# Household Payment Behavior and the Optimal Quantity of CBDC in Japan*

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*Preliminary*

**Abstract**

We analyze Japanese household payment behavior and estimate the welfare-maximizing circulation of a central bank digital currency (CBDC) using a dynamic stochastic general equilibrium (DSGE) model. Based on a nationwide survey of 8,000 individuals conducted in 2024, we find that households would convert approximately 15.6% of their cash and 12.7% of their deposits into CBDC, suggesting a digital yen would partially substitute for existing means of payment. Calibrating the model with Japanese macroeconomic and financial data, and incorporating household liquidity preferences derived from the survey, we find that the optimal quantity of CBDC ranges between 28% and 203% of quarterly GDP, and equals 92% when the CBDC interest rate is set to zero—higher than the euro area's 64% (Burlon et al., 2024). The larger optimal quantity of CBDC in Japan may reflect not only differences in household liquidity preferences, but also structural characteristics of its financial system, including a larger central bank balance sheet and a higher ratio of corporate lending to GDP.

*Keywords*: CBDC; Household Survey; Optimal Monetary Policy; Japan.

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# 1 Introduction

In recent years, the introduction of central bank digital currencies (CBDCs) as a means of payment for individuals has been actively discussed around the world. This growing interest reflects the accelerating digitalization of finance, driven by advances in information and communication technology, the spread of mobile payments, and the rise of private cashless payment systems. Many central banks view CBDCs as a new public payment infrastructure to complement or substitute cash and have been examining their design, advantages, and possible side effects from both theoretical and empirical perspectives.

However, there are relatively few studies which combine behavioral survey data with macroeconomic models to analyze how CBDCs affect social welfare through household payment behavior. Japan provides an interesting case: the use of cash remains high, the expansion of private cashless payments has been gradual by international standards, and the central bank's balance sheet is relatively large. These characteristics make it both a policy-relevant and academically important task to evaluate the welfare implications of introducing CBDC in Japan.

The purpose of this study is to analyze Japanese household payment behavior and their willingness to hold CBDC, and to estimate the welfare-maximizing CBDC circulation using a dynamic stochastic general equilibrium (DSGE) model. Specifically, based on a nationwide survey of 8,000 individuals conducted in March 2024, we first provide descriptive evidence on how Japanese households choose among payment instruments and how they view the potential introduction of CBDC. We then incorporate household liquidity preferences implied from this survey into a DSGE model and compute the welfare-maximizing CBDC circulation under either a quantity or interest-rate-based rule for CBDC issuance. The model builds on the New Keynesian banking framework of Burlon et al. (2024), which analyzes the optimal quantity of CBDC for the euro area, and is recalibrated to match Japanese macroeconomic and financial data.

Our main results are as follows. First, the household survey reveals that while cashless payments have become more prevalent—particularly among younger generations—cash usage remains deeply rooted. Only about 12% of respondents expressed a willingness to hold CBDC, and they would convert about 15.6% of their cash and 12.7% of their deposits into CBDC. These findings suggest that CBDC would partially substitute for existing payment methods. Second, the model-based welfare analysis shows that the welfare-maximizing CBDC circulation ranges between 28% and 203% of quarterly GDP, with the corresponding optimal CBDC interest rate between –0.17% and 0.10%.[1] Assuming a zero CBDC interest

[1] The welfare-maximizing circulation of CBDC ranges from 28% to 129% of quarterly GDP under the

rate, the optimal quantity of CBDC in Japan is estimated at 92% of quarterly GDP—higher than the 64% benchmark for the euro area.[2] The larger optimal quantity of CBDC in Japan may reflect not only differences in household liquidity preferences, but also structural characteristics of its financial system, including a larger central bank balance sheet and a higher ratio of corporate lending to GDP.

The remainder of this paper is organized as follows. Section 2 describes the design of the household survey and presents the key descriptive results. Section 3 outlines the model structure. Section 4 presents data and calibration procedure. Section 5 conducts the welfare analysis of CBDC introduction and discusses policy implications. Section 6 concludes. The appendix provides details on Japanese macro-financial data sources and the model specification.

# 2 Household Payment Behavior: Survey Results

This section provides an overview of Japanese households' payment behavior and their willingness to hold CBDC, based on the survey results (Nomura Research Institute, 2024). The analysis does not delve into underlying factors or behavioral mechanisms; instead, these aspects are examined in the next section through the estimation of liquidity demand parameters in the model.

## 2.1 Overview of the Survey and Sample Characteristics

The outline of the survey is as follows. The survey was conducted on March 25–26, 2024, targeting individuals aged 18 and over across Japan. A total of 8,000 valid responses were collected through an online survey.[3] The survey items are broadly classified into three categories:

(1) current choice of payment methods, their reasons, and associated challenges;

(2) understanding of the differences between existing payment methods and CBDC; and

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quantity rule, and from 62% to 203% under the interest rate rule, with the corresponding interest rates ranging between -0.17% and 0.04%, and -0.05% and 0.10%, respectively.

[2]Burlon et al. (2024) show that in the euro area, social welfare is maximized when CBDC circulation amounts to 15%–45% of quarterly GDP, with the corresponding CBDC interest rate ranging from -3% to -1%. When the CBDC interest rate is set at 0%, the equilibrium level of CBDC circulation equals 65% of quarterly GDP.

[3]To assess respondents' knowledge and understanding of CBDC, we conducted a quiz and related survey questions. Since public awareness of CBDC remains limited in Japan, a subset of respondents was randomly selected and provided with information about CBDC, allowing us to compare their responses before and after the information provision to evaluate the impact of information disclosure.

(3) choice of payment methods and reasons when a CBDC is issued.

Regarding the distribution of sample attributes, gender, age group, occupation, and place of residence (prefecture) are well balanced and broadly consistent with Japan's population structure (Figure 1). In terms of financial literacy, we ask respondents the "Big 3" financial knowledge questions. We find that men tend to score higher, and financial literacy improves with age (Figure 2).[4]

We also find that younger male respondents own crypto assets more often. However, among younger generations, there is also a high proportion of individuals who are unaware of cryptocurrencies, indicating a polarization (Figure 3).

