Credible Forward Guidance*

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Abstract

How can the central bank credibly implement a "lower-for-longer" strategy? To answer this question, we analyze a series of optimal sustainable policy problems—indexed by the duration of reputational loss—in a sticky-price model with an effective lower bound (ELB) constraint on nominal interest rates. We find that, even when the central bank lacks commitment, the central bank can still credibly keep the policy rate at the ELB for an extended period—though not as extended as under the optimal commitment policy—and meaningfully mitigate the adverse effects of the ELB constraint on economic activity.

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1 Introduction

In order to anchor longer-term inflation expectations at this level, the Committee seeks to achieve inflation that averages 2 percent over time, and therefore judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time.

Board of Governors of the Federal Reserve System [1]

To maintain the symmetry of its inflation target, the Governing Council recognises the importance of taking into account the implications of the effective lower bound. In particular, when the economy is close to the lower bound, this requires especially forceful or persistent monetary policy measures to avoid negative deviations from the inflation target becoming entrenched. This may also imply a transitory period in which inflation is moderately above target.

European Central Bank [2]

Developing effective strategies to manage the adverse consequences of the effective lower bound (ELB) constraint on nominal interest rates is an important task for economists and central bankers. In forward-looking models with an ELB, the commitment to keeping the policy rate at the ELB for an extended period—and temporarily overshooting inflation and output targets—is known to be effective in stimulating economic activity during a deep recession, as the anticipation of an overheated economy leads forward-looking households and firms to increase consumption and set higher prices.[3] We refer to this type of policy as overheating commitment or lower-for-longer policy in this paper.

Recently, as shown in the quotes above, some central banks such as the Federal Reserve Board and the European Central Bank have become more willing to allow temporary overshooting of inflation above the target. (The Bank of Japan has also adopted this type of policy since November 2016.) While these policies cannot be strictly interpreted as a form of commitment, they share common elements with overheating commitment or lower-for-longer policy.

One key argument against overheating commitment policy is its potential time-inconsistency. Ex ante, it is desirable to promise to overheat the economy in the future, as the expectations of future overheating stimulate inflation and output when the economy faces headwinds and

[1] "2020 Statement on Longer-Run Goals and Monetary Policy Strategy" https://www.federalreserve.gov/monetarypolicy/review-of-monetary-policy-strategy-tools-and-communications-statement-on-longer-run-goals-monetary-policy-strategy.htm.

[2] "The ECB's monetary policy strategy statement" https://www.ecb.europa.eu/home/search/review/html/ecb.strategyreview_monpol_strategy_statement.en.html.

[3] See, for example, Reifschneider and Williams (2000); Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006).

the ELB is a binding constraint. However, once the headwinds dissipate, the central bank will have an incentive to renege on the promise of overheating the economy by raising the policy rate, because the overheating is ex post undesirable. Then, a natural question would be how much overheating is needed to stimulate the economy while keeping monetary policy credible.

In this paper, we study credible overheating commitment policies in a sticky-price model with the ELB with an eye towards understanding the best allocations the central bank can credibly achieve when the optimal commitment policy is not credible. Specifically, we formulate and solve a series of *optimal sustainable policy problems* in which the central bank chooses state-contingent allocations to maximize welfare subject to not only private-sector equilibrium conditions, but also an incentive compatibility constraint—known as the sustainability constraint. The sustainability constraint requires that the continuation value associated with the chosen state-contingent allocation has to be at least as large as the continuation value associated with deviating from that allocation—and falling into a discretionary regime for N periods—at any time and after any history of shocks. Under certain conditions discussed in Nakata (2018), the sustainability constraint does not bind and the optimal sustainable policy coincides with the optimal commitment policy.[4] Our main interest is to characterize optimal sustainable policies when the sustainability constraint occasionally binds.

Our main result is that, even when the optimal commitment policy is not credible, the central bank can still credibly keep the policy rate at the ELB for an extended period in the aftermath of a crisis—though not as extended as under optimal commitment policy. As in optimal commitment policy, such lower-for-longer policy generates a temporary post-crisis overheating of the economy and mitigates the declines in output and inflation in a crisis through expectations. Under reasonable assumptions regarding how long the central bank suffers from a loss of reputation after reneging on the promise of lower-for-longer, the welfare cost of the ELB constraint is substantially lower under an optimal sustainable policy than under optimal discretionary policy.

One key feature of optimal sustainable policies is that they are less history dependent than optimal commitment policy. As discussed in detail by Eggertsson and Woodford (2003), a key feature of optimal commitment policy is history dependence. In particular, under optimal commitment policy, the additional period at which to keep the policy rate at the ELB in the aftermath of a crisis—as well as the magnitudes of the output and inflation overshoots—increases as the realized crisis shock duration increases. When the reputational

[4]Specifically, in Nakata (2018), if the central bank were to renege on the promise of overheating the economy in the aftermath of a crisis, it would lose reputation and private-sector agents would not believe similar promises in future crises. If private-sector agents do not believe the central bank's promise to overheat the economy, future ELB episodes will be associated with large declines in inflation and output. Thus, concern for maintaining reputation gives the central bank an incentive to fulfill the promise of keeping the lower-for-longer promise. According to Nakata (2018), this incentive to maintain reputation dominates the short-run incentive to eliminate the overheating of the economy—and as a result, the optimal commitment policy is credible—if that the policy rate is expected to fall into the ELB in the future with sufficient frequency and the loss of reputation lasts for a sufficiently long duration.

force is strong, the policy rate path under optimal sustainable policies exhibit qualitatively similar history dependence, though the degree of history dependence is weaker than under optimal commitment policy. When the reputational force is sufficiently weak, the policy rate paths under optimal sustainable policies do not feature any history dependence. That is, the additional ELB duration does not depend on the realized crisis shock duration.

We also analyze similarities between optimal sustainable policies and average inflation targeting policies. Average inflation targeting can be seen as a way of implementing overheating commitment policies (see Budianto, Nakata, and Schmidt, 2023). When the averaging window is one period, average inflation targeting coincides with the standard inflation targeting, which acts similarly to the optimal discretionary policy. When the averaging window is infinity, average inflation targeting coincides with the price-level targeting, which acts similarly to the optimal commitment policy. With the averaging window in between one and infinity, average inflation targeting can capture a variety of outcomes with differing degrees of commitment, just as optimal sustainable policies can capture a variety of outcomes with the punishment duration. Therefore, our optimal sustainable policies can be interpreted as offering a microfoundation of average inflation targeting policies.

Our optimal sustainable policies are of interest to central banks for two reasons. First, by construction, the optimal sustainable policies are time-consistent; thus, it is immune to the criticism that the promised overshoot of inflation and output associated with any lower-for-longer strategies may not be credible. Second, when the duration of reputational loss is sufficiently short, the policy rate path under optimal sustainable policies are not history dependent. Thus, it overcomes the criticism that, because the policy rate path associated with a lower-for-longer strategy is complex, it is difficult for central banks to clearly explain these strategies to the public.

Our paper builds on the literature on optimal monetary policy in the New Keynesian model with the ELB. This literature has demonstrated the effectiveness of lower-for-longer policies in stimulating the economy at the ELB, assuming that the central bank is equipped with an explicit commitment technology (Eggertsson and Woodford (2003); Jung, Teranishi, and Watanabe (2005); Adam and Billi (2006); and Nakov (2008)). Our paper contributes to this body of work by characterizing optimal sustainable policies in a model with the ELB and showing that the central bank can credibly engage in lower-for-longer policies even in the absence of an explicit commitment technology.

Within the literature on optimal policy and the ELB, some authors have explored ways to implement lower-for-longer policies at the ELB in a time-consistent way. Eggertsson (2006) and Burgert and Schmidt (2014) show that in models with non-Ricardian fiscal policy, a discretionary government can provide incentives to a future government to keep the policy rate at the ELB for longer by adopting expansionary fiscal policy and raising the nominal level of government debt. Jeanne and Svensson (2007), Berriel and Mendes (2015), and Bhattarai, Eggertsson, and Gafarov (2013) show that central banks' balance sheet policies can act as a commitment device that allows the central bank to credibly implement lower-for-longer policies. Billi (2017) and Nakata and Schmidt (2019b) analyze policy delegation in models with the ELB, showing that lower-for-longer policies can be implemented in a time-consistent way if the discretionary central bank's standard dual-mandate objective function is replaced by a nominal-income stabilization objective or augmented with an interest-rate smoothing objective, respectively. Unlike these papers that either introduce a new policy instrument or modify the central bank's objective function, we use reputation to achieve lower-for-longer policies in a time-consistent way.

Our paper is closely related to Nakata (2018) and Walsh (2018). Nakata (2018) has shown that optimal commitment policy in the New Keynesian model with the ELB can be made time-consistent by a particular trigger strategy capturing the reputational concern of the central bank. Our paper is different from Nakata (2018) because we study the best allocations the central bank can credibly achieve when the optimal commitment policy is not credible, whereas Nakata (2018) characterizes the conditions under which the optimal commitment policy is credible. Walsh (2018) examines credibility of simple policy rules with forward guidance—those that keep the policy rate at the ELB for a fixed number of periods after crises—and reaches a conclusion similar to that of Nakata (2018).[5] Our paper is different from Walsh (2018) because we characterize the optimal allocation the central bank can credibly achieve subject to a sustainability constraint, whereas Walsh (2018) studies credibility of simple policy rules that may or may not be optimal.[6] It turns out that there is an interesting relationship between our optimal sustainable policies and the forward guidance policy of Walsh (2018), which will be discussed in detail in Section 4.3.[7]

This paper is also closely related to the work of Dong and Young (2019) which uses the recursive method of Abreu, Pearce, and Stacchetti (1990), Chang (1998), and Phelan and Stacchetti (2001) to characterize the entire set of sustainable plans in a fully nonlinear New Keynesian model with the ELB. We discuss this paper in detail in Section 4.2.

Finally, our paper is related to a set of papers that characterize optimal allocations in macroeconomic models with a sustainability constraint. Kehoe and Perri (2002) characterize the optimal allocation in an international business cycle model in which a deviation from the promised plan would push the economy to autarky. Fujiwara, Kam, and Sunakawa (2019b) study the optimal sustainable policy in a two-country model in which the deviation from the promised cooperative plan would push the countries into a non-cooperative regime. The most

[5]See also Sukeda (2018), which extends the analysis of Walsh (2018) to a model with a discounted Euler equation and a discounted Phillips curve.

[6]Nakata (2018) and Walsh (2018) in turn build on earlier work of reputation in macroeconomics, including Barro and Gordon (1983), Rogoff (1987), Chari and Kehoe (1990), Chang (1998), and Phelan and Stacchetti (2001), among many others. Recent contributions include Kurozumi (2008) and Loisel (2008).

[7]See also Barthélemy and Mengus (2018) who examine sustainability of optimal commitment policy in a model with the ELB constraint in which the central bank's objective function—either a benevolent or conservative kind—is unknown to private-sector agents and there is an inflationary bias. In their model, the benevolent central bank can make the optimal commitment policy sustainable by raising inflation prior to a liquidity trap and signaling its type to private-sector agents.

closely related to our paper is Sunakawa (2015) who characterizes the optimal sustainable policy in a New Keynesian model with cost-push shocks but without the ELB constraint. Our paper applies the same analytical framework and methodology used in these papers to models with the ELB.

The rest of the paper is organized as follows. Section 2 describes the model and the central bank's optimization problems. Section 3 presents the results. Section 4 provides additional discussions and results. Section 5 concludes.

