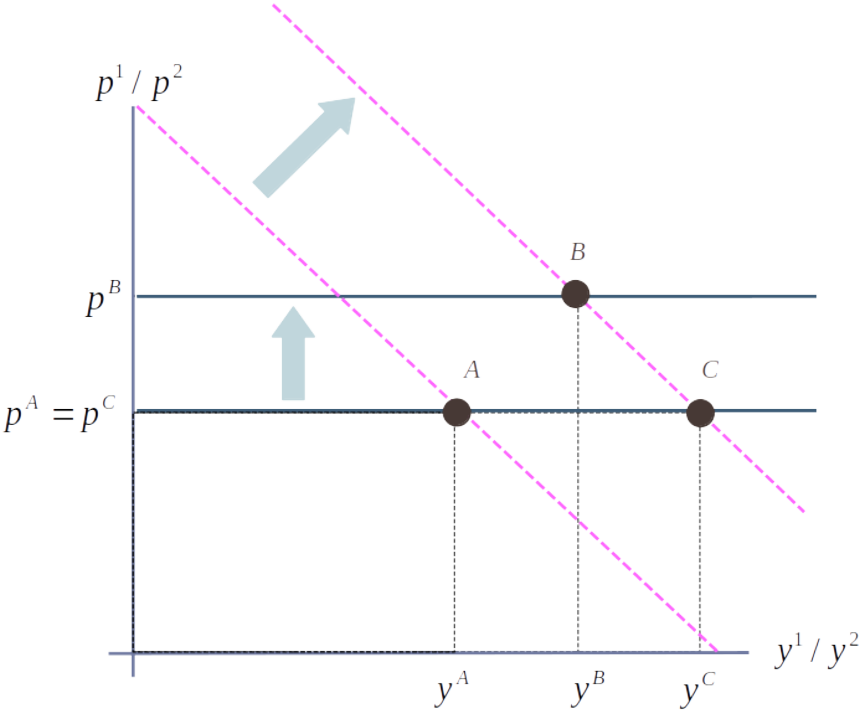
$\tilde{p}_t = \frac{1-\varphi_t}{\varphi_t}\left(\frac{\tilde{c}_t^1}{\tilde{c}_t^2}\right)^{-1}$. The same discussion applies to Figure 3.

Figure 3: Equivalent problem.



goods produced in sector 1, φ_t, using nonlinear least squares following Herrendorf et al. (2013):

$$\frac{p_t c_t^1}{p_t c_t^1 + c_t^2} = \varphi_t = n_t^w \left[1 + (1/\mu^w - 1)(p_t)^{\epsilon-1}\right]^{-1} + (1 - n_t^w) \left[1 + (1/\mu^r - 1)(p_t)^{\epsilon-1}\right]^{-1}, \tag{33}$$

where n_t^w is the working population ratio and p_t is the relative price.[15] See Appendix A for details of the data used. It is difficult to identify the parameter for the degree of substitution. In fact, when we jointly estimate (ϵ, μ^w, μ^r), we obtain the elasticity of substitution $\epsilon = 0$, which implies the two goods are perfect complements.[16] Therefore we also fix the value of $\epsilon = \{0.25, 0.5, 1.0\}$ for each and estimate (μ^w, μ^r).

The estimation results are shown in the left panel of Table 3. We find that μ^w is not significantly

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[15] When $\epsilon \neq 1$, there is also an effect of p_t on φ_t. We find such an effect is small in our estimation. $\epsilon \neq 1$ also violates the balanced growth path when p_t has a trend in the model.

[16] Buera and Kaboski (2009); Herrendorf et al. (2013) find that the elasticity of substitution is almost zero when using the value-added data for estimation.

different from zero for all values of ϵ. We also find that μ^r is significant and its value is around 0.2–0.3. That is, the elderly spend about 20–30% of their expenditure on hygiene and medical services. Most importantly, the estimates are consistent with the assumption made (Assumption 3). The left-hand side panel of Figure 4 also shows the fitted values of the consumption share. For all the values of ϵ and the associated estimates, the fitted values follow the data well. That is, they capture the upward trend exhibited in the aging industry.