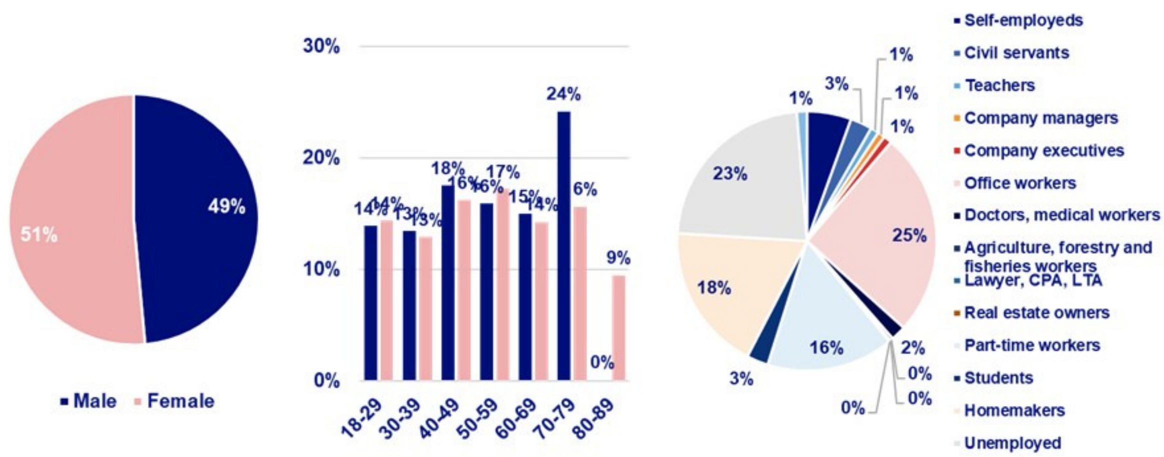


**Figure 1. Sample Characteristics: Gender, Age, and Occupation**

## 2.2 Household Payment Behavior

In Japan, the use of cashless payment methods such as credit cards and QR code payments has been expanding. Nevertheless, only 4.8% of respondents reported that they do not usually carry any cash, indicating that most people still keep some cash on hand. Regarding the amount of cash typically carried (cash on hand), about 30% of respondents reported

[4]The following three questions—known as the "Big 3"—were asked to assess respondents' financial literacy:

Q1. "Suppose you deposit 1 million yen in a savings account with an annual interest rate of 2%. After five years, how much will be in the account? Please ignore taxes on interest."

Q2. "If the inflation rate is 2% and the interest rate you receive on your savings account is 1%, how much will you be able to buy with the money in this account after one year?"

Q3. "Buying a single company's stock usually provides a safer return than buying a stock mutual fund (a financial product that invests in several companies). Is this statement true or false?"

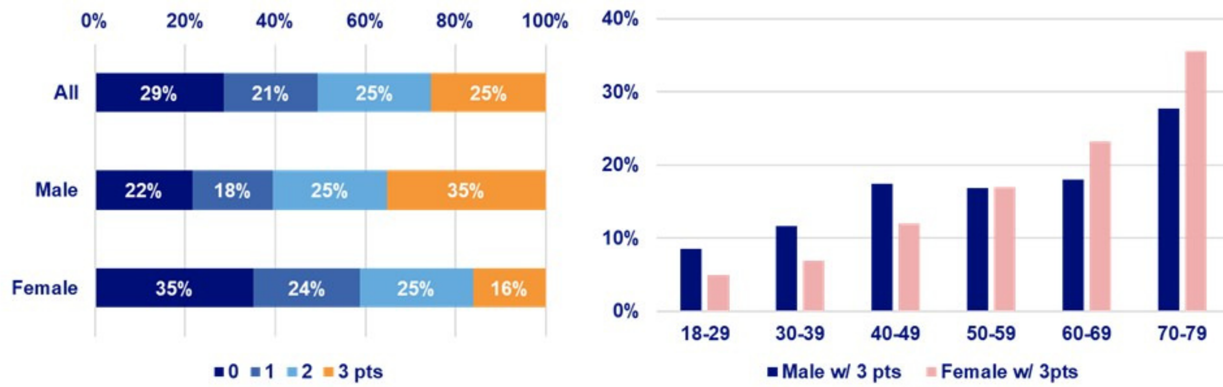


**Figure 2. Sample Characteristics: Financial Literacy**

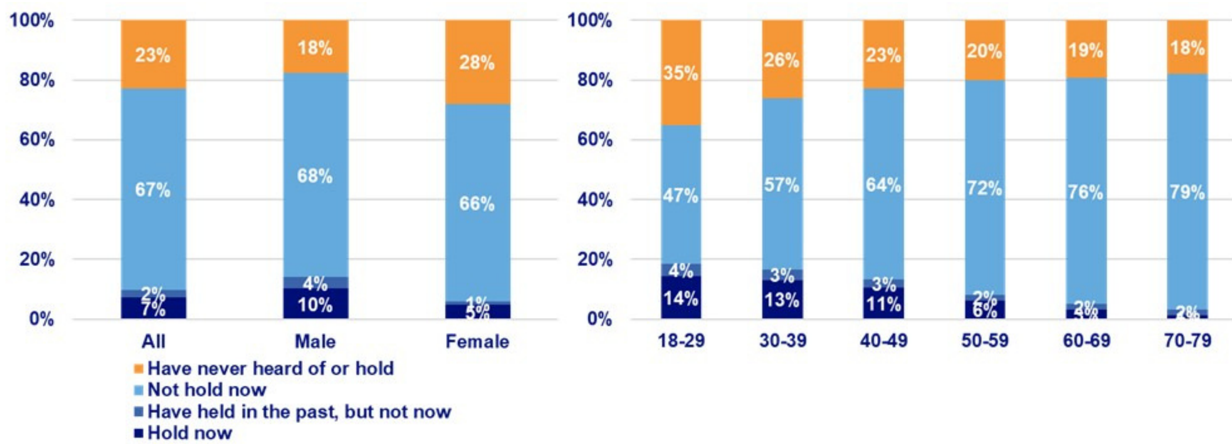


**Figure 3. Sample Characteristics: Cryptocurrency Ownership and Awareness**

holding between 10,000 and 50,000 yen, followed by slightly more than 20% each who hold between 5,000 and 10,000 yen, and between 1,000 and 5,000 yen.

In contrast, when asked whether they keep cash at home, more than 25% answered "no." Only slightly more than 10% reported holding 100,000 yen or more, suggesting that most households keep only the amount necessary for daily life (Figure 4).

Across all generations, cash is the most widely used payment method, with about 90% of respondents reporting that they are using it (Figure 5). Credit cards are the second most common method, used by around 60% of younger respondents and nearly 90% of older respondents. Among other payment methods, older respondents frequently use retail e-money (issued by major distribution companies) and deposit transfer in addition to credit cards, whereas middle-aged respondents tend to use QR code payments, and younger respondents are more likely to use public transportation and deferred payment e-money.