2 Model

2.1 Private sector

Our main model is a semi-loglinear New Keynesian model with a static Phillips curve. The private-sector equilibrium conditions of this model are given by:

$$y_t(s^t) = E_t y_{t+1}(s^{t+1}) - \sigma(i_t(s^t) - E_t \pi_{t+1}(s^{t+1}) - r^*) + s_t \tag{1}$$

$$\pi_t(s^t) = \kappa y_t(s^t) + \beta E_t \pi_{t+1}(s^{t+1}) \tag{2}$$

$$i_t(s^t) \geq i_{ELB} \tag{3}$$

where y_t is output, π_t is inflation, and i_t is the policy rate. Equations (1) and (2) are the Euler equation and the Phillips curve, respectively. Inequality (3) imposes the ELB constraint, denoted by i_{ELB}, on the policy rate. σ is the intertemporal elasticity of substitution, $r^* > 0$ is the natural rate of interest at the deterministic steady state, and κ is the slope of the Phillips curve.[8]

The exogenous shock in the Euler equation, s_t, is the natural rate shock and s^t denotes a history of shocks up to time t. That is, $s^t := \{s_k\}_{k=1}^t$. Because there is uncertainty, allocations are state-contingent and depend on s^t. We refer to the state-contingent sequence of consumption, inflation, and the nominal interest rate, $\{y_t(s^t), \pi_t(s^t), i_t(s^t)\}_{t=1}^{\infty}$, as *an outcome*. Given a process for s_t, an outcome is said to be *competitive* if, for all $t \geq 1$ and $s^t \in \mathbb{S}^t$, (i) $y_t(s^t) \in \mathbb{R}$, $\pi_t(s^t) \in \mathbb{R}$, $i_t(s^t) \in \mathbb{R}$, where $\mathbb{R}$ denotes a set of real numbers, and (ii) equations (1)-(3) are satisfied.

We assume that s_t follows a two-state Markov process with $s_t = r^* > 0$ in the "high" or "normal" state and $s_t = r_c < 0$ in the "low" or "crisis" state. The probability of moving from the high/normal state to the low/crisis state is denoted by p_H and will be referred to as the crisis frequency, whereas the probability of moving from the low/crisis state to the low/crisis

[8]In the working paper version, we used the model with a static Phillips curve as well as models with discounted Euler equation and discounted Phillips curve. See also Bilbiie (2019) who uses a model with a static Phillips curve. See Nakata, Ogaki, Schmidt, and Yoo (2019) and Levin and Sinha (2020a,b) for a systematic analysis of optimal commitment policies with varying degrees of discounting in the Euler equation and Phillips curve.

state is denoted by p_L and will be referred to as the crisis persistence. Following Nakata (2018), we allow p_H to be non-zero, which opens up the possibility for a reputational concern to make lower-for-longer policies credible.

The central bank's *value* at period t is given by

$$V_t(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u\left(\pi_{t+j}(s^{t+j}), y_{t+j}(s^{t+j})\right) \tag{4}$$

where the per-period objective function is given by the following function.

$$u(\pi, y) := -\frac{1}{2}\left[\pi^2 + \lambda y^2\right] \tag{5}$$

This quadratic objective function can be obtained as the second-order approximation to the household's welfare.[9] For any outcome, there is an associated state-contingent sequence of values, $\{V_t(s^t)\}_{t=1}^{\infty}$, which will be referred to as the value sequence.

2.2 Central bank

We will consider three classes of competitive outcomes that differ in how the central bank sets its interest rate policy: the discretionary outcome, the commitment outcome, and the sustainable outcomes. Appendix B explains the optimization problem the central bank faces in detail.

2.2.1 Discretionary outcome

At each time t, the discretionary central bank's optimization problem is to choose $\{y_t, \pi_t, i_t\}$ to maximize the value today, taking as given the value function ($W_{t+1}(\cdot)$) and policy functions for inflation and output ($\pi_{t+1}(\cdot)$ and $y_{t+1}(\cdot)$) in the next period. That is,

$$W_t(s_t) = \max_{\pi_t, y_t, i_t} \quad u(y_t, \pi_t) + \beta E_t W_{t+1}(s_{t+1}), \tag{6}$$

subject to equations (1), (2), and (3).

Let $\{W_{do}(\cdot),\ \pi_{do}(\cdot),\ y_{do}(\cdot),\ i_{do}(\cdot)\}$ be the set of time-invariant value and policy functions that solve the Bellman equation above and in which the ELB binds only in the crisis state.[10] They are functions of today's shock realization, s_t. The discretionary outcome is defined as, and denoted by, the state-contingent sequence of output, inflation, and the policy rate, $\{y_{do,t}(s^t),\ \pi_{do,t}(s^t),\ i_{do,t}(s^t)\}_{t=1}^{\infty}$ such that $y_{do,t}(s^t) = y_{do}(s_t)$, $\pi_{do,t}(s^t) = \pi_{do}(s_t)$, and $i_{do,t}(s^t) = i_{do}(s_t)$ and the discretionary value sequence is defined as, and denoted by,

[9] See, for example, Woodford (2003) and Galí (2015).

[10] There also exists a time-invariant solution to this discretionary government's problem in which the ELB binds in both states. See Armenter (2017), Nakata (2018), and Nakata and Schmidt (2019a) for extensive analyses of such deflationary Markov-perfect equilibrium.

$\{V_{do,t}(s^t)\}_{t=1}^{\infty}$ such that $V_{do,t}(s^t) = W_{do}(s_t)$. We will also refer to the discretionary outcome as the outcome under the optimal discretionary policy (ODP).

2.2.2 Commitment outcome

At the beginning of time one, the central bank with commitment technology chooses a state-contingent allocation, $\{y_t(s^t),\ \pi_t(s^t),\ i_t(s^t)\}_{t=1}^{\infty}$, to maximize the time-one value. That is,

$$V_{c,1}(s_1) = \max_{\{y_t(s^t),\pi_t(s^t),i_t(s^t)\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t(s^t), \pi_t(s^t)), \tag{7}$$

subject to equations (1), (2), and (3) for all $t \geq 1$ and after all histories of shocks s^t. The commitment outcome, or the Ramsey outcome, is defined as the solution to this optimization problem. In other words, the commitment outcome is a competitive outcome with the highest time-one value. We denote the commitment outcome by $\{y_{co,t}(s^t),\ \pi_{co,t}(s^t),\ i_{co,t}(s^t)\}_{t=1}^{\infty}$. The value sequence associated with the commitment outcome is denoted by $\{V_{co,t}(s^t)\}_{t=1}^{\infty}$ and will be referred to as the commitment value sequence. We will also refer to the commitment outcome as the outcome under the optimal commitment policy (OCP).

2.2.3 Sustainable outcomes

At the beginning of time one, the central bank chooses a state-contingent allocation, $\{y_t(s^t),\ \pi_t(s^t),\ i_t(s^t)\}_{t=1}^{\infty}$, to maximize the time-one value:

$$V_{so,1}(s_1) = \max_{\{y_t(s^t),\pi_t(s^t),i_t(s^t)\}_{t=1}^{\infty}} \quad E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(y_t, \pi_t), \tag{8}$$

subject to equations (1), (2), and (3), and the following sustainability constraint,

$$E_t \sum_{k=0}^{\infty} \beta^k u(y_{t+k}(s^{t+k}), \pi_{t+k}(s^{t+k})) \geq W_{do}^N(s_t), \tag{9}$$

for all $t \geq 1$ and after all histories of shocks, s^t. The left-hand side of the sustainability constraint is the continuation value of implementing a chosen state-contingent allocation at time t after s^t. The right-hand side, $W_{do}^N(s_t)$, is the continuation value if the central bank deviates from the chosen state-contingent allocation, with N indicating how many periods it takes for the central bank to restore its lost reputation ("punishment" duration). During the periods of reputational loss, the central bank cannot engage in state-contingent policies. That is, the central bank has to act under discretion.

$W_{do}^N(s_t)$ is recursively defined as follows. For $N = 0$,

$$W_{do}^0(s) := V_{so,1}(s), \quad \pi_{do}^0(s) := \pi_{so,1}(s), \quad y_{do}^0(s) := y_{so,1}(s).$$

In this case with $N = 0$, the punishment duration is zero and the central bank is not allowed to deviate from the sustainable outcome. For any $N > 0$,

$$W_{do}^{N}(s) = \max_{\pi,y,i} \quad u(\pi, y) + \beta E[W_{do}^{N-1}(s')|s]$$

where the maximization is subject to the private-sector equilibrium conditions, taking as given the value and policy functions for the next period (that is, $W_{do}^{N-1}(\cdot)$, $\pi_{do}^{N-1}(\cdot)$, and $y_{do}^{N-1}(\cdot)$).

Note that the sustainability constraint has to be respected each period and for each history of shocks, just as the Euler equation, the Phillips curve, and the ELB constraint have to be respected each period and for each history of shock. *The sustainable outcome with N-period reputational loss* is defined as the solution to this infinite-horizon optimization problem. We will also refer to the sustainable outcome with N-period reputational loss as the outcome under the optimal sustainable policy (OSP) with N-period reputational loss.

Note that the punishment value, $W_{do}^{N}(s_t)$, is determined jointly with the sustainable outcome, except when $N = \infty$. When $N = \infty$, the punishment lasts forever and its value is given by the discretionary value, $W_{do}(s)$, which is independent of the sustainable outcome. For any finite N, the central bank eventually restores its reputation and the economy returns to the allocations consistent with the sustainable outcome. Thus, the punishment value and the sustainable outcome are not independent of each other. All else equal, an increase (decrease) in the value associated with the sustainable outcome implies an increase (decrease) in the punishment value.

As described in detail in Appendix A, once the sustainable outcome is computed from the optimization problem above, we can construct a plan—a pair of central bank and private-sector strategies—that induces the sustainable outcome and that has a trigger-type structure. In particular, we can construct a revert-to-discretion plan in which (i) the economy follows the sustainable outcome as long as the central bank has never deviated from the policy rate path consistent with the sustainable outcome in the past, and (ii) the economy follows the discretionary outcome, or a temporary deviation to a discretionary regime, otherwise. By construction, such a revert-to-discretion plan is credible, meaning that neither the central bank nor private-sector agents have incentives to deviate from the instructions given by the strategies. The central bank does not have an incentive to deviate from the policy rate path consistent with the sustainable outcome because the sustainability constraint is imposed on the central bank's optimization problem, ensuring that the continuation value under the sustainable outcome is at least as large as the punishment continuation value. Private-sector agents do not have incentives to deviate from the private-sector strategy because the Euler equation and the Phillips curves are satisfied, meaning that the output and inflation are consistent with their optimizing behaviors given the central bank strategy. Even though the deviation does not occur in equilibrium, the specification of what would happen if the central bank were to deviate from the sustainable outcome does affect what happens under the sustainable outcome.

If the sustainability constraint does not bind at any time t and after any histories of shocks, the sustainable outcome coincides with the commitment outcome. Also, if the sustainability constraint always binds—which happens, for example, when the punishment length (N) is zero or when the crisis frequency (p_H) is zero—the sustainable outcomes coincides with the discretionary outcome. Our main interest is those cases in which the sustainability constraint occasionally binds.

2.3 Parameter values

Table 1 shows the baseline parameter values. The quarterly frequency of crises is set to 0.5/100 (=2/400). This choice is motivated by the fact that, in the United States, there have been two large crises that pushed the short-term nominal interest rate to the ELB over roughly the last 100 years (400 quarters) since the creation of the Federal Reserve System. The crisis shock persistence is set to 2/3, which implies the expected duration of the crisis shock of 3 quarters. The intertemporal elasticity of substitution, σ, is set to 1. The natural rate of interest in the crisis state, r_c, and the slope of the forward-looking Phillips curve, κ, are chosen so that output declines 7 percent and inflation declines 1 percentage point (annualized) in the crisis state under the optimal discretionary policy. This severity of the crisis is consistent with that considered in Boneva, Braun, and Waki (2016) and Nakata (2018), and is intended to capture the severity of the Great Recession of 2007-2009 in the United States. λ is set to 0.0012, a value consistent with a microfounded objective function when the elsticity of substitution across intermediate goods is set to 10.

Table 1: Baseline Parameter Values

Parameter	Description	Parameter Value
β	Discount rate	$\frac{1}{1+0.0075} \approx 0.9925$
σ	Intertemporal elasticity of substitution	1
κ	Slope of the Phillips curve	0.012
λ	Relative weight on output volatility	0.0012
p_H	Crisis shock frequency	0.5/100
p_L	Crisis shock persistence	2/3
r^*	Natural rate in the normal state	3/400
r_c	Natural rate in the crisis state	-0.030
N	Punishment length	[16, 40, ∞]

We consider three values for the duration of reputational loss (16, 40, and ∞), which are chosen to cover qualitatively distinct cases that can arise. We put these values into perspective in Section 3.3.