To estimate the parameters for firm technology $(\varepsilon, \chi^1, \chi^2)$, we first obtain the ratio of MRPK x_t from the relative price p_t. That is,

$$x_t = \left(\frac{\chi^1 - \hat{p}_t^{1-\varepsilon} \chi^2 - 1 + \hat{p}_t^{1-\varepsilon}}{\chi^1 - \hat{p}_t^{1-\varepsilon} \chi^2} \right)^{\frac{1}{(1-\varepsilon)\alpha}} ,$$

where $\hat{p}_t = (\zeta_t^1/\zeta_t^2)[(1-\tau_t^1)/(1-\tau_t^2)]^\alpha p_t$. We assume the term from exogenous wedges $s \equiv (\zeta_t^1/\zeta_t^2)[(1-\tau_t^1)/(1-\tau_t^2)]^\alpha$ is a constant to be estimated. Then, given x_t, we have

$$z_t^1/z_t^2 = \left[\frac{\left(\frac{1-\chi^1+\chi^1 x_t^{(1-\varepsilon)\alpha}}{1-\chi^2+\chi^2 x_t^{(1-\varepsilon)\alpha}} \right)^{1-\frac{(\varepsilon-1)(1-\alpha)}{\varepsilon}}}{\left(\frac{1-\chi^1+\chi^1 x_t^{(1-\varepsilon)\alpha-1}}{1-\chi^2+\chi^2 x_t^{(1-\varepsilon)\alpha-1}} \right)^{\frac{(\varepsilon-1)\alpha}{\varepsilon}}} \right]^{\frac{\varepsilon}{\varepsilon-1}} . \tag{34}$$

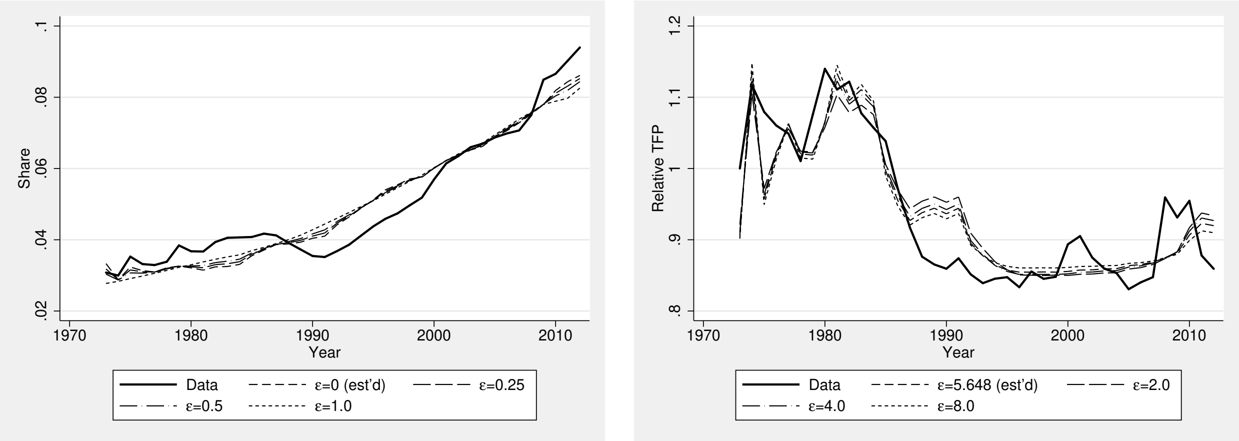
The right-hand side panel of Table 3 provides the estimation results. We include the restriction of $\varepsilon > 1$ so that differentiated goods in the same factor are substitutes and the sectoral production exhibits decreasing returns to scale. When we jointly estimate $(\varepsilon, \chi^1, \chi^2)$, we obtain $\varepsilon = 5.648$. However, the estimate of ε has a large standard error. Therefore, as before, we also fix the value of $\varepsilon = \{2.0, 4.0, 8.0\}$ and estimate (χ^1, χ^2). For each value of ε, the steady-state markup $1 + \tau_k = \varepsilon/(\varepsilon - 1)$ is 2, 1.33, and 1.2. The estimates of (χ^1, χ^2) are stable for different values of ε. When setting $\varepsilon = 4$, we obtain $(\chi^1, \chi^2) = (0.300, 0.007)$, which implies that 30% of firms are borrowers in sector 1 whereas only 0.7% of firms are borrowers in sector 2. Once again, the estimation results support the assumption made (Assumption 2). As the balanced growth path (BGP) is obtained only if $\epsilon = 1$, we use the estimates with $\epsilon = 1$, $(\mu^w, \mu^r) = (0.333, 0.003)$ in the simulation presented hereafter. We also use the estimates $(\varepsilon, \chi^1, \chi^2) = (4.0, 0.300, 0.007)$.