In terms of the amount of average monthly household payments, credit cards account for the highest share—about 70% of monthly expenditures—followed by cash, QR code

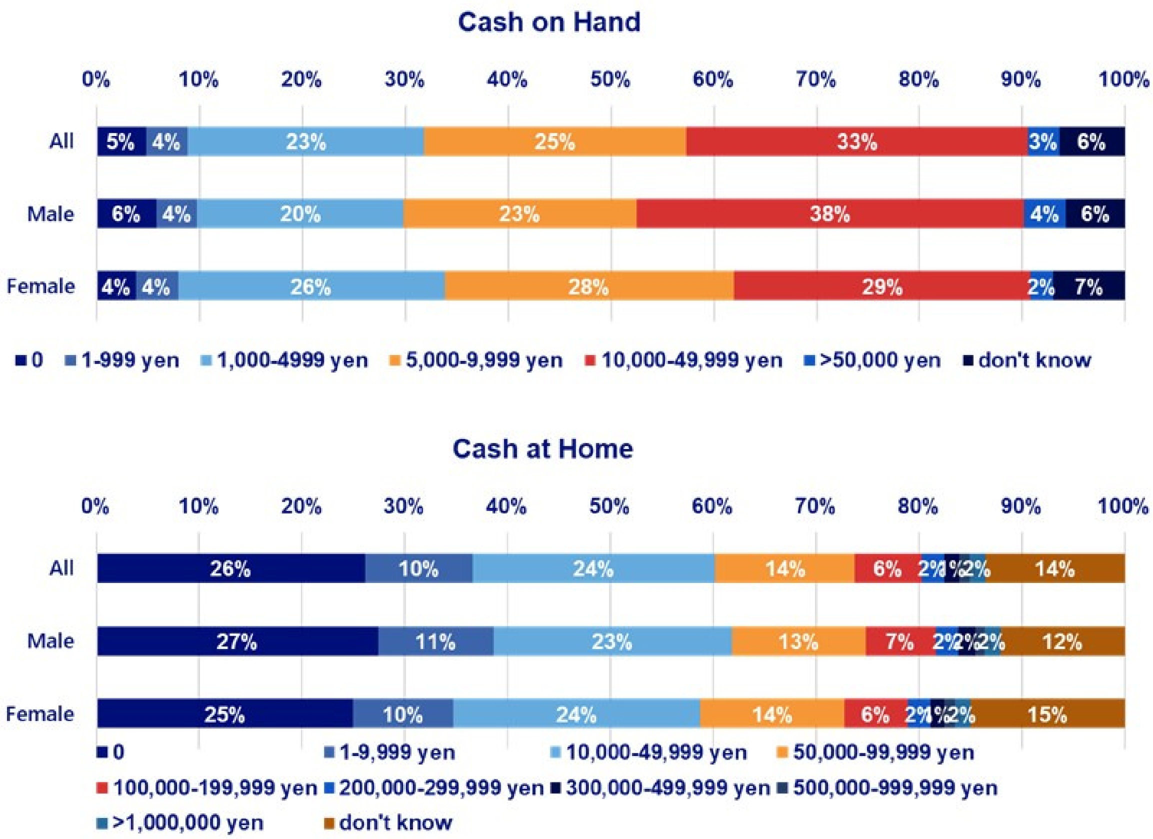


**Figure 4. Household Cash on Hand and at Home**

payments, and deposit transfer. The amounts paid via QR codes and other methods below that rank are typically less than 10,000 yen per month, indicating that they are mainly used for small-value transactions (Figure 6).

When asked about the most frequently used payment method by payment scene, cash remains the most common overall, followed by credit cards, QR code payments, and deposit transfer (Figure 7). Different payment methods are used depending on the context: cash for person-to-person payments, credit cards for irregular or high-value purchases, QR codes for daily shopping, and deposit transfer primarily for local tax payments.

Among various payment scenes, the largest average monthly payment amounts are for daily shopping, followed by housing expenses, utilities, local taxes, national taxes, high-value purchases, entertainment, person-to-person transfers, medical and nursing care, and education.