2.4 Solution method

The model is highly nonlinear, featuring two inequality constraints—the ELB constraint and the sustainability constraint—and cannot be solved analytically. Following Marcet and

Marimon (2019), Kehoe and Perri (2002) and Sunakawa (2015), we recursify the infinite-horizon optimization problem of the central bank into a saddle-point functional equation using the Lagrange multipliers on the Euler equation and the Phillips curve as pseudo-state variables. We then apply a projection method to find the set of time-invariant policy functions that solve the saddle-point functional equation. Appendix C describes the details of the solution method as well as its accuracy.

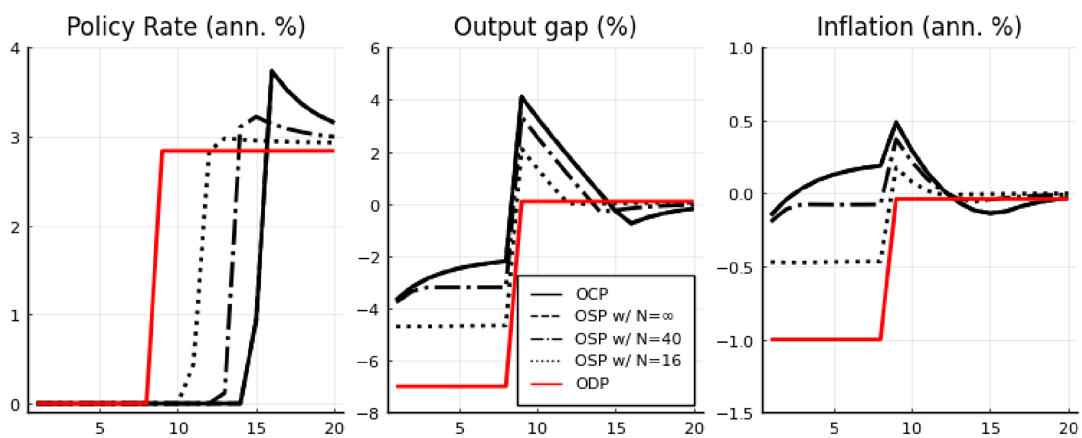
3 Results

3.1 Dynamics

Figure 1 shows the dynamics of the economy under the ODP, the OCP, and OSPs with $N = [16, 40, \infty]$.

In this figure, the crisis shock hits the economy at time 1 and stays there until time 8. The crisis shock disappears at time 9 and the economy is in the normal state from that point on.

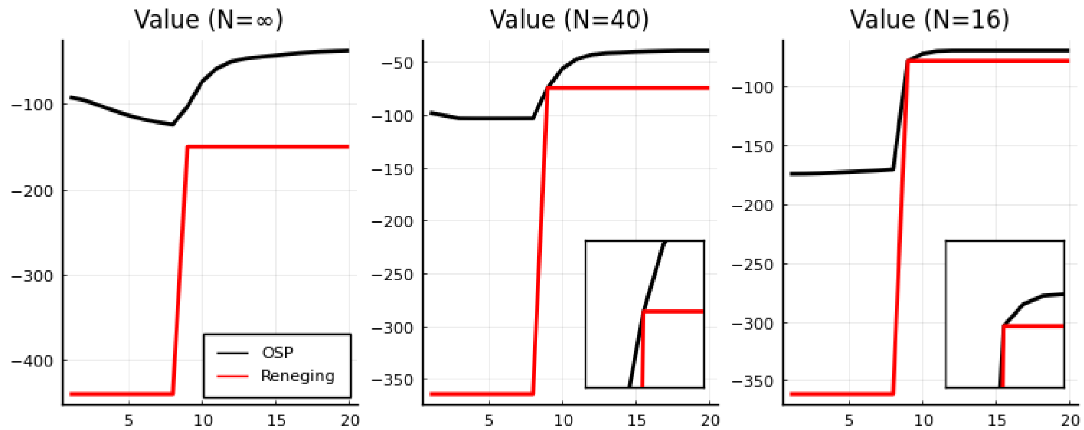
Figure 1: Dynamics



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

Under the ODP—shown by the solid red lines—the central bank keeps the policy rate at the ELB as long as the crisis shock persists and raises the policy rate immediately after the crisis shock disappears. Under the OCP—shown by the solid black lines—the central bank keeps the policy rate at the ELB even after the crisis shock disappears, engineering the overshooting of inflation and output above their targets. Since households are forward looking, the anticipation of high inflation and high output in the aftermath of the crisis stimulates economic activity during the crisis. The declines in inflation and output are substantially smaller under the OCP than under the ODP.

Figure 2: Value of fulfilling versus reneging on the promised allocations



Note: OSP stands for optimal sustainable policy.

The allocations under the OSP with $N = \infty$ are identical to those under the OCP in this crisis scenario. As shown in the left panel of Figure 2, the value under the OSP with $N = \infty$—shown by the solid black—is always above the value in case the central bank deviates from the OSP with $N = \infty$—shown by the dashed red line. That is, the sustainability constraint does not bind. In our calibration, the crisis shock is sufficiently frequent so that the cost of being unable to use lower-for-longer policies in the future forever outweighs the benefit of eliminating the temporary overshooting of inflation and output targets. This result is consistent with the finding of Nakata (2018) that a very small probability of being hit by the crisis shock suffices to make the OCP credible.

When the loss of reputation is not as long, the cost of reneging on the lower-for-longer promise in the aftermath of the crisis shock is smaller. In other words, the continuation value in case of deviation is higher with a smaller N. The middle panel of Figure 2 shows the value of the OSP with $N = 40$ and the value of deviating from the OSP with $N = 40$ (solid black and dashed red lines, respectively). According to the panel, the sustainability constraint binds right after the crisis shock disappears, limiting the magnitude of the inflation and output overshooting in the aftermath of the crisis. The smaller overshoot means that inflation and output decline by more during the crisis under the OSP with $N = 40$ than under the OCP and the OSP with $N = \infty$. However, the declines in inflation and output are still much smaller under the OSP with $N = 40$ than under the ODP.

Similarly, the sustainability constraint binds right after the crisis shock disappears under the OSP with $N = 16$, as can be seen in the right panel of Figure 2, limiting the magnitudes of the inflation and output overshoots. The overshoots in the aftermath of the crisis are smaller—and as a result, the declines in inflation and output are larger—under the OSP with $N = 16$ than under the OSP with $N = 40$. Even with $N = 16$, the declines in inflation and output are still much smaller under the OSP than under the ODP.

Reflecting the less severe crisis under the OSP with $N = 16$ and $N = 40$, the welfare cost of the ELB—shown in Table 2—is substantially lower under these OSPs than under the ODP. With $N = 40$, welfare cost of the ELB is about 20 percent of that under the ODP and is only slightly larger than under the OCP. Even with $N = 16$, the welfare cost of the ELB is only about one third of that under the ODP.

Table 2: Welfare Cost of the ELB

	abs($E[V]$)
Optimal commitment policy	38.1 (0.18)
Optimal sustainable policy	
with $N = \infty$	38.1 (0.18)
with $N = 40$	40.2 (0.19)
with $N = 16$	71.3 (0.34)
Optimal discretionary policy	207.7 (1.00)

Note: Numbers in parentheses are the welfare cost of the ELB relative to that under the optimal discretionary policy.

3.2 History Dependence

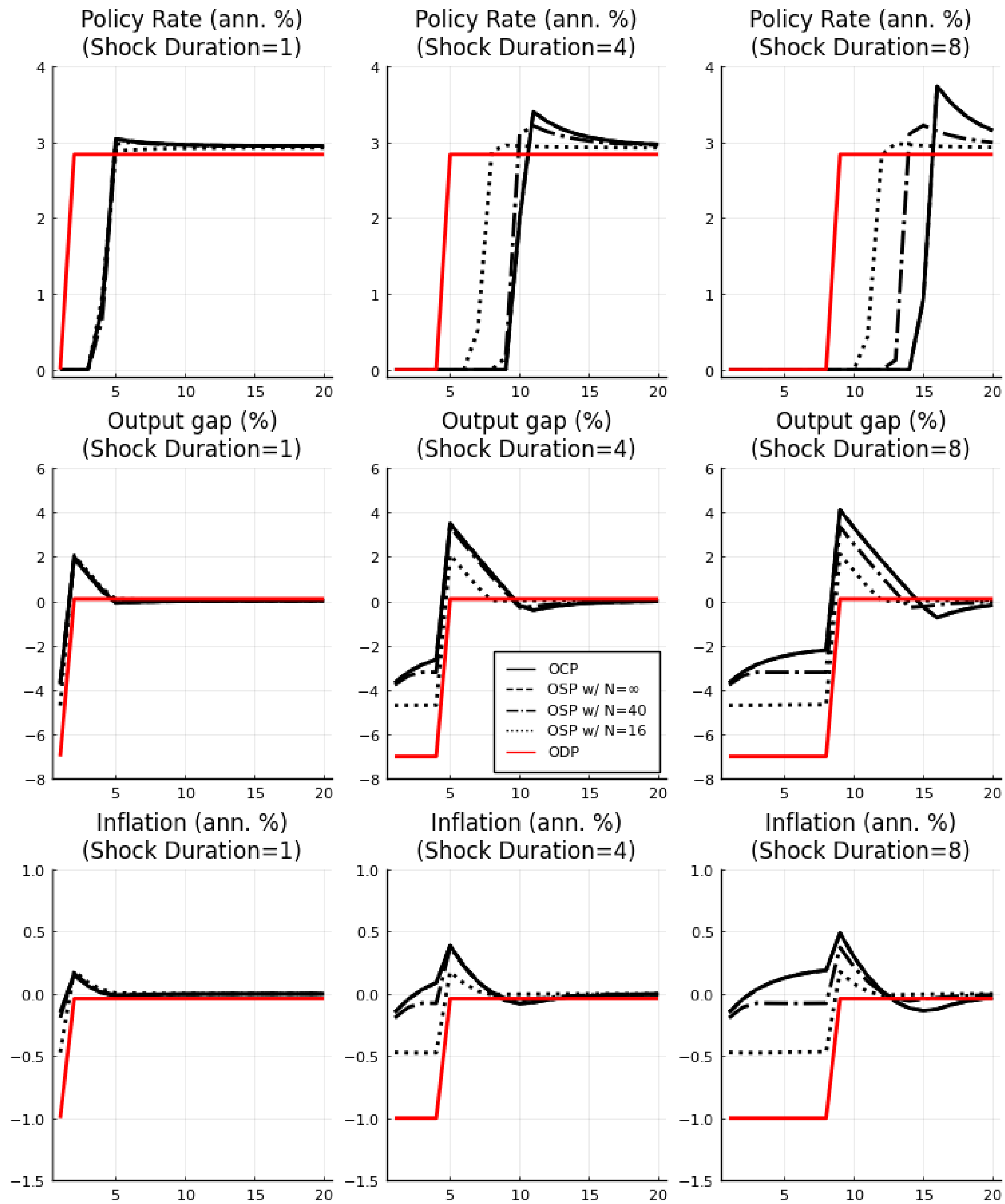
Figure 3 shows the dynamics of the policy rate, the output gap, and inflation—displayed in the top, middle, and bottom rows, respectively—under three alternative realized durations of the crisis shock. The first, second, and third columns are for the realized crisis shock duration of 1, 4, and 8 quarters, respectively.

Under the OCP—shown by the solid black lines—the additional ELB duration is 2, 5, and 6 quarters when the realized crisis shock duration is 1, 4, and 8 quarters, respectively, as can be seen in the bottom panels of Figure 3. The magnitude of the output (inflation) overshoot is 1.9 (0.15) percentage points, 3.5 (0.39) percentage points, and 4.1 (0.49) percentage points when the realized crisis shock duration is 1, 4, and 8 quarters, respectively, as can be seen in the middle (bottom) panels of Figure 3. Thus, both the additional ELB duration and the sizes of the inflation and output overshoots depend on the realized crisis shock duration. This dependence can be seen in Figure 4, which shows how the additional ELB duration and the size of the inflation and output overshoot vary with the realized crisis shock duration.[11] The OSP with $N = \infty$ exihibits history dependence that is identical to the OCP, as the sustainability constraint does not bind in any state and the allocations under the OCP and the OSP with $N = \infty$ are identical.

The history dependence of the OCP is in sharp contrast with the lack of history dependence in the ODP. Under the ODP—shown by the solid red lines in Figure 3 and 4—the additional

[11] In computing the additional ELB duration and the sizes of the output overshoots, we assume that, prior to the crisis shock, the economy has been in the normal state for some time and the Lagrange multipliers on the Euler equation and the Phillips curve is zero in the period right before the crisis shock materializes.