Table 3: Estimation results.

	Value-added share: φ_t in (33)					Relative TFP: z_t^1/z_t^2 in (34)			
	(1)	(2)	(3)	(4)		(1)	(2)	(3)	(4)
ϵ	0.034	0.25	0.5	1	ε	5.648	2.0	4.0	8.0
	(0.08)					(1.92)			
μ^w	0.018	0.015	0.011	0.003	χ^1	0.293	0.296	0.300	0.264
	(2.29)	(5.47)	(3.96)	(1.13)		(5.20)	(5.22)	(5.59)	(6.21)
μ^r	0.209	0.239	0.272	0.333	χ^2	0.002	0.029	0.007	0.000
	(3.45)	(15.71)	(17.90)	(22.08)		(0.44)	(9.67)	(4.26)	(1.66)
					s	0.600	0.607	0.604	0.595
						(17.23)	(17.84)	(18.40)	(15.15)
RMSE	.005	.005	.005	.006	RMSE	.050	.052	.050	.050

Notes: t-values in parentheses. RMSE is the root mean squared error.

Figure 4: Fit of estimation.



The other parameters are calibrated. The discount factor for savers and households is set to $\beta = 0.96$ following Hayashi and Prescott (2002), which calibrate a one-sector neoclassical growth model to the Japanese economy of the 1980s. Note that a period in the model corresponds to a year. The credit-to-value ratio is 70%, $\theta = 0.7$, which is consistent with the literature, including Iacoviello (2005). The capital share $\alpha = 0.452$ and the depreciation rate $\delta = 0.081$ are calibrated to match the detrended steady-state values to the data. We use the means of the capital–output and the investment–capital ratios for 1980–2012 as calibration targets.[17] $\beta_b = 0.649$ is set so that the relative TFP in the steady state hits the target value of 0.9398, being the mean for 1973–2012 (the value in 1973 is normalized to one). Table 4 summarizes the parameters.

Table 4: Parameter values.

	Description	Value	Source or target
β	discount factor	.96	Hayashi and Prescott (2002)
θ	credit-to-value ratio	.7	Iacoviello (2005)
ϵ	elasticity of intermediate goods	1.0	BGP restriction
μ^w	workers bias for sector 1 goods	.000	estimated by (33)
μ^r	retirees bias for sector 1 goods	.333	
ε	elasticity of final goods	4.0	markup: $1 + \tau_k = 1/(1 - \varepsilon^{-1}) = 1.33$
χ^1	ratio of borrowers in sector 1	.300	estimated by (34)
χ^2	ratio of borrowers in sector 2	.007	
δ	depreciation rate	.0812	$\tilde{i}/\tilde{k} = 0.0935$
α	capital share	.4528	$\tilde{k}/\tilde{y} = 2.4948$ and $n^w = 0.8436$
β_b	discount factor for borrowers	.6495	relative TFP: $\eta_1/\eta_2 = 0.9398$

4.2 Steady state

Table 5 summarizes the detrended steady-state values in both cases (with and without financial frictions).[18] We compute two sets of steady-state values. One is with the working population ratio $n^w = 0.909$ for the Japanese economy in 1980 and the other is with $n^w = 0.759$ in 2012. The trend growth rate in sector 2 $\nu_t/\nu_{t-1} = (\zeta_t^2/\zeta_{t-1}^2)^{1/(1-\alpha)}$ is set to 1.012, being the mean for 1973–2012. In the case without financial frictions, i.e., $\theta = \bar{\theta} = 1/(\beta\beta_b)$ and $\beta_b = \beta$, capital allocation among firms is efficient, the tightness of collateral constraint is $\mu = 0$ and the ratio of MRPK is $x = 1$. The (detrended) efficiency wedges are $\eta^1 = \eta^2 = 1$, the capital wedges are $(1 + \tau_k^1)^{-1} = (1 + \tau_k^2)^{-1} = 1 - \varepsilon^{-1}$ (when $1 - \tau_t^i = 1$), and the relative price is $\tilde{p} = 1$. While the efficiency and capital wedges and related variables are independent of the size and structure of the economy, some aggregate variables and sectoral compositions depend on the size and demand

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[17] The capital–output ratio also depends on the working population ratio given financial frictions, and we set its value to the mean for 1980–2012.