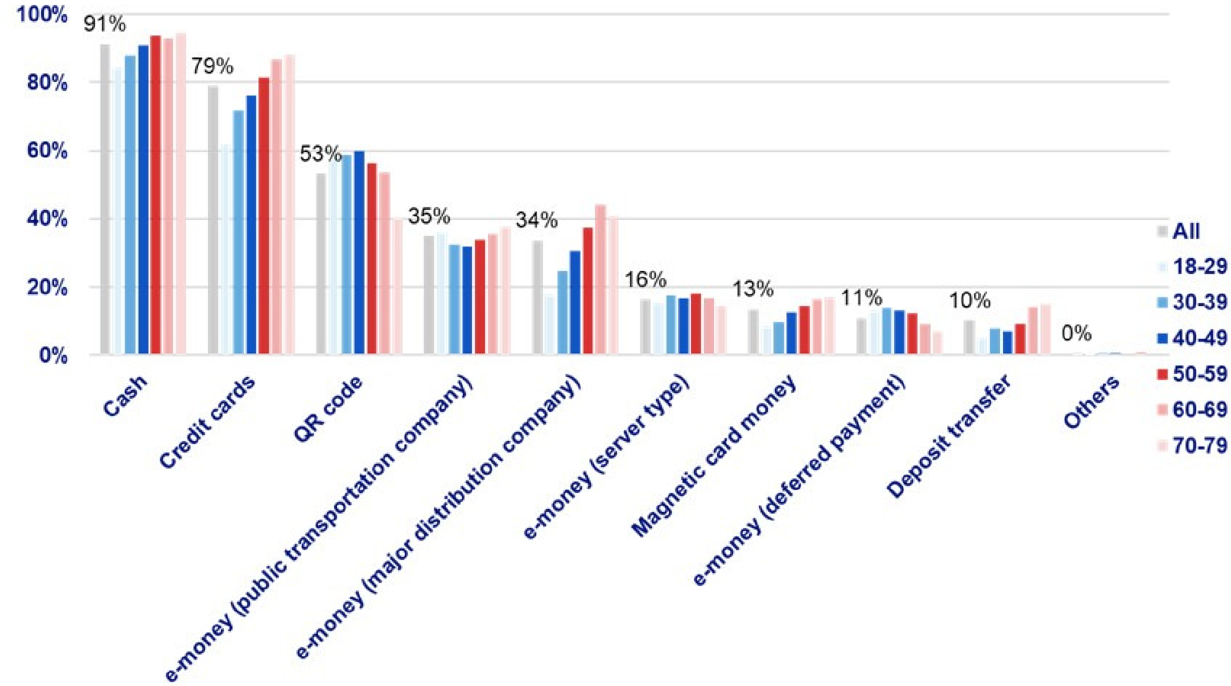


**Figure 5. Household Payment Methods**

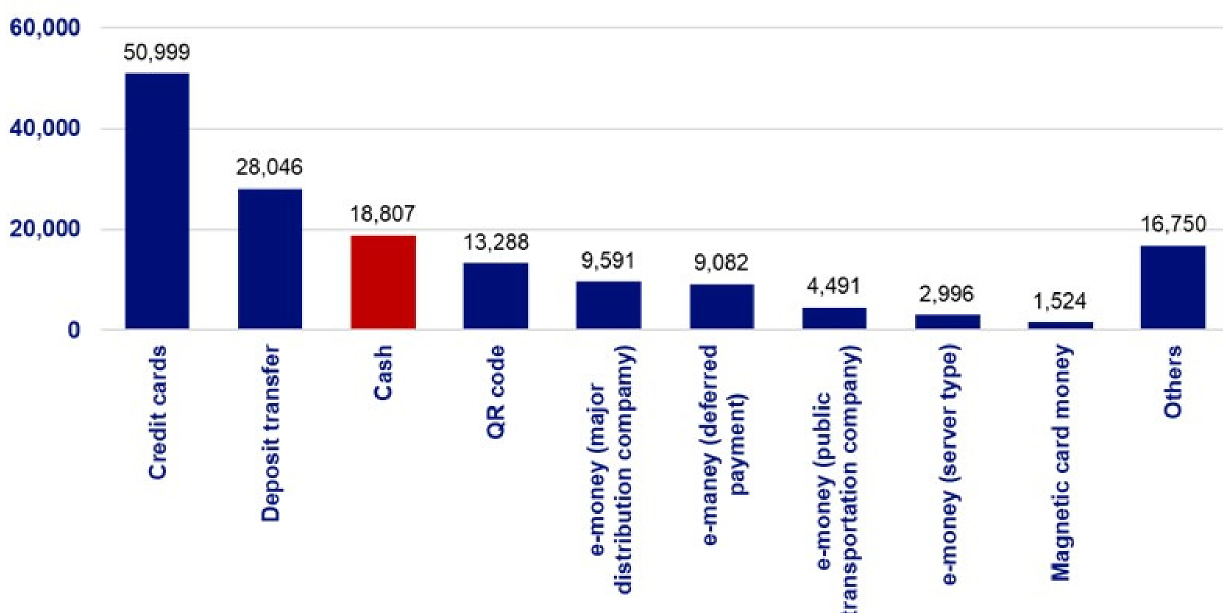


**Figure 6. Household Payment Amounts by Payment Methods**

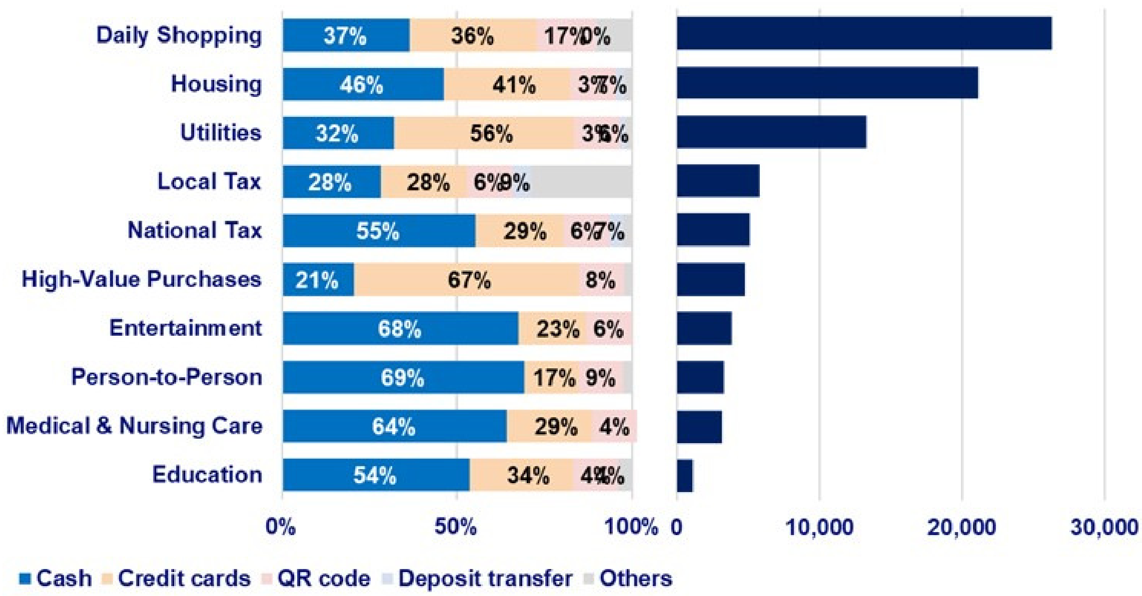


**Figure 7. Household Payment Methods and Amounts by Payment Scenes**

Note: "Daily shopping" denotes irregular purchases under 10,000 yen, whereas "High-value purchases" denote those over 10,000 yen.

## 2.3 Household Willingness to Hold CBDC

Among the survey respondents described above, 11.6% of the total sample ($n = 8,000$) answered that they "would like to hold" or "somewhat would like to hold" CBDC (Figure 8).[5]

By gender, men showed a stronger willingness to hold CBDC, with 16.0% responding positively ("would like to hold" or "somewhat would like to hold"). By age group, respondents aged 25–49 were relatively more proactive, with 14–18% in each age group indicating a willingness to hold CBDC. In contrast, among those aged 50 and above, the willingness to hold CBDC declines markedly with age. Moreover, respondents with higher financial literacy scores tend to have a stronger willingness to hold CBDC (Figure 9). Other factors such as place of residence (prefecture), household income, and interest in crypto assets also appear to influence CBDC holding intentions.[6]

[5]Public awareness of CBDC remains very limited at present in Japan: only 16% of respondents have heard any term related to CBDC, and among them, just 13% report that they actually know what CBDC is. Furthermore, when respondents were given a short quiz about CBDC, many lacked understanding of its basic characteristics—such as its convertibility with cash and its legal tender status. On the other hand, awareness of CBDC is higher among those with greater financial literacy or stronger interest in crypto assets.