Figure 3: History Dependence (I)
—Dynamics with Alternative Realized Crisis Shock Durations—

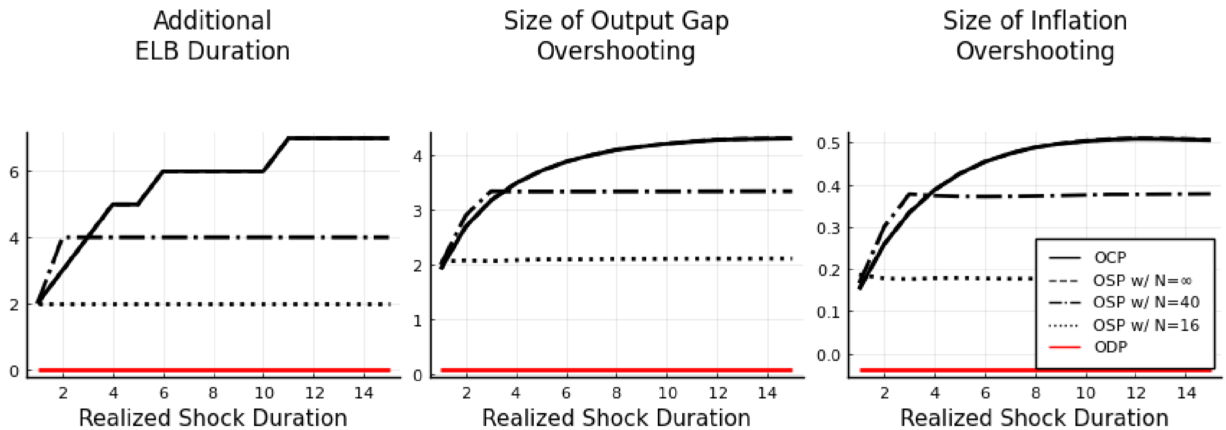


Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent.

ELB duration and the sizes of the inflation and output overshoots do not depend on the realized crisis shock duration.

The OSP with $N = 40$ is history-dependent, but less so than the OCP or the OSP with $N = \infty$. Under the OSP with $N = 40$, as the realized crisis shock duration increases

Figure 4: History Dependence (II)



Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The output gap is expressed in percent.

from 1 quarter to 2 quarters, the additional ELB duration increases from 2 quarters to 4 quarters. Thereafter, the additional ELB duration stays at 4 quarters. The sizes of the inflation and output overshoots increase initially as the realized crisis shock duration increases, similarly to what happens under the OCP. However, as the shock duration exceeds above 3, the sustainability constraint starts binding and the sizes of the output and inflation overshoots stays stable, albeit not perefectly constant. Overall, the policy rate path as well as the sizes of the inflation and output overshoots are less history dependent than under the OCP.

Under the OSP with $N = 16$, the additional ELB duration is 2 quarters regardless of the realized shock duration. In other words, the policy rate path is not history dependent at all. While the sizes of the output and inflation overshoots are not perfectly constant but remain stable as the realized shock duration increases.

3.3 On "reasonable" duration of reputational loss

When the loss of reputation lasts for a long time, the power of reputation is strong and OSPs resemble the OCP. When the loss of reputation lasts for a short time, the power of reputation is weak and OSPs resemble the ODP. A natural question that arises is what reasonable values of the duration of reputational loss are.

We discuss two ways to think about this question.

One way to think about the reasonable duration of reputational loss is to hypothetically ask how long it might take for a central bank to restore its reputation once it loses it. The reasonable value of the duration of reputational loss based on this thought experiment may depend on various factors: whether one believes the central bank's reputation is individual-specific or institution-specific and the tenure duration of the central bank's governors or chairs, if one believes in the individual specific nature of reputation.

The assumption that the central bank can restore its reputation after a finite number of periods can be motivated by the fact that the tenure of governorship at central banks is finite as well as the possibility that reputation may be specific to the leader of the central bank, as opposed to the institution. As shown in Table 3, the average tenure of the governorship in central banks in economies that have recently faced, or are currently facing, the ELB ranges from about 5 years (20 quarters) in the Bank of Japan to about 10 years (40 quarters) for the Bank of Canada. The maximum tenure duration exceeds 15 years (60 quarters) at several central banks (the Federal Reserve, Bank of Canada, Bank of England, and Sveriges Riksbank), as shown in Table 4.

Table 3: Average Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Year of foundation	No. of leaders since foundation	No. of leaders since 1946	Average tenure since foundation	Average tenure since 1946
Federal Reserve System	1914	16	10	7.0	8.1
European Central Bank	1998	4	4	7.0	7.0
Bank of Canada	1934	10	9	9.6	9.4
Bank of Japan	1882	32	16	4.5	5.2
Bank of England	1694	121	10	2.7	8.3
Sveriges Riksbank	1901	15	12	9.0	7.1
Swiss National Bank	1907	14	10	8.1	7.4

Note: In computing the average tenure duration, we exlude the current chairperson/governor/president whose tenure is yet to be concluded.

Table 4: Maximum Tenure Duration of Chairpersons in Select Central Banks

Central Bank	Max duration since foundation	Max duration since 1946
Federal Reserve System	18 yrs and 10 months (Martin)	18 yrs and 10 months (Martin)*
European Central Bank	8 yrs (Trichet and Draghi)	8 yrs (Trichet and Draghi)
Bank of Canada	20 yrs (Towers)	14 yrs (Boey)
Bank of Japan	10 yrs (Kuroda)	10 yrs (Kuroda)
Bank of England	24 yrs (Norman)	12 yrs (Cobbold)
Sveriges Riksbank	19 yrs (Rooth)	18 yrs (Asbrink)
Swiss National Bank	14 yrs (Bachmann)	11 yrs (Jordan)

Note: The tenure of Alan Greenspan lasted for 18 years and 6 months.

Another way to think about the reasonable duration of reputational loss is to theoretically refine the concept of sustainability. In models in which the commitment and discretionary outcomes are different, there are multiple—typically infinitely many—sustainable plans. In this paper, we study infinitely many (countable) sustainable plans indexed by the duration of lost reputation. By imposing further restrictions on the set of sustainable plans and thus refining the concept of sustainability, we can select one of these sustainable plans as being more reasonable than others.

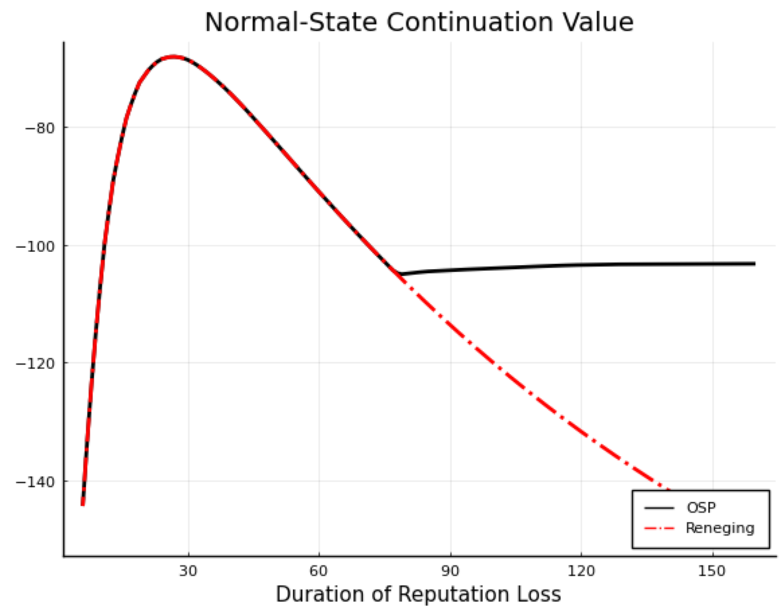
One refinement concept for any sequential equilibria—a sustainable plan in our setup—developed by game theorists in the context of two-player games is renegotiation proofness. Roughly speaking, renegotiation proofness requires that, even if the deviation from an equilibrium were to occur hypothetically, two players would have no incentives to renegotiate the contract—strategies in our setup—that they have initially agreed on. According to one definition of renegotiation-proofness proposed by Pearce (1987), a sustainable plan is renegotiation-proof if the punishment value associated with that plan is higher than the punishment value of any other sustainable plans.[12]

To apply this concept of renegotiation proofness to our model, the dash-dotted red line in Figure 5 shows the punishment value associated with deviating from the OSPs with different values for N. According to the figure, the punishment continuation value is non-monotonic. When N is large, a reduction in the punishment duration increases the punishment value: all else equal, it is good to stay in the discretionary regime for a shorter duration, as the value under the discretionary regime is lower than the value under the OSP. However, when the punishment duration is sufficiently short and the sustainability constraint binds, a shorter punishment duration lowers the value associated with the OSPs. This non-monotonicity arises because a shorter punishment duration limits the sizes of the inflation and output overshoots in the aftermath of crises and lowers the value associated with OSPs. As a result, when the punishment regime ends, the economy will return to a sustainable outcome that is not as good as the sustainable outcome with a longer punishment duration. When N is sufficiently small, this second effect dominates the first effect, and a shorter punishment duration lowers the punishment value.

According to Figure 5, in our model, the sustainable outcome with the least severe punishment value is the sustainable outcome with $N = 27$. The dash-dotted red lines in Figure 6 show the dynamics of the economy under the OSP with $N = 27$, together with those under the OCP and the ODP. The post-crisis ELB duration is 3 quarters under the OSP with $N = 27$, 3 quarter shorter than that under the OCP. The size of the post-crisis overshooting is smaller—and output and inflation decline by more—under the OSP with $N = 27$ than under the OCP. However, the declines in inflation and output are much smaller under the OSP with $N = 27$ than under the ODP. The welfare cost of the ELB constraint is about 23 percent of that under the ODP.

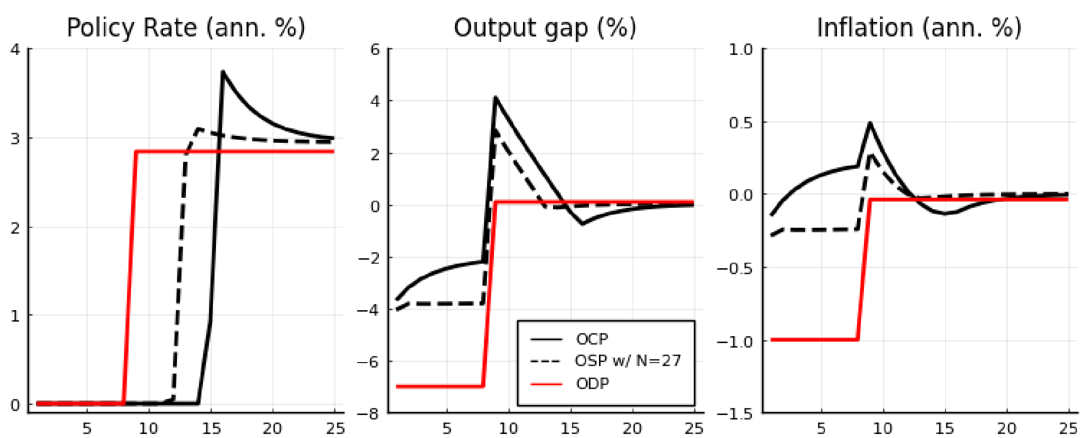
[12] Farrell and Maskin (1989) proposed an alternative definition of renegotiation-proofness whereby a sustainable plan is renegotiation proof if there is no Pareto-improving move to another sustainable plan after deviating from the on-equilibrium path at any point in time. As pointed out by Matsuyama (1997), in models with benevolent government where the government's objective function and the private-sector's objective function coincide, this definition rules out any chance for the economy to achieve allocations better than the ODP.

Figure 5: Values in case of reneging



Note: Continuation value under the sustainable outcome after a crisis that has lasted for 8 quarters.