[18] The steady-state values are analytically obtained in Appendix B.2.

structure of households, and aggregate capital decreases as the working population ratio and labor input declines. However, the capital–output ratio is constant across different values of n^w in the absence of financial frictions, as capital allocation between firms and sectors is efficient. At the same time, a demand shift from sector 2 to 1 arises given the estimate $\mu^r > \mu^w$, and the preference wedge (equivalent to the consumption share, $pc^1/(pc^1 + c^2)$) and relative output in both sectors increases.

In the case with financial frictions, as $\theta < \bar{\theta}$ holds in our calibration, only borrowers are constrained and there is capital misallocation among firms, resulting in a tightness of collateral constraints of $\mu = \beta - \beta_b = 0.307$ and a ratio of MRPK of $x = 3.133$. As a consequence, the sectoral TFP (the efficiency wedges) in each is less than one, $\eta^1 = 0.939$, and $\eta^2 = 0.999$, respectively. Note that $\eta^1 < \eta^2$ given the estimate $\chi^1 > \chi^2$. Similarly, $(1 + \tau_k^1)^{-1} = 0.708 < 1 - \varepsilon^{-1}$, $(1 + \tau_k^2)^{-1} = 0.749 < 1 - \varepsilon^{-1}$ and $\tilde{p} = 1.092 > 1$, showing that capital allocation between sectors is inefficient compared to that without financial frictions.[19] The values of aggregate capital and relative output also decrease with financial frictions and capital misallocation. Interestingly, as relative output increases, the capital–output ratio decreases with financial frictions, as we assume that sector 1 is more financially constrained and $(1 + \tau_k^2)/(1 + \tau_k^1) > 1$ holds.

4.3 Transition dynamics

4.3.1 An example

To demonstrate the transmission mechanism for the effect of the decline in the working population ratio on sectoral TFP and structural change, we perform a perfect-foresight simulation and observe the transition dynamics as the labor input permanently declines and the economy migrates to the new steady state. We assume that the working population ratio declines from 0.909 to 0.759 in period 1, which corresponds to the size of the reduction in the working population ratio the Japanese economy has experienced over recent years.

Figure 5 depicts the transition dynamics for each sector. When the working population ratio

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[19] The value of the steady-state relative price in the model is close to the mean of the relative price in the data for 1973–2012 of 1.079, even though this is not a calibration target. As the relative TFP is a calibration target, the capital wedges are important for pushing the relative price higher.

Table 5: Steady-state values.

	Description	Value			
		w/ fric.		w/o fric.	
n^w	working population ratio	0.925	0.759	0.925	0.759
μ	tightness of borrowing constraint	0.307	0.307	0	0
x	MRPK ratio	3.133	3.133	1.000	1.000
η^1	efficiency wedge in sector 1	0.939	0.939	1.000	1.000
η^2	efficiency wedge in sector 2	0.999	0.999	1.000	1.000
$(1+\tau_k^1)^{-1}$	capital wedge in sector 1	0.708	0.708	0.750	0.750
$(1+\tau_k^2)^{-1}$	capital wedge in sector 2	0.749	0.749	0.750	0.750
$\tilde{p}$	relative price	1.092	1.092	1.000	1.000
φ	preference wedge	0.028	0.083	0.028	0.083
$\tilde{y}^1/\tilde{y}^2$	relative output	0.020	0.062	0.022	0.068
$\tilde{k}/\tilde{y}$	capital–output ratio	2.498	2.492	2.503	2.503