[6]Fujiki (2024) shows that there is a significant correlation between knowledge or willingness to hold

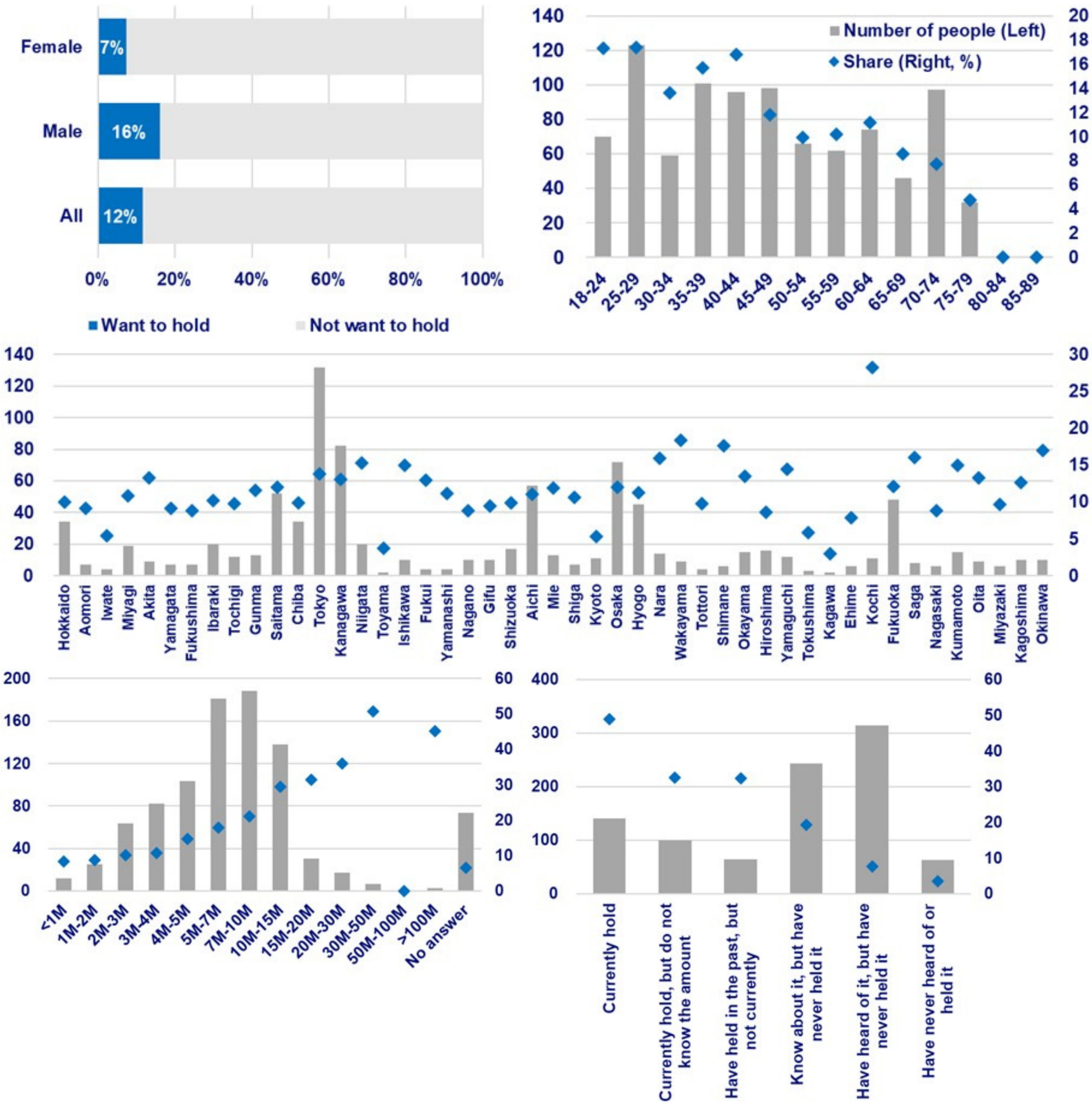


**Figure 8. Household Willingness to Hold CBDC: Gender, Age, Place of Residence, Household Income, and Crypto Assets**

Note: "Share" refers to the ratio of respondents who answered that they would like to hold CBDC to the total number of survey participants in each category. "Want to hold" includes both "would like to hold" and "somewhat would like to hold," whereas "Not want to hold" includes "unwilling to hold" and "somewhat unwilling to hold."

CBDC and interest in crypto assets. The study also suggests that individuals who hold crypto assets tend to carry significantly less cash on hand and are more likely to rely on cashless payment methods.

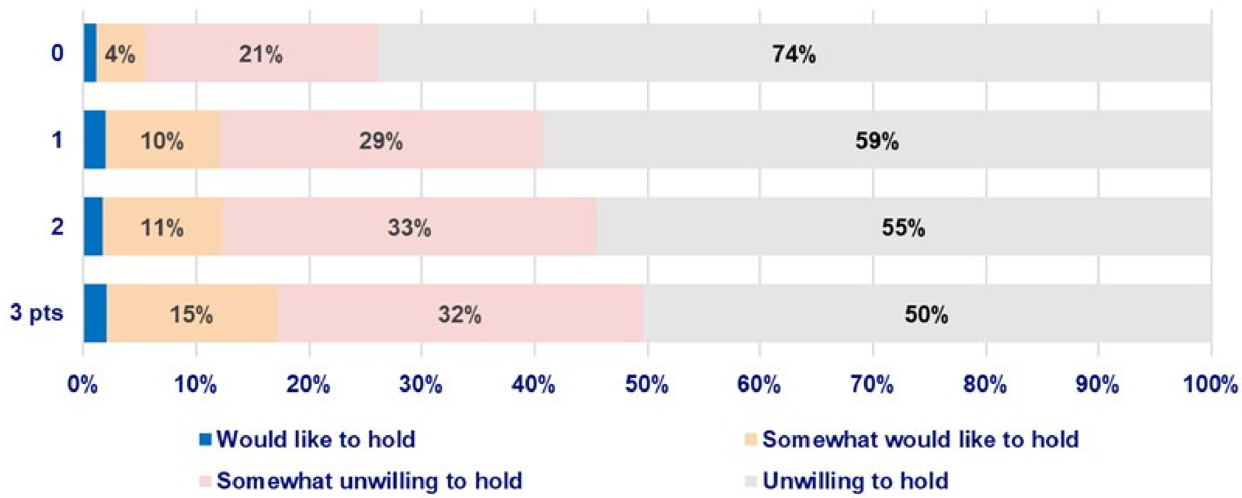


**Figure 9. Household Willingness to Hold CBDC: Financial Literacy**

Next, among the total sample of 8,000 respondents, we asked the 924 individuals (11.6%) who indicated that they "would like to hold CBDC" or "somewhat would like to hold" how much of their existing assets they would be willing to convert into CBDC.[7] Excluding the top and bottom 5% of responses, the average amount converted from cash was 11,664 yen—equivalent to 15.6% of their current cash holdings—and the average amount converted from ordinary deposits was 120,926 yen—equivalent to 12.7% of their deposit holdings (Table 1).

| | From cash | From deposits |
|---|---|---|
| Amount (yen) | 11,664 | 120,926 |
| % of current assets | 15.6% | 12.7% |

**Table 1. Amounts Households Would Transfer from Existing Assets to CBDC**

Note: Figures show average amounts converted from cash and deposits, excluding the top and bottom 5 percent of responses.