Figure 6: Dynamics with $N = 27$

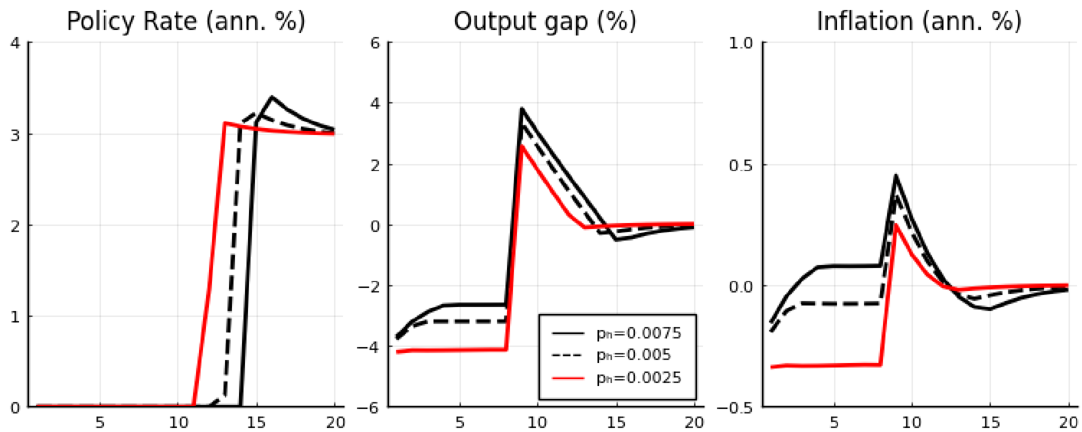


Note: ODP, OCP, and OSP stand for optimal discretionary policy, optimal commitment policy, and optimal sustainable policy, respectively. The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

3.4 The role of p_H

A key force that makes the lower-for-longer promise sustainable in our model is that the crisis shock can hit the economy in the future. Behind the sustainable outcome, there is a plan—a pair of central-bank and private-sector strategies—specifying that, if the central bank does not fulfill the lower-for-longer promise in the aftermath of the current recession, private-sector will not believe the same type of promise in future recessions ("the central bank loses its reputation"). If the central bank cannot effectively implement lower-for-longer policies, the economy will suffer large declines in inflation and output. Thus, the threat of punishment by private-sector agents to distrust lower-for-longer promises in future recession gives the central bank an incentive to fulfill the promise.

Figure 7: Dynamics under alternative crisis frequencies



Note: The output gap is expressed in percent.

This incentive for the central bank to fulfill the lower-for-longer promise is larger when the probability of a future crisis is higher. In the extreme case in which the crisis shock is a one-time event and the normal state is an absorbing state, as typically assumed in the literature, the incentive for the central bank to fulfill the promise does not exist, making any degree of lower-for-longer promises unsustainable. See Nakata (2015) and Nakata (2018) for more detailed exposition of the mechanism as well as the discussion of how other parameters—including p_L—affect the reputational cost of reneging on the lower-for-longer promise as well as its short-run stabilization benefit.

Figure 7 illustrates the role of p_H in our model. In this numerical simulation, we consider two alternative values of p_H (0.0025 and 0.0075) and the crisis shock is set to last for 8 quarters. According to the figure, the additional duration of being at the ELB in the aftermath of a crisis is longer—and the magnitudes of the inflation and output overshoots is larger—the higher the crisis probability (p_H) is. This numerical result is consistent with the mechanism described in the preceding paragraphs.

4 Additional discussion and results

4.1 Relation to the loose commitment approach of Bodenstein, Hebden, and Nunes (2012)

Under OSPs, the central bank achieves crisis-state allocations that are "in between" that under the ODP and that under the OCP. This feature of OSPs is reminiscent of the optimal policy obtained in a loose commitment approach in which the central bank reoptimizes with a constant probability every period regardless of the incentive to renege on the prior commitment.[13] While these two approaches differ from each other in many ways, both approaches share the same spirit that they are intended to shed light on what the central bank may be able to achieve when no explicit commitment technology is available. Indeed, recent work by Fujiwara, Kam, and Sunakawa (2019a) shows that, when using a model without the ELB, the allocations under the loose commitment approach with an appropriately chosen re-optimization probability can approximate the allocation under the OSP with N-period punishment reasonably well for any N. While we believe their result is likely to extend to the model with ELB, it would be useful to verify the validity of their claim in our model in future research.

4.2 Relation to Dong and Young (2019)

Our work is closely related to Dong and Young (2019). In this subsection, we discuss commonalities and differences between our work and Dong and Young (2019).

Dong and Young (2019) and our analysis both aim to characterize a set of outcomes that are credible. i.e. that do not suffer from the time-inconsistency problem. The difference is that we aim to characterize a subset of sustainable outcomes that are easy to interpret and that can be found using a methodology that is a variant of a widely used the method of Marcet and Marimon (2019), whereas Dong and Young (2019) aim to characterize the entire set of sustainable outcomes using a rigorous computational method that is based on Chang (1998). Our approach is more tractable than theirs in the sense that, for any punishment duration N, we obtain the associated sustainable outcome as the solution of the standard Ramsey problem with one additional incentive-compatibility constraint. Our tractability comes at the cost of not being able to explore the entire set of sustainable outcomes.

The best sustainable outcome (BSO) in Dong and Young (2019) looks quite different from any of our sustainable outcomes. Their BSO features (i) less aggressive lowering of the policy rate at the ELB, (ii) the policy rate never touching the ELB constraint, and (iii) larger inflation overshooting in the aftermath of a recession than under the Ramsey policy. These results are intriguing because (i) the policy rate is lowered to the ELB immediately upon the arrival of the shock both under the Markov Perfect policy and under the Ramsey policy and (ii) larger inflation overshooting—considered to be the key source of time-inconsistency in this model—suggests more severe time-inconsistency at least at a first glance. In other words, their BSO is not "in between" the Markov perfect policy and the Ramsey policy.

The exact reason for why the BSO looks the way it does is yet to be explored in the literature. The literature on sustainable outcomes suggests that the set of sustainable outcomes is large and features a sophisticated strategy of the government in many models. In contrast,

[13] See Bodenstein, Hebden, and Nunes (2012) for an analysis of optimal monetary policy under loose commitment in the model with ELB. See Levin and Sinha (2020a,b) for a more recent treatment.

for any given N, our sustainable outcome is "in between" the Markov perfect policy and the Ramsey policy, in a way that is reminiscent of the equilibrium in the imperfect commitment literature. Thus, some readers may find the result in our sustainable outcomes intuitive.

4.3 Relation to the simple forward guidance policies of Walsh (2018)

While our OSPs may be theoretically interesting, they may be hard to implement in reality. In this subsection and the next, we contrast our OSPs with two alternative simpler monetary policies that may be easier to communicate with the public: (i) policy in which the central bank keeps the policy rate at the ELB for a fixed additional duration after the crisis shock is gone and (ii) an average inflation targeting policy.

We have shown that, when N is sufficiently small, the policy rate paths under OSPs are not history dependent. That is, the policy rate path after the crisis shock disappears does not depend on the realized duration of the crisis shock. Thus, the OSPs bear some resemblance to the simple forward guidance policies considered by Walsh (2018). Under the simple forward guidance policies of Walsh (2018), the central bank keeps the policy rate at the ELB for a fixed number of periods after the crisis shock disappears, regardless of the realized duration of the crisis shock, and lets the policy rate return to the steady-state level immediately thereafter. The only (minor) difference is that, under the optimal sustainable policy, the policy rate does not return to the steady state level immediately after liftoff. Instead, there is typically one period after liftoff in which the policy rate is still below the steady-state level.

The similarity between the simple forward guidance policies and OSPs with small Ns points to one benefit of OSPs over the OCP; it may be easier for central banks to explain these OSPs to the public than the OCP. One key criticism against the OCP is that it is complex. As Walsh (2018) argues, because of its complexity, it may be difficult for the central bank in practice to steer the private-sector agents' expectations in a way consistent with the OCP. One dimension of complexity is history dependence. The OSPs have an advantage over the OCP because they are less history-dependent and thus simpler.

Note that, in our model, the policy rate path under the ODP and the OSPs with small Ns are history independent but state-contingent. They are state-contingent because the policy rate path—in particular the liftoff date—depends on the realized crisis shock duration. Thus, these policies are different from so-called calendar-based forward guidance that specifies the likely liftoff date, if that guidance were to be narrowly or mistakenly interpreted as a non-state-contingent commitment to raising the policy rate from the ELB at a particular date regardless of the evolution of the economy.[14]

[14]Even when central banks indicate a likely date of liftoff from the ELB, they typically emphasize that the liftoff date will depend on the evolution of the economic outlook. That is, if the economy were to recover faster or more slowly than in the baseline economic projection, the central bank will raise the policy rate from the ELB earlier or later than the most likely liftoff date under the baseline projection. In practice, it is unlikely that any central bank will ever engage in non-state-contingent forward guidance, though market participants may not interpret the forward guidance specifying the likely date of liftoff as state-contingent as the central bank intends.

4.4 Results from the model with average inflation targeting

In this section, we analyze similarities between optimal sustainable policies and average inflation targeting (AIT) policies—policies that feature overheating commitment as in OSPs and that have recently attracted a lot of attentions because of new monetary policy frameworks at the Fed and the ECB.

We consider a policy rule

$$i_t = s_t + \phi\omega^{-1}\hat{\pi}_t, \tag{10}$$

where

$$\hat{\pi}_t = \omega\pi_t + (1-\omega)\hat{\pi}_{t-1}, \tag{11}$$

ϕ is the degree of response to inflation, and $\omega \in [0,1]$ is a weight on the current inflation rate in an exponential moving average inflation rate $\hat{\pi}_t$. The central bank sets the policy rate by referring to the natural rate of interest s_t as in Cúrdia, Ferrero, Ng, and Tambalotti (2015). In addition, the central bank responds to an average of the past inflation rates $\hat{\pi}_t$. The rest of the equilibrium conditions consist of the Euler equation and Phillips curve, which are given by Equations (1) and (2). We also consider the ELB constraint (3) as before.

This policy rule (10) and (11) nests the following different strategies (See also Budianto, Nakata, and Schmidt, 2023):

- When $\omega = 1$, the Equation (10) becomes the standard Taylor rule

  $$i_t = s_t + \phi\pi_t.$$

  The central bank follows a standard flexible inflation targeting strategy. We assume that the central bank knows the natural rate of interest s_t so that the standard inflation targeting resembles the ODP.

- When $\omega = 0$, the Equation (10) becomes

  $$i_t = s_t + \phi(p_t - p_{-1}).$$

  The central bank aims to stabilize the price level $p_t \equiv \pi_t + p_{t-1}$.[15] It is well known that

[15] Note that

$$\omega^{-1}\hat{\pi}_t = \sum_{j=0}^{\infty}(1-\omega)^j\pi_{t-j} + (1-\omega)^t\omega^{-1}\hat{\pi}_{-1}.$$

Assuming $\hat{\pi}_{-1} = 0$, we have

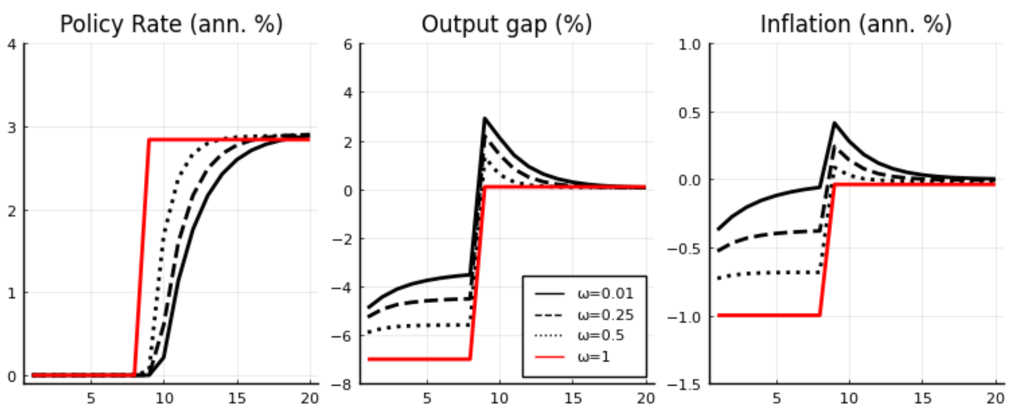
$$i_t = s_t + \phi\sum_{j=0}^{\infty}(1-\omega)^j\pi_{t-j}$$

the OCP has such a property of price level targeting.[16]

- When $\omega \in (0, 1)$, the central bank aims to stabilize an exponential moving average inflation rate $\hat{\pi}_t$ as given in Equation (11). We are especially interested in these intermediate cases because they can be similar to OSPs which are also intermediate between the OCP and the ODP.