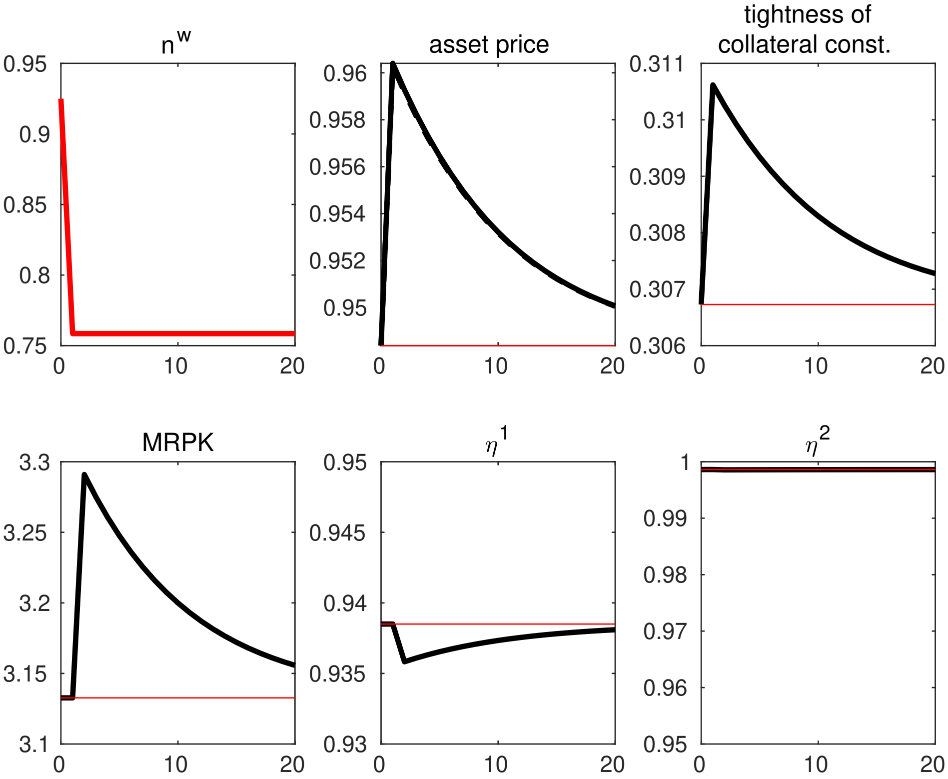
drops in period 1, the collateral constraint becomes tighter as borrowers want to borrow more in the face of low real interest rates (the inverse of the risk-free bond price). The MRPK ratio increases in the next period as there is a one-period lag for new capital to be installed. The sectoral TFP (the efficiency wedges) also decreases in the following period, and more so in the more constrained sector (sector 1), as shown in Proposition 1. Note that as the real interest rate moves back to the steady state, all variables within each sector also revert to their steady-state values. Note that there is almost no difference in the real interest rate between the case with and without financial frictions, as there is little feedback from the efficiency wedges to the real interest rate via financial frictions.

4.3.2 Japanese economy 1973–2012

We simulate the model for the Japanese economy for 1973–2012 using the estimated parameters in Section 4.1. That is, we feed the sequences of the working population ratio and the sectoral TFP for each sector into the model and compute the perfect-foresight path by simulating the model and taking the initial value of the capital–output ratio as given.

Figure 6 illustrates the exogenous path of the working population ratio and sectoral TFP, as well as the endogenous perfect-foresight path of the relative price and the consumption share in

Figure 5: Responses of sectoral variables.



the simulation. The fit of the model with the data is quite reasonable. The simulated path of the relative price is close to the data, although it fails to generate the upward trend in the data toward the end of the sample period.[20] As we set $\epsilon = 1$ and the relative price has no effect on the consumption share, the simulated path of consumption share is exactly the same as in the estimation. The model also has a good fit with the capital–output data, as we use its 1973 value and the mean for 1980–2012 as the initial and the steady-state values for each in the model, respectively.

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[20] We can rectify this by adding time-varying exogenous capital wedges $1 - \tau_t^i$, as shown in Equation (31).

Figure 6: Simulation for the Japanese economy, 1973–2012.