When respondents were asked in which situations they would like to use CBDC, the most frequently selected options—ranked from most to least common—were: in-store and online daily purchases, monthly fixed payments, taxes and subsidilies, financial transactions,

[7] In the survey, respondents were asked how much of their existing assets—specifically "cash," "bank deposits (including ordinary, time, and foreign-currency deposits)," and "financial instruments"—they would convert into CBDC. Responses indicating amounts exceeding their current holdings in each asset category were treated as invalid, and the averages were calculated using only the valid responses.

salary receipt, domestic person-to-person transfers, real estate transactions, and cross-border transfers / remittances (Figure 10).[8][9]

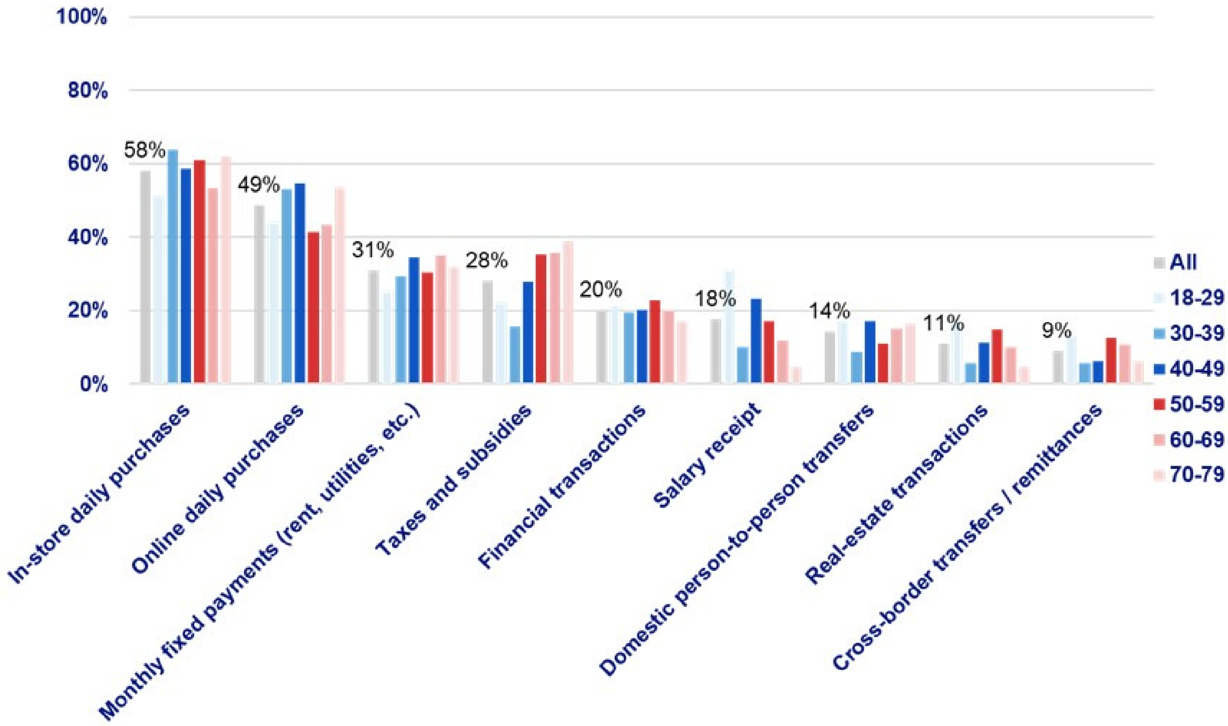


**Figure 10. Household Usages of CBDC by Payment Scenes**

By age group, younger respondents expressed a broader range of intended uses—including salary receipt, financial transactions, and real-estate transactions—while across all generations, strong demand was observed for tax payments and receiving subsidies.

In summary, the survey of Japanese households shows that while the use of cash remains deeply rooted, cashless payments are expanding, particularly among younger generations. Male, younger, and higher-income respondents, as well as those who own crypto assets tend to be more willing to hold CBDC, with an average intention to shift 15.6% of their cash holdings and 12.7% of their deposits into CBDC. The introduction of CBDC could therefore provide households with a new payment option within their diverse payment behaviors and

[8] Compared with the average household savings reported in the 2023 Family Income and Expenditure Survey (Savings and Liabilities) for two-or-more-person households—where the average balance of ordinary deposits is approximately 6.6 million yen—the survey respondents in this study reported a slightly lower average balance of around 6.12 million yen.

[9] When focusing only on respondents who indicated they would transfer at least one yen from each asset category, the average amount converted is 83,067 yen (48.6%) from cash and 450,374 yen (32.3%) from ordinary deposits. However, when including all respondents—including those who stated they would not hold CBDC—the corresponding averages decline to 7,825 yen (1.9%) from cash and 36,637 yen (1.6%) from ordinary deposits.

potentially influence their liquidity preferences and payment choices.[10]

# 3 Model

The model employed in this section is based on Burlon et al. (2024). We provide a brief overview of the model structure and the transmission channels through which CBDC influences the economy. For further details, readers are referred to Appendix B of this paper or to Burlon et al. (2024).

## 3.1 Related Literature

Since no advanced economy has yet introduced a central bank digital currency (CBDC), theoretical models are used to examine its potential effects on the economy. Existing studies can be broadly classified into the following three categories:

1. New Monetarist (Search) Models (Lagos and Wright, 2005): These studies explore the design of CBDC as a means of payment.

2. Banking Models (Diamond and Dybvig, 1983): These examine the potential implications of CBDC for financial instability, such as the risk of bank runs.

3. Quantitative Dynamic Stochastic General Equilibrium (DSGE) Models: These assess the quantitative macroeconomic effects of CBDC issuance.