The parameter values are set to the values in Table 1. We set ϕ to 5.0 so that the central bank responds to inflation aggressively. We have checked that our results are robust to different values of ϕ.

Figure 8: Dynamics under AIT policies



Note: The policy rate and the inflation rate are expressed in annualized percent. The output gap is expressed in percent.

Figure 8 shows the dynamics for different values of $\omega \in [0, 1]$ when the crisis shock hits the economy at time 1 and remains there for 8 quarters as in the main exercise in Section 2 (see Figure 1). The dynamics under the AIT policy with $\omega = 0.01$—shown by the solid black lines—are similar to those under the OCP and the OSP with the punishment duration $N = \infty$, whereas the dynamics under the AIT policy with $\omega = 1.0$—shown by the solid red lines—are similar to those under the ODP. As we increase the value of ω, the paths of inflation, the output gap, and the policy rate diverge from those under OCP and converge to those under ODP. This pattern is similar to what happens when we decrease N of OSPs in Figure 1.

As $\omega \to 0$, we have

$$i_t = s_t + \phi(p_t - p_{-1}).$$

[16] Eggertsson and Woodford (2003) shows that, even in the presence of ELB, a modified price level targeting by setting the price level to an output gap-adjusted price index can mimic the OCP. The policy rule considered here only intends to set the price level constant and does not completely replicate the OCP.

4.5 Cost-push shock

Throughout the paper, we focus on the time-inconsistency problem that arises from the demand shock—more precisely speaking, the interaction of the ELB constraint and the demand shock—and abstract from the cost-push shock—another shock commonly studied in the literature. The cost-push shock makes the commitment outcome time-inconsistent in the absence of a reputational mechanism, as pointed out by Walsh (2003), among many others. Sunakawa (2015) characterizes OSPs in a New Keynesian model with cost-push shocks but without the ELB constraint.

Nakata (2018) examines how the introduction of a cost-push shock affects the credibility of the OCP in the model featuring the ELB and a demand shock. He finds that the introduction of the cost-push shock increase or decreases the threshold crisis frequency above which the OCP is credible, reflecting two opposing forces. On the one hand, the introduction of a cost-push shock lowers the continuation value associated with the ODP, making the reneging on the lower-for-longer promise more costly. On the other hand, the introduction of a cost-push shock increases the magnitudes of the inflation and output overshoots, increasing the temptation to renege on the lower-for-longer promise.

We leave the task of simultaneously examining the demand and cost-push shock in our model to future research.

5 Conclusion

In this paper, we have characterized OSPs in models with the ELB constraint. We find that, even when the OCP is not credible, the central bank can still credibly commit to keeping the policy rate at the ELB in the aftermath of a crisis—though not as long as under the OCP—and meaningfully mitigate the adverse consequences of the ELB constraint on economic activity in crises.

By construction, our OSPs are time-consistent and thus overcome the criticism that the temporary overheating of the economy associated with lower-for-longer strategies is not credible. When the loss of reputation is sufficiently short-lived, these OSPs are less history dependent than the OCP. Thus, it overcomes the criticism that the implied policy rate path is too complex for the central bank to be able to explain to the public, making the OSPs even more attractive.

Although we focus on the time-consistency aspect of lower-for-longer policies in this paper, there are other aspects of these policies that could make them less attractive in reality than in theory. For example, the public may not understand the temporary nature of the inflation overshooting, resulting in unanchoring of the long-run inflation expectations (Kohn (2012) and Yellen (2018)). The overheating of the economy may be less desirable for policymakers in reality than what's implied by our model if the overheating of the economy leads to financial instability (Yellen (2018)). It would be useful to formally analyze how these factors affect the effectiveness and implementability of lower-for-longer strategies. We leave such analysis to future research.

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Technical Appendix for Online Publication

This technical appendix is organized as follows:

- Appendix A defines some key concepts.
- Appendix B describes the equilibrium conditions characterizing the sustainable outcome of the model in detail.
- Appendix C describes the numerical solution method and reports the solution accuracy.
- Appendix D collects policymakers' speeches in which the time-inconsistency problem of the lower-for-longer policy is discussed.

A Definition of a plan and credibility

This section defines a plan, credibility, and the revert-to-discretion plan. The definitions closely follow Chang (1998) and Nakata (2018).

A.1 Plan

A government strategy, denoted by $\sigma_g := \{\sigma_{g,t}\}_{t=1}^{\infty}$, is a sequence of functions that maps a history of the nominal interest rates up to the previous period and a history of states up to today into today's nominal interest rate. Formally, $\sigma_{g,t}$ is given by $\sigma_{g,1} : \mathbb{S} \to \mathbb{R}_{\geq 0}$ and $\sigma_{g,t} : \mathbb{R}_{\geq 0}^{t-1} \times \mathbb{S}^t \to \mathbb{R}_{\geq 0}$ for all $t \geq 2$.[17] Given a particular realization of $\{s_t\}_{t=1}^{\infty}$, a sequence of nominal interest rates will be determined recursively by $i_1 = \sigma_{g,1}(s_1)$ and $i_t = \sigma_{g,t}(i^{t-1}, s^t)$ for all $t > 1$ and for all $s^t \in \mathbb{S}^t$. A government strategy is said to *induce* a sequence of the nominal interest rates. *A private-sector strategy*, denoted by $\sigma_p := \{\sigma_{p,t}\}_{t=1}^{\infty}$, is a sequence of functions mapping a history of nominal interest rates up to today and a history of states up to today into today's consumption and inflation. Formally, $\sigma_{p,t}$ is given by $\sigma_{p,t} : \mathbb{R}^t \times \mathbb{S}^t \to (\mathbb{R}, \mathbb{R})$ for all t.

Given a government and private-sector strategy, a sequence of consumption and inflation will be determined recursively by $(y_t, \pi_t) = \sigma_{p,t}(i^t, s^t)$ for all $t \geq 1$ and for all $s^t \in \mathbb{S}^t$. A private sector strategy, together with a government strategy, is said to *induce* a sequence of consumption and inflation.[18] *A plan* is defined as a pair of government and private sector strategies, (σ_g,σ_p). Notice that a plan induces an outcome—a state-contingent sequence of consumption, inflation, and the nominal interest rate. As discussed earlier, there is a value sequence $\{w_t(s^t)\}_{t=1}^{\infty}$, associated with any outcome.

[17] The first period is a special case, as there is no previous policy action.

[18] Note that, while the nominal interest rate today depends on the history of nominal interest rates up to the previous period, consumption and inflation today depend on the history of nominal interest rates up to today. The implicit within-period-timing protocol behind this setup is that the government moves before the private sector does.

A.2 Credibility

Let us use $CE_t^I(s)$ to denote a set of state-contingent sequences of the nominal interest rate consistent with the existence of a competitive equilibrium when $s_t = s$. Formally, for each $s \in \mathbb{S}$, $CE_t^I(s) := \{\boldsymbol{i}_t(s) \in \mathbb{I}^\infty | \ \exists \ (\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s)) \text{ s.t. } (\boldsymbol{y}_t(s), \boldsymbol{\pi}_t(s), \boldsymbol{i}_t(s)) \in CE_t(s)\}$. σ_g is said to be *admissible* if, after any history of policy actions, i^{t-1}, and any history of states, s^t, $\boldsymbol{i}_t(s)$ induced by the continuation of σ_g belongs to $CE_t^I(s_t)$.

A plan, (σ_g, σ_p), is *credible* if (i) σ_g is admissible, (ii) after any history of policy actions, i^t, and any history of states, s^t, the continuation of σ_p and σ_g induce a $(\boldsymbol{y}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t)) \in CE_t(s_t)$, and (iii) after any history i^{t-1} and s^t, $\boldsymbol{i}_t(s_t)$ induced by σ_g maximizes the government's objective over $CE_t^I(s_t)$ given σ_p. In plain languages, a plan is said to be credible if neither the private sector nor the government has incentive to deviate from the strategies associated with it.

An outcome is said to be credible if there is a credible plan that induces it. When a certain plan A is credible and the plan A induces a certain outcome α, we say that the outcome α can be made credible, or time-consistent, by the plan A.

A.3 The revert-to-discretion plan

I now define a key object of this paper, the revert-to-discretion plan, and discuss the condition under which this plan is credible.

The revert-to-discretion plan, $(\sigma_g^{rtd}, \sigma_p^{rtd})$, consists of (i) the following government strategy: $\sigma_{g,1}^{rtd} = i_{so,1}(s_1)$ for any $s_1 \in \mathbb{S}$, $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{so,t}(s^t)$ if $i_j = i_{so,j}(s^j)$ for all $j \leq t-1$, and $\sigma_{g,t}^{rtd}(i^{t-1}, s^t) = i_{do,t}(s^t)$ otherwise, and (ii) the following private-sector strategy: $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{so,t}(s^t), \pi_{co,t}(s^t))$ if $r_j = i_{co,j}(s^j)$ for all $j \leq t$, $\sigma_{p,t}^{rtd}(i^t, s^t) = (y_{br}(s_t, i_t), \pi_{br}(s_t, i_t))$ otherwise,[19] where

$$y_{br}(s_t, r_t) = E_t y_{do,t+1}(s^{t+1}) - \sigma\Big[\big[i_t - E_t \pi_{do,t+1}(s^{t+1}) - r^*\big] + s_t\Big] \tag{12}$$

$$\pi_{br}(s_t, r_t) = \kappa y_{br}(s_t, r_t) + \beta E_t \pi_{do,t+1}(s^{t+1}) \tag{13}$$

The government strategy instructs the government to choose the nominal interest rate consistent with the sustainable outcome, but chooses the interest rate consistent with the discretionary outcome if it has deviated from the sustainable outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the sustainable outcome as long as the government has never deviated from the sustainable outcome. If the government has ever deviated from the nominal interest rate consistent with the sustainable outcome, the private sector strategy instructs the household and firms to choose output and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the discretionary outcome. By construction, the revert-to-discretion plan induces the sustainable outcome, and the implied value sequence is identical to the sustainable value sequence.

It is relatively straightforward to show that the revert-to-discretion plan is credible. By construction, $V_{so,t}(s^t) \geq V_{do,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$, making sure that the government does not have an incentive to deviate from the instruction given by the government strategy after any history i^{t-1} and s^t in which the optimal sustainable policy has been followed.

[19]Subscript *br* stands for *b*est *r*esponse.

The revert-to-discretion plan that induces the optimal sustainable outcome with a finite period punishment is defined in a similar way (see Nakata (2018) for rigorous exposition). It is also straightforward to show that such a plan is credible.

B Model Details

The policymaker maximizes

$$
\begin{aligned}
& V_1 = -E_1 \sum_{t=1}^{\infty} \beta^t \left(\pi_t^2 + \lambda y_t^2\right), \\
& \text{subject to} \\
& y_t = E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1} - r^*\right) + s_t, && (14) \\
& \pi_t = \beta E_t \pi_{t+1} + \kappa y_t, && (15) \\
& i_t \geq i_{ELB}, && (16) \\
& V_t = -E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2\right) \geq W_{do}^N(s_t), && (17)
\end{aligned}
$$

for all $t \geq 0$. The shock, s_t, follows two-state Markov chain, $s_t \in \{s_H, s_L\}$ where $s_H > s_L$. Transition probability matrix is given as $P = \begin{bmatrix} 1-p_H & p_H \\ 1-p_L & p_L \end{bmatrix}$, where p_H is the frequency of the crisis and p_L is the persistence of the crisis. $W_{do}^N(s_t)$ is the continuation value under N-period punishment.