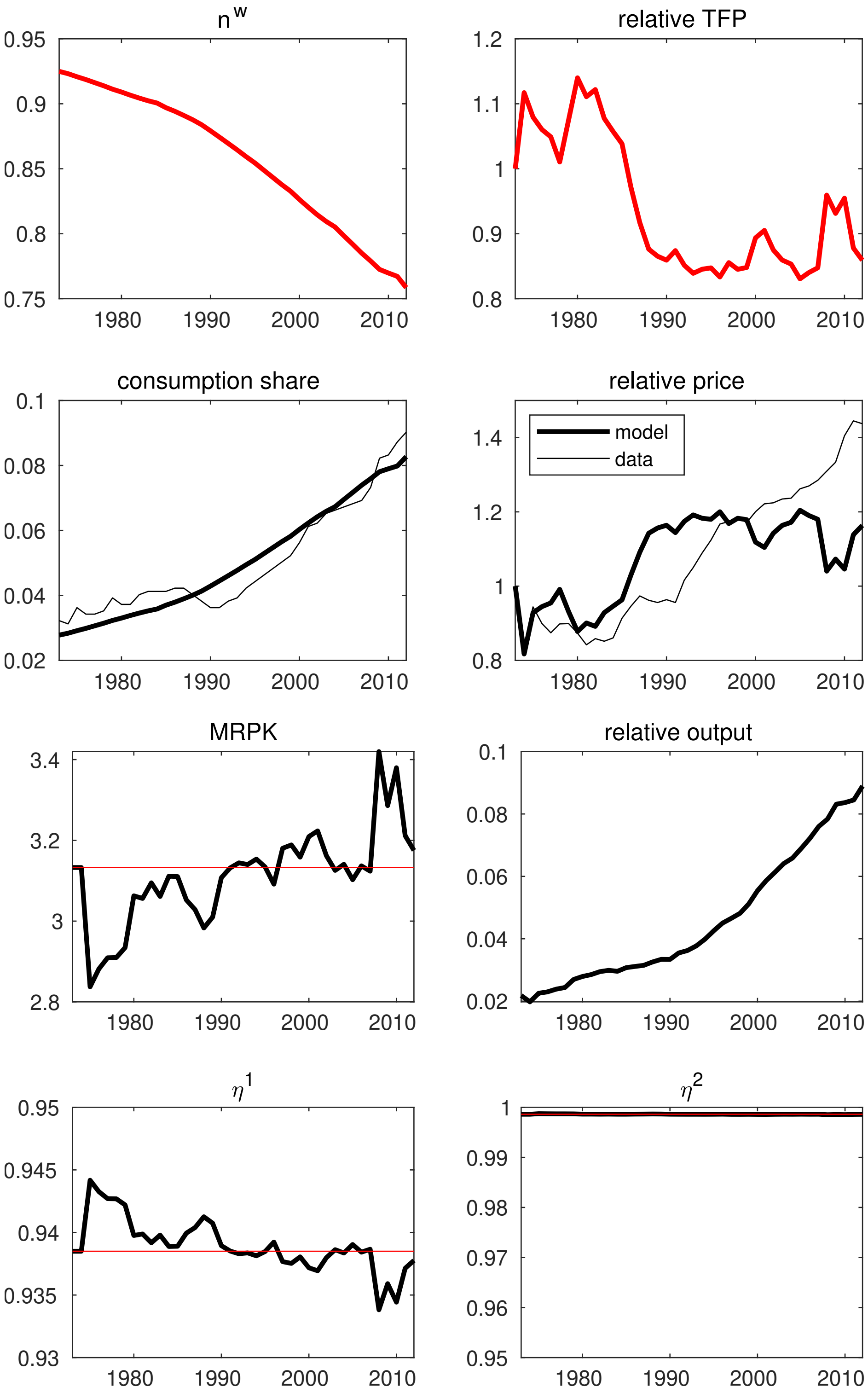
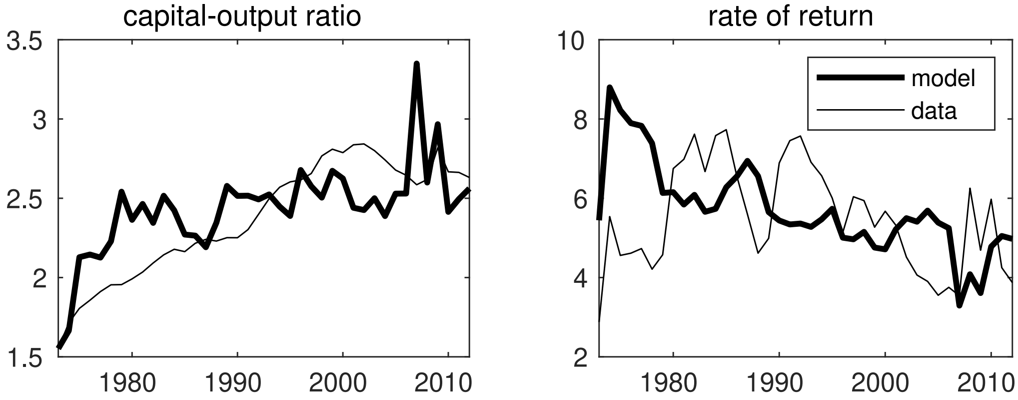


Figure 7: Aggregate capital and rate of return.