This study falls into the third category. It constructs a quantitative DSGE model incorporating the banking sector under certain assumptions about CBDC design, and calibrates the model parameters using macro-financial data. The model is then used to estimate the optimal quantity of CBDC that maximizes social welfare, defined as the aggregate utility of households. In the quantitative DSGE model employed in this study, the welfare costs and benefits of CBDC are derived from the following three components:

[10] CBDC adoption may do more than simply diversify payment options; it could stimulate household spending and potentially reveal latent consumption demand. Prior studies can be reviewed from two perspectives: (i) the effect of CBDC substituting for cash payments, and (ii) the effect of adding CBDC as a public digital payment instrument alongside existing private cashless methods. The key findings are summarized below. For (i), empirical evidence shows that the shift to cashless payments reduces the psychological "pain of paying" and increases spending (Feinberg, 1986; Runnemark et al., 2015; Falk et al., 2016; Fujiwara, 2023). For (ii), Nocciola and Zamora-Pérez (2024) show, using EU data, that the diffusion of mobile payments may facilitate CBDC adoption. In Japan, Wakamori and You (2025) find that cashless payments have continued to expand even after the end of the government's point-rebate program. Therefore, CBDC adoption could help broaden consumer markets by enhancing the diversity and complementarity of payment instruments.

1. Liquidity Service Effect: The issuance of CBDC improves depositor welfare by enhancing liquidity provision. In the household utility function, cash, deposits, and CBDC are assumed to be imperfect substitutes.

2. Bank Intermediation Effect: The issuance of CBDC reduces bank deposits and tightens lending conditions, thereby lowering borrower welfare.

3. Stabilization Effect: The issuance of CBDC stabilizes fluctuations in deposits and lending over the business cycle, improving borrower welfare.

Social welfare is defined as the sum of the benefits from (1) and (3) minus the cost from (2).

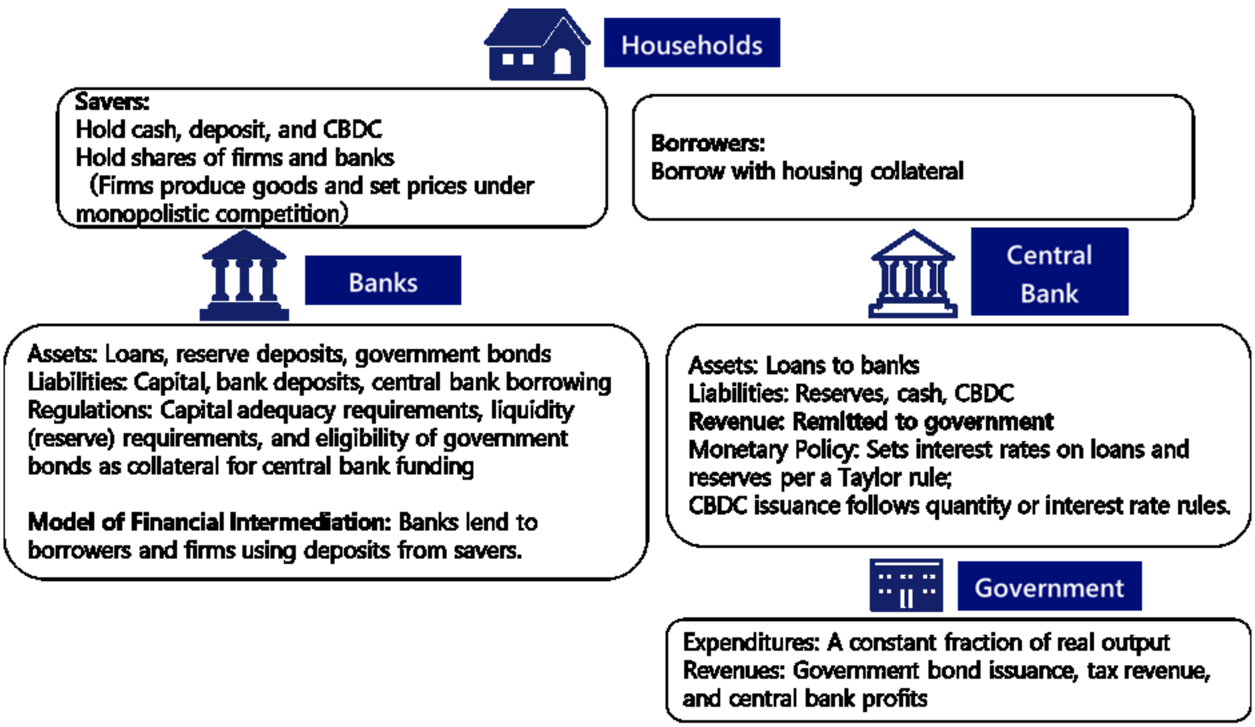


**Figure 11. Agents in the model and their behavior**

## 3.2 Setup

The model consists of households (net savers and net borrowers), firms, banks, the government, and the central bank (Figure 11).

There are two types of households: patient net savers and impatient net borrowers.[11] Net

[11] The "patience" of households is determined by their discount factor for future utility: a higher discount factor indicates greater patience, as households allocate more resources to future consumption rather than current spending.

savers hold cash, bank deposits, and CBDC, as well as government bonds.[12] The holdings of cash, deposits, and CBDC provide positive utility to households through improved liquidity services.[13] We assume that cash, deposits, and CBDC are imperfect substitutes in the household utility function. Net savers consume goods and housing services, supply labor to firms, and earn income from wages and financial assets.[14] Net borrowers, by contrast, borrow from banks using existing housing as collateral and use the borrowed funds to purchase consumption goods and housing services. Like net savers, they supply labor to firms and earn wage income.

Private banks hold government bonds, reserves, and loans as assets, and central bank borrowing, equity, and deposits as liabilities (Figure 12). We impose a capital adequacy constraint requiring equity to exceed a fixed share of total loans, defined as the risk-weighted average of household and corporate lending.[15] We also incorporate a liquidity requirement that sets a lower bound on reserves as a fraction of deposits, while we assume that central bank borrowing is subject to an upper limit determined by the collateral value of government bonds.[16]

The government levies lump-sum taxes on net savers and banks and spends a fixed share of GDP. Expenditures and interest payments on existing government bonds are financed by taxes, central bank profits, and new bond issuance. The central bank sets the lending rate and the interest rate on reserves: the lending rate follows a Taylor-type policy rule based on inflation and GDP growth, while the reserve rate is maintained at a fixed margin below the lending rate (the interest rate corridor).