The analytical solution for the discretionary outcome: The optimality condition for the discretionary outcome is given by

$$\lambda y_t = -\kappa \pi_t - \psi_t,$$

where ψ_t is the Lagrange multiplier on the Euler equation. As we have no endogenous state variable and the exogenous shock takes only two values in this case, each variable also takes two values. That is, $(y_t, \pi_t, i_t, \psi_t) = (y_H, \pi_H, i_H, \psi_H)$ in the high state and $(y_t, \pi_t, i_t, \psi_t) = (y_L, \pi_L, i_L, \psi_L)$ in the low state. We assume that the ZLB is slack in the high/normal state, $\psi_H = 0$, and the ZLB is binding in the low/crisis state, $\psi_L = \psi > 0$. Thus, the equilibrium conditions become

$$
\begin{aligned}
& -\sigma^{-1} y_H + \sigma^{-1}(1-p_H) y_H + \sigma^{-1} p_H y_L - i_H + (1-p_H)\pi_H + p_H \pi_L + r^* + s_H = 0, \\
& -\pi_H + \kappa y_H + \beta(1-p_H)\pi_H + \beta p_H \pi_L = 0, \\
& -\lambda y_H - \kappa \pi_H = 0, \\
& -\sigma^{-1} y_L + \sigma^{-1}(1-p_L) y_H + \sigma^{-1} p_L y_L - r^* + (1-p_L)\pi_H + p_L \pi_L + r^* + s_L = 0, \\
& -\pi_L + \kappa y_L + \beta(1-p_L)\pi_H + \beta p_L \pi_L = 0, \\
& -\lambda y_L - \kappa \pi_L - \psi = 0.
\end{aligned}
$$

These equations can be solved for $(y_H, \pi_H, i_H, y_L, \pi_L, \psi)$. Also, having these values at hand, the value functions become

$$
\begin{aligned}
V_H &= -\pi_H^2 - \lambda y_H^2 + \beta(p_H V_L + (1-p_H)V_H), \\
V_L &= -\pi_L^2 - \lambda y_L^2 + \beta(p_L V_L + (1-p_L)V_H),
\end{aligned}
$$

which can be solved for (V_H, V_L).

Solving for the sustainable outcome: The Lagrangean is given by

$$
\begin{aligned}
\mathcal{L} \equiv\ & E_1 \sum_{t=1}^{\infty} \beta^t \left\{ -\left(\pi_t^2 + \lambda y_t^2\right) - 2\phi_t \left(-\pi_t + \beta E_t \pi_{t+1} + \kappa y_t\right) \right. \\
& +2\psi_t \left(-y_t + E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1} - r^*\right) + s_t\right) \\
& \left. +\varphi_t \left(-E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2\right) - W_{do}^N(s_t) \right) \right\}, \\
=\ & E_1 \sum_{t=1}^{\infty} \beta^t \left\{ -\Psi_t \left(\pi_t^2 + \lambda y_t^2\right) - 2\kappa\phi_t y_t + 2\left(\phi_t - \phi_{t-1}\right)\pi_t \right. \\
& \left. +2\psi_t \left(-y_t - \sigma i_t + s_t\right) + 2\beta^{-1}\psi_{t-1}\left(\sigma\pi_t + y_t\right) - \varphi_t W_{do}^N(s_t) \right\},
\end{aligned}
$$

where $\Psi_t = \varphi_0 + \varphi_1 + ... + \varphi_t < \infty$ is the sum of the Lagrange multipliers on the sustainability constraint. The FOCs are given by

$$
\begin{aligned}
\partial y_t: &\quad \lambda \Psi_t y_t = -\kappa\phi_t - \psi_t + \beta^{-1}\psi_{t-1}, \\
\partial \pi_t: &\quad \Psi_t \pi_t = \phi_t - \phi_{t-1} + \sigma\beta^{-1}\psi_{t-1}, \\
\partial \phi_t: &\quad -\pi_t + \kappa y_t + \beta E_t \pi_{t+1} = 0, \\
\partial \psi_t: &\quad -y_t + E_t y_{t+1} - \sigma\left(i_t - E_t\pi_{t+1} - r^*\right) + s_t = 0.
\end{aligned}
$$

Normalizing the first and second equations by Ψ_t, we have

$$
\begin{aligned}
\pi_t &= \tilde{\phi}_t - z_t \tilde{\phi}_{t-1} + \sigma\beta^{-1} z_t \tilde{\psi}_{t-1}, \\
\lambda y_t &= -\kappa\tilde{\phi}_t - \tilde{\psi}_t + \beta^{-1} z_t \tilde{\psi}_{t-1},
\end{aligned}
$$

where $\tilde{\phi}_t = \phi_t/\Psi_t$, $\tilde{\psi}_t = \psi_t/\Psi_t$, and $z_t = \Psi_{t-1}/\Psi_t \in (0,1]$. The Karush-Kuhn-Tucker conditions (KKTCs) must be satisfied as well

$$
\begin{aligned}
&\psi_t \left(i_t - i_{ELB}\right) = 0, \\
&\psi_t \geq 0, \\
&\varphi_t \left(V_t - W_{do}(s_t)\right) = 0, \\
&\varphi_t \geq 0.
\end{aligned}
$$

The initial conditions on the Lagrange multipliers are such that $\phi_0 = \psi_0 = 0$ and $\varphi_0 = 1$, which implies $\tilde{\phi}_0 = \tilde{\psi}_0 = 0$ and $\Psi_0 = 1$.

C Projection method

In this section, we describe the projection method that we use to solve the model. In particular, we employ a collocation method. Our goal is to construct a parametric approximation to each function in the set $\{V, \phi', i, z, y, \pi, \psi', W\}$. Let ξ be the vector of the state variables (ϕ, ψ, s) and $\theta \equiv \left(\theta_V, \theta_{\phi'}, \theta_i, \theta_z, \theta_y, \theta_\pi, \theta_{\psi'}, \theta_W\right)$ denote the vector of parameters indexing candidate approximations.

Collocation nodes. Functions of the state vector are continuous along the ϕ and ψ dimensions, and therefore, we need to select collocation nodes along these two dimensions. In the ϕ dimension, we construct 20 evenly-spaced collocation nodes between -9 and 1. We then add additional collocation nodes at $\phi = (-3000,\ -2500,\ -2000,\ -1500,\ -1000,\ -500,\ -100,\ -75,\ -50,\ -25,\ 10,\ 100,\ 200)$. In the ψ dimension, we construct 26 geometrically spaced collocation nodes between $\psi_{\min} = 0$ and $\psi_{\max} = 300$. We have two values for the exogenous state variable $s \in \{s_H, s_L\}$. Combined across the three dimensions, this yields a total of $n = 33 \times 26 \times 2 = 1716$ collocation nodes per function.

Parameterization of the approximation. For a given basis function Φ, we construct our approximations as

$$\begin{aligned}
V_\theta(\xi) &\equiv W_\theta(\xi) + \left|\Phi(\xi)' \theta_V\right| \\
\phi'_\theta(\xi) &\equiv \Phi(\xi)' \theta_{\phi'} \\
i_\theta(\xi) &\equiv \left|\Phi(\xi)' \theta_i\right| \\
z_\theta(\xi) &\equiv \Phi(\xi)' \theta_z \\
y_\theta(\xi) &\equiv \Phi(\xi)' \theta_y \\
\pi_\theta(\xi) &\equiv \Phi(\xi)' \theta_\pi \\
\psi'_\theta(\xi) &\equiv \left|\Phi(\xi)' \theta_{\psi'}\right| \\
W_\theta(\xi) &\equiv 1\{s = s_L\}\,\theta_{W,1} + 1\{s = s_H\}\,\theta_{W,2}
\end{aligned}$$

Empirically, we find that the piecewise linear basis function works best to ensure stability. We also find it convenient to approximate expectation functions. To this end, we construct a vector of auxiliary parameters $\theta_A = \left(\theta_{EV'}, \theta_{E\pi'}, \theta_{Ey'}\right)$ as follows

$$\begin{aligned}
\theta_{EV',j} &\equiv p(s_H \mid s_j)\, V_\theta\left(s_H, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right) + p(s_L \mid s_j)\, V_\theta\left(s_L, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right) \\
\theta_{E\pi',j} &\equiv p(s_H \mid s_j)\, \pi_\theta\left(s_H, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right) + p(s_L \mid s_j)\, \pi_\theta\left(s_L, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right) \\
\theta_{Ey',j} &\equiv p(s_H \mid s_j)\, y_\theta\left(s_H, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right) + p(s_L \mid s_j)\, y_\theta\left(s_L, \phi'_\theta(\xi_j), \psi'_\theta(\xi_j)\right)
\end{aligned}$$

where ξ_j denotes the j-th collocation node and s_j denotes the associated exogenous state. The validity of this construction depends on using the piecewise linear basis function to deal with kinks due to occasionally binding constraints. Also, note that these parameters are implicitly functions of θ. Hence, we can naturally express approximations to the expectation functions as

$$
\begin{aligned}
EV'_\theta(\xi) &\equiv \Phi(\xi)'\,\theta_{EV'} \\
E\pi'_\theta(\xi) &\equiv \Phi(\xi)'\,\theta_{E\pi'} \\
Ey'_\theta(\xi) &\equiv \Phi(\xi)'\,\theta_{Ey'}
\end{aligned}
$$

Our parameterization choice imposes that $V_\theta(\xi) \geq W(\xi)$, $i_\theta(\xi) \geq r_{ELB}$, and $\psi'_\theta(\xi) \geq 0$, but doesn't require that $0 \leq z_\theta(\xi) \leq 1$. Empirically, we find that imposing constraints on this last function via the choice of parameterization occasionally destabilizes our algorithm. We therefore proceed by verifying that our candidate solution satisfies this constraint.

Construction of the residual function. We now describe the construction of the residual function. For a given function $h_\theta \in \{V_\theta, \phi'_\theta, i_\theta, z_\theta, y_\theta, \pi_\theta, \psi'_\theta, W_\theta(\xi), EV'_\theta, E\pi'_\theta, Ey'_\theta\}$, we use the shorthand notation $h_j \equiv h_\theta(\xi_j)$. For each j, we construct the following residuals:

$$
\begin{aligned}
e_1(\xi_j) &\equiv -\pi_j + \phi'_j - z_j\phi_j + \frac{\sigma}{\beta} z_j\psi_j \\
e_2(\xi_j) &\equiv -\lambda^{-1}y_j - \kappa\phi'_j - \psi'_j + \frac{z_j\psi_j}{\beta} \\
e_3(\xi_j) &\equiv -\pi_j + \kappa y_j + \beta E\pi'_j \\
e_4(\xi_j) &\equiv -y_j + Ey'_j - \sigma\left(i_j - i_{ELB} - r^* - E\pi'_j - s_j\right) \\
e_5(\xi_j) &\equiv -V_j - \pi_j^2 - \lambda^{-1}y_j^2 + \beta EV'_j \\
e_6(\xi_j) &\equiv \psi'_j\,(i_j - i_{ELB}) \\
e_7(\xi_j) &\equiv (1 - z_j)(V_j - W_j)
\end{aligned}
$$

Note that we use a system of equations that is slightly different than the one described in the previous section. In particular, the functions $\{\phi', \psi', V, W\}$ and state variables are scaled by λ^{-1} compared to the previous section. We use this parametrization because we find that it is more stable.

Another object that we need to compute is $W_{do}^N(s_j)$. We compute it recursively as

$$
\begin{aligned}
&W_{do}^0(s_j) = V(s_j,0,0), \quad y_{do}^0(s_j) = y(s_j,0,0), \quad \pi_{do}^0(s_j) = \pi(s_j,0,0), \\
&W_{do}^N(s_j) = \max_{y,\pi,i}\left(-y^2 - \lambda^{-1}\pi^2 + \beta\sum_{k=1}^{N_s} p(s_k|s_j)W_{do}^{N-1}(s_k)\right), \\
&y_{do}^N(s_j) = \arg\max_{y,\pi,i}\left(-y^2 - \lambda^{-1}\pi^2 + \beta\sum_{k=1}^{N_s} p(s_k|s_j)W_{do}^{N-1}(s_k)\right), \\
&\pi_{do}^N(s_j) = \kappa y_{do}^N(s_j) + \beta\sum_{k=1}^{N_s} p(s_k|s_j)\pi_{do}^{N-1}(s_k) = 0
\end{aligned}
$$

subject to

$$
\begin{aligned}
&-y-\sigma\left(i-r^{*}\right)+s_j+\sum_{k=1}^{N_s} p(s_k|s_j) y_{do}^{N-1}(s_k)+\sigma \sum_{k=1}^{N_s} p(s_k|s_j) \pi_{do}^{N-1}(s_k)=0, \\
&i \geq i_{ELB}.
\end{aligned}
$$

Letting $\boldsymbol{e}_j=\left[e_1\left(\xi_j\right) \cdots e_7\left(\xi_j\right)\right]'$, we then construct our residual function as

$$
R(\theta, N)=\begin{bmatrix}
\boldsymbol{e}_1 \\
\vdots \\
\boldsymbol{e}_n \\
W_\theta-W_{do}^N
\end{bmatrix}
$$

where we use the fact that both W_θ and W_{do}^N can be represented as vectors of equal length, $N_s=2$.