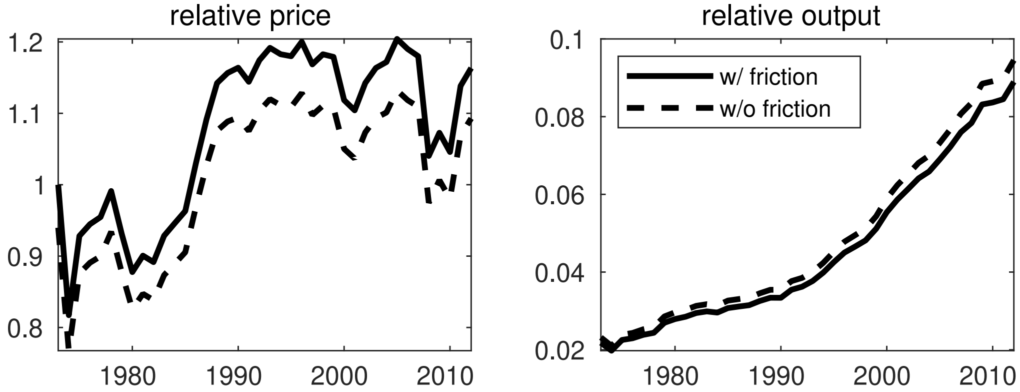
Notes: The rate of return is adjusted for corporate tax by multiplying it by 0.5.

Having obtained a good fit of the model with the data, we also evaluate some other endogenous variables. The endogenous component of sectoral TFP displays a downward trend as both the asset price (i.e., the capital–output ratio) and the MPRK exhibit upward trends. Sector 1 TFP declines by 1% from the beginning to the end of the sample period (from 0.945 to 0.935). Recall that the sectoral TFP already falls by 6% in the steady state (shown by the red line). Relative output also has an upward trend, as does the consumption share.

Finally, we perform a counter-factual simulation to evaluate the role of financial frictions on sectoral TFP and structural change. Recall that sectoral TFP is decomposed as $z_t^i = \zeta_t^i \eta_t^i$. z_t^i is given by the data and η_t^i is endogenously obtained depending on the MRPK. Therefore, we can trace out the sequence of $\zeta_t^i = z_t^i/\eta_t^i$ in the model with financial frictions, which can be interpreted as sectoral TFP in the model without financial frictions. We simulate the model without financial frictions using $z_t^i = \zeta_t^i$ as the exogenous sequence of sectoral TFP that we feed into the model and setting $\theta = 1/(\beta\beta_b)$ and $\beta_b = \beta$.

Figure 8 depicts the relative prices and outputs for the models with and without financial frictions. As the new sector (sector 1) is more constrained, its efficiency wedge (sectoral TFP) and capital wedge decrease more. Therefore, the relative price increases and the supply curve shifts upwards (as shown in Figure 3), which dampens the relative output in the model with financial frictions. As the endogenous component of sectoral TFP involves additional downward pressure owing to demographic change, the gap in the relative price and output widens toward the end of the sample period.

Figure 8: Relative price and output.



5 Concluding Remarks

We examined the effects of a decline in the working population ratio by inducing changes in labor input and the structure of household demand in a two-sector model with financial frictions. We assumed that firms have differing financial needs and that the new sector in which demand increases given structural change is more constrained than the old sector in which demand declines. The original two-sector model with financial frictions is equivalent to a prototype two-sector model with wedges in terms of allocations and prices, while the wedges themselves depend on the degree of capital misallocation among firms and the structure of household demand.

We found that the decline in labor input strengthens the collateral constraints of borrowers and generates capital misallocation and lower sectoral TFP. With respect to the implications for structural change, as the new sector is more constrained than the old, the new sector's TFP is also lower than that in the old sector, and the supply of goods produced in the new sector is much less than in the case with no financial frictions, thereby impeding structural change.

We also demonstrated that our theoretical findings for a two-sector economy with *endogenous* wedges can explain the actual Japanese data for 1973–2012. We endogenized the wedges and considered the role of capital misallocation in structural change to investigate the effects of technology shocks and structures in our stylized model. We found that financial frictions and capital misallocation may have indeed impeded structural change.

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