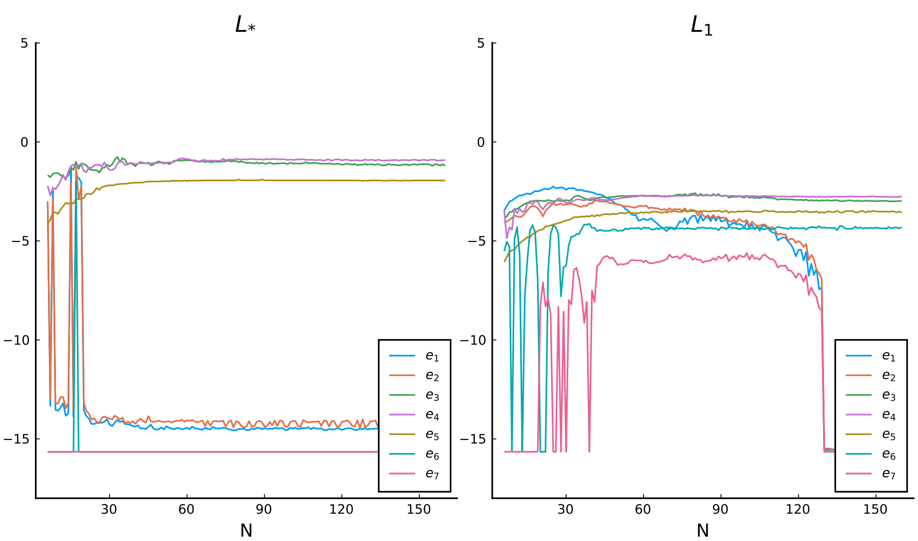
An approximate solution to the model is a solution to the following root-finding problem.

$$
R(\theta, N)=0
$$

Solving the root-finding problem. For each N, we want to solve a system of $n \times 7+2=$ 12014 nonlinear equations in 12014 unknowns. To solve these systems, we loop over values of N starting at $N=160$. At each step, we use a trust-region algorithm initialized at the solution of the previous step. We also solve for the case of the optimal commitment policy without the sustainability constraint.

Error Analysis. To check the solution accracuy, Figure 9 shows the average and 99.5 percentile of the residuals in absolute terms (log 10 units) based on a 100,000-period stochastic simulation as a function of N.

Figure 9: Simulation Residuals



Note: L_1 is the average and L_* is the 99.5 percentile of the residuals in absolute terms (log 10 units) based on a 100,000-period stochastic simulation.

D Time-inconsistency of the commitment policy in the words of policymakers

The time-inconsistency of the commitment policy at the ELB is not a mere theoretical curiosity. Policymakers in many central banks have pointed out the potential time-inconsistency of the commitment-type forward guidance policy. Some have argued that the time-inconsistency is one key reason for why most central banks refrained from making the overheating commitment. Below are some examples:

D.1 Bean (2013)

"In particular, we signalled our intention not to countenance tightening policy until unemployment has fallen to at least 7 percent."

"**This guidance is intended primarily to clarify our reaction function and thus make policy more effective, rather than to inject additional stimulus by pre-committing to a time-inconsistent 'lower for longer' policy path in the manner of Woodford (2012). While such a time-inconsistent policy may be desirable in theory, in an individualistic committee like ours, with a regular turnover of members, it is not possible to implement a mechanism that would credibly bind future members in the manner required.**"

D.2 Bullard (2013)

"The New Keynesian, sticky price literature has been influential in U.S. monetary policymaking. The literature has been led by Michael Woodford. This line of research argues that policy accommodation can be provided even when the policy rate is near zero. The extra accommodation comes from a promise to maintain the near zero policy rate into the future, beyond the point when ordinary policymaker behavior would call for an increase in the policy rate. This promise must be credible to have an impact.

The "Woodford period" approach to forward guidance relies on a credible announcement made today that future monetary policy will deviate from normal. The central bank does not actually behave differently today. One might argue that such an announcement is unlikely to be believed. Why should future monetary policy deviate from normal once the economy is growing and inflation is rising? But if the announcement is not credible, then the private sector will not react with more consumption and investment today. That is, any effects would be minimal."

D.3 Carney (2012)

"Today, to achieve a better path for the economy over time, a central bank may need to commit credibly to maintaining highly accommodative policy even after the economy and, potentially, inflation picks up. **Market participants may doubt the willingness of an inflation-targeting central bank to respect this commitment if inflation goes temporarily**

above target. These doubts reduce the effective stimulus of the commitment and delay the recovery."

D.4 Clarida (2019)

"The benefits of the makeup strategies rest heavily on households and firms believing in advance that the makeup will, in fact, be delivered when the time comes–for example, that a persistent inflation shortfall will be met by future inflation above 2 percent. **As is well known from the research literature, makeup strategies, in general, are not time consistent because when the time comes to push inflation above 2 percent, conditions at that time will not warrant doing so. Because of this time inconsistency, any makeup strategy, to be successful, would have to be understood by the public to represent a credible commitment. That important real-world consideration is often neglected in the academic literature, in which central bank "commitment devices" are simply assumed to exist and be instantly credible on decree. Thus, one of the most challenging questions is whether the Fed could, in practice, attain the benefits of makeup strategies that are possible in models.**"

D.5 Cœuré (2013)

"Most notably, the central bank may try to convince markets that it would keep interest rates low, even if this would imply inflation well above its previous objective, at least temporarily. The promise of higher future inflation, if credible, induces private agents to substitute future for current consumption, hence providing additional stimulus today. This type of forward guidance is closer to the academic concept of forward guidance in the strict sense—as discussed, for example, in Woodford (2012).

The main challenge of such guidance is its inherent inconsistency over time and thus lack of credibility. When the time comes, the central bank may be tempted to deviate from its prior commitment: once the benefits of higher inflation expectations in terms of front-loaded spending have been reaped, the central bank may not be willing to pay the bill in terms of higher inflation afterward. If the public foresees this temptation, expectations might remain unaffected in the first instance and the desired inter-temporal substitution of spending might not materialise. This is a possible explanation why, in practice, central banks have refrained from using forward guidance in a way that implies a major change in strategy. Therefore, central banks' forward guidance has rather aimed at providing greater clarity on the reaction function and the assessment of future economic conditions."

D.6 Dudley (2013)

"With respect to forward guidance, it is important to distinguish between two specific forms that this guidance may take. In the first form the central bank provides its forecast for the future path of the policy rate and, possibly, some sense of the degree of uncertainty around this path. In the second, the central bank pre-commits to a specific future path for its policy rate.

Providing a forecast for the policy rate by itself does not create any budget or reputational risk for the Federal Reserve, so I generally do not see the first form of forward guidance as posing much risk to central bank independence.

The second form of forward guidance—pre-commitment to a policy rate path—could create more risk for the central bank. In particular, consider a scenario in which the central bank decided to increase monetary accommodation by committing to maintain a low short-term interest rate for a long time even if this commitment resulted in inflation overshooting the central bank's objective in the future. I could see how this could create a potential threat to the central bank's independence. That is because the commitment could force the central bank in the future to conduct monetary policy in a way that was inconsistent with the inflation portion of its mandate.

Although this second form of forward guidance could create greater risk for the central bank with respect to its future independence, this is not a policy that has been adopted by the Federal Reserve. There are implementation challenges with this approach. **In particular, it is difficult for a monetary policy committee today to institutionally bind future monetary policy committees to follow actions that could conflict with their objectives in the future. Without such a credible forward commitment, such policies would likely be ineffective in affecting expectations in the manner needed to provide additional monetary policy accommodation.**"

D.7 George (2019)

"Third, a price-level targeting strategy is time inconsistent unless policymakers can credibly commit to following it. If the goal is to have inflation of 2 percent on average, a period of below 2 percent inflation would require an equal period of inflation above 2 percent. **But once inflation has moved up to 2 percent, policymakers might be tempted to renege on their prior commitment and not allow inflation to go higher. This would undermine the future credibility of the price-level targeting strategy. To the extent the public understood this time inconsistency problem, price-level targeting would not be credible to begin with, absent a commitment device. With regular turnover among members of the FOMC, it would be difficult for one Committee to commit a future Committee to a particular course of action.**"

D.8 Lacker (2013)

"Designing such conditional guidance involves trade-offs, however. Credibility requires consistency, over time, between a central bank's statements and its actual subsequent actions. A central bank's statements will have greater immediate effect on the public's expectations the more they are seen as limiting the central bank's future choices. Yet there are likely to be circumstances, ex post, in which the central bank feels constrained by past statements. Yielding to the temptation to implicitly renege by reworking decision criteria or citing unforeseen economic developments may have short-term appeal, but widely perceived discrepancies between actual and foreshadowed behavior will inevitably erode the faith people place in future central bank statements. So central banks face an ex ante trade-off, as well, between the short-run value of exercising discretion and the ability to communicate effectively and credibly in the future."

D.9 Plosser (2013)

"Note, however, that the central bank's ability to influence the public's belief about the future path of policy and the economy depends critically on the bank's commitment to that policy path and the credibility of that commitment in the eyes of the public. **The public must believe that even after the economy begins to strengthen, the central bank will hold rates lower than it otherwise might have found desirable to do had it not been at the zero bound in the past."**

D.10 Powell (2019)

"By the time of the crisis, there was a well-established body of model-based research suggesting that some kind of makeup policy could be beneficial. In light of this research, one might ask why the Fed and other major central banks chose not to pursue such a policy. The answer lies in the uncertain distance between models and reality. For makeup strategies to achieve their stabilizing benefits, households and businesses must be quite confident that the "makeup stimulus" is really coming. This confidence is what prompts them to raise spending and investment in the midst of a downturn. **In models, confidence in the policy is merely an assumption. In practice, when policymakers considered these policies in the wake of the crisis, they had major questions about whether a central bank's promise of good times to come would have moved the hearts, minds, and pocketbooks of the public. Part of the problem is that when the time comes to deliver the inflationary stimulus, that policy is likely to be unpopular–what is known as the time consistency problem in economics.**"

D.11 Ueda (2013)

"If Max (the Taylor rule rate, zero) describes the usual central bank's reaction function to the macroeconomic environment, the central bank can generate easing effects by offering a new reaction function to the market with a promise of a longer period at the zero rate than the above rule suggests. **To the extent that the Taylor rule represents an optimal response of the central bank to macroeconomic environment, however, this forward guidance strategy amounts to "irresponsible" central bank behaviour. In other words, the strategy is time-inconsistent. This means that when the economy no longer requires a zero rate, it is better to raise the interest rate, reneging on the promise made. If people foresaw this ex ante, however, the strategy would become ineffective.** Thus, the central bank would be sending a confusing signal if it was using forward guidance in this sense and insisted that it was still behaving in a "responsible" way. Also, the central bank does not seem to get much mileage out of a vague promise, such as the maintenance of a low policy rate "for an extended period," unless there is much confusion in the market as to where the policy rate would go in the short term.

The BOJ seems to have faced the time-inconsistency problem in 2000."

D.12 Williams (2012)

"Although forward policy guidance has proven to be a very useful policy tool, it's not a perfect substitute for the kind of monetary stimulus that comes from lower interest rates. One issue is that, for the forward guidance policy to work as desired, the public has to believe that the FOMC will really carry out the policy as it says it will. But, the Fed doesn't have the ability to tie its hands that way. This point was made by Finn Kydland and Edward Prescott in the late 1970s. Let me explain. For forward policy guidance to have its maximum effect, the Fed must commit to keeping the short-term policy rate lower than it otherwise would to compensate for the fact that the short-term interest rate cannot be lowered today. **But when the time comes to carry out the commitment made in its forward guidance, it may no longer want to do so. For instance, it might be hard to resist raising rates earlier than promised to head off an increase in inflation. So, even when central bankers say they will keep rates unusually low for a set time, the public may worry that the central bank will raise rates earlier to fight budding inflation pressures.**"