When the central bank issues a CBDC, its supply or interest rate is determined by a policy rule. The central bank holds central bank lending as assets, and cash, reserves, and CBDC as liabilities. All assets and liabilities, except for cash, bear interest. The resulting net profit is transferred to the government when positive, whereas losses are covered by the

[12] While deposits, CBDC, and government bonds generate income through interest payments, holding cash involves a holding cost.

[13] This corresponds to the so-called money-in-the-utility assumption, under which the model does not endogenously explain the reason households choose to hold cash, deposits, or CBDC.

[14] The model includes four types of firms: intermediate goods producers, final goods producers, capital goods producers, and entrepreneurs, who own the intermediate goods firms. Intermediate goods producers combine capital, housing services, and labor to produce differentiated intermediate goods and set prices under monopolistic competition. Final goods producers aggregate these intermediate goods and sell the resulting composite good to households. Capital goods producers manufacture the capital required for production and sell it to entrepreneurs. Entrepreneurs borrow from banks using existing capital as collateral and purchase capital and housing services with the borrowed funds. The capital and housing services owned by entrepreneurs, together with labor supplied by households, are used as inputs in the production of intermediate goods.

[15] We assume that the risk weights assigned to government bonds and reserve holdings are zero.

[16] Banks choose the composition of their balance sheets to maximize the discounted present value of future dividends.

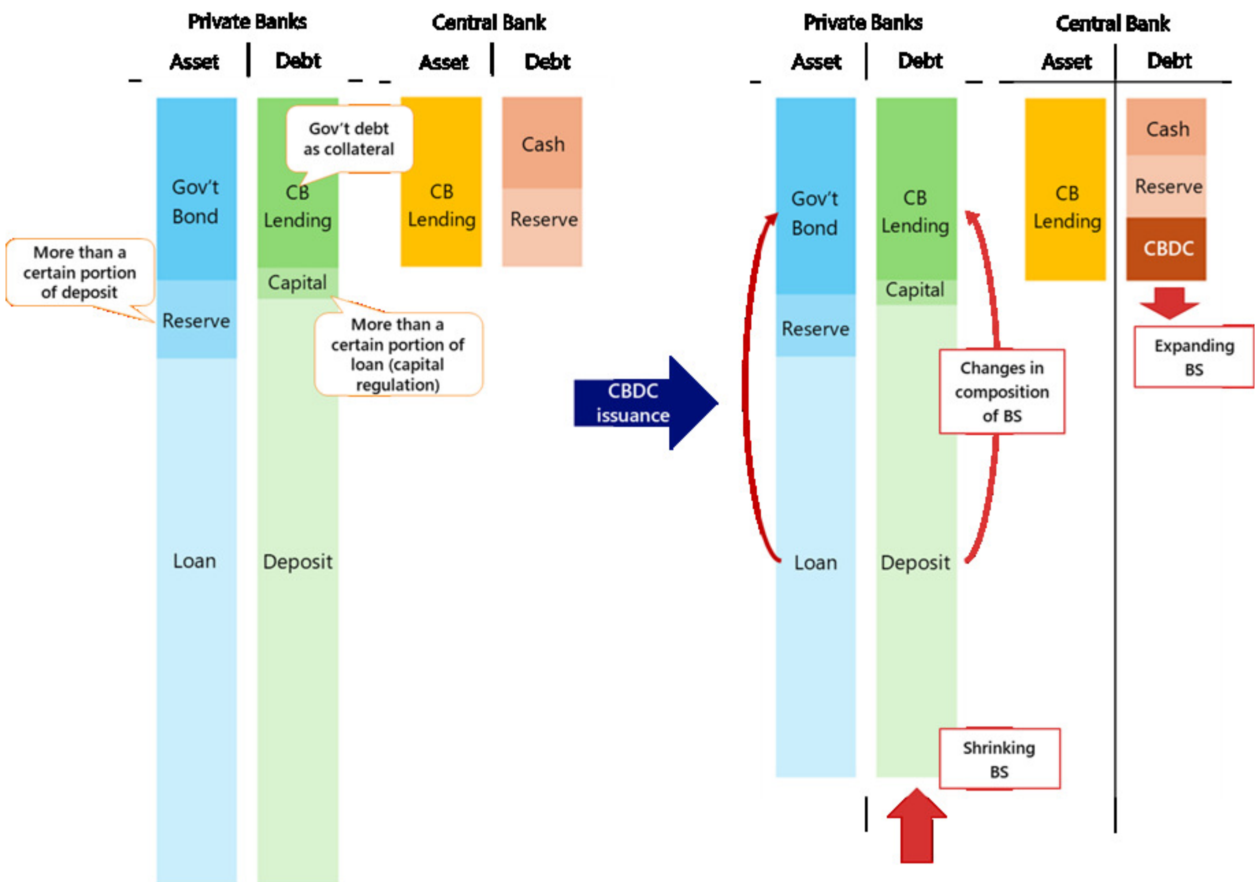


**Figure 12. Private and Central Bank Balance Sheets**

government when negative.

Let us now examine how the issuance of CBDC affects GDP through changes in the balance sheets of private banks and the central bank (Figure 13). Whether CBDC issuance reduces GDP depends on the relative magnitudes of its positive and negative effects, which are determined by the model's parameter values.

Under the assumption that CBDC is an imperfect substitute for other liquid assets held by households—namely, cash and deposits—the issuance of CBDC increases the total liquidity in the economy (cash + deposits + CBDC), thereby expanding household liquidity. This expansion leads to an enlargement of the central bank's balance sheet and an increase in its profit through seigniorage (monetary issuance gains).[17] The resulting profits are transferred to the government, strengthening its fiscal space and contributing positively to GDP.

[17] A decline in deposits also reduces reserve holdings, but to a lesser extent, owing to the lower bound imposed by the liquidity requirement. Since CBDC pays less interest than reserves, the shift from reserves to CBDC on the liability side of the balance sheet improves the central bank's profitability.

At the same time, although household liquidity increases, the share of bank deposits in household liquidity declines depending on the degree of imperfect substitutability between deposits and CBDC. This results in a contraction of private banks' balance sheets. In terms of composition, on the liability side, part of the reduced deposits is replaced by borrowing from the central bank, which in turn increases the amount of government bonds held as collateral on the asset side, while reducing the share of loans in total assets. These balance-sheet adjustments—from relatively low-cost funding (deposits) to higher-cost funding (central bank borrowing) on the liability side, and from higher-yield assets (loans) to lower-yield assets (government bonds) on the asset side—compress banks' profitability.[18] Thus, the decline in financial intermediation, accompanied by reduced lending, exerts downward pressure on GDP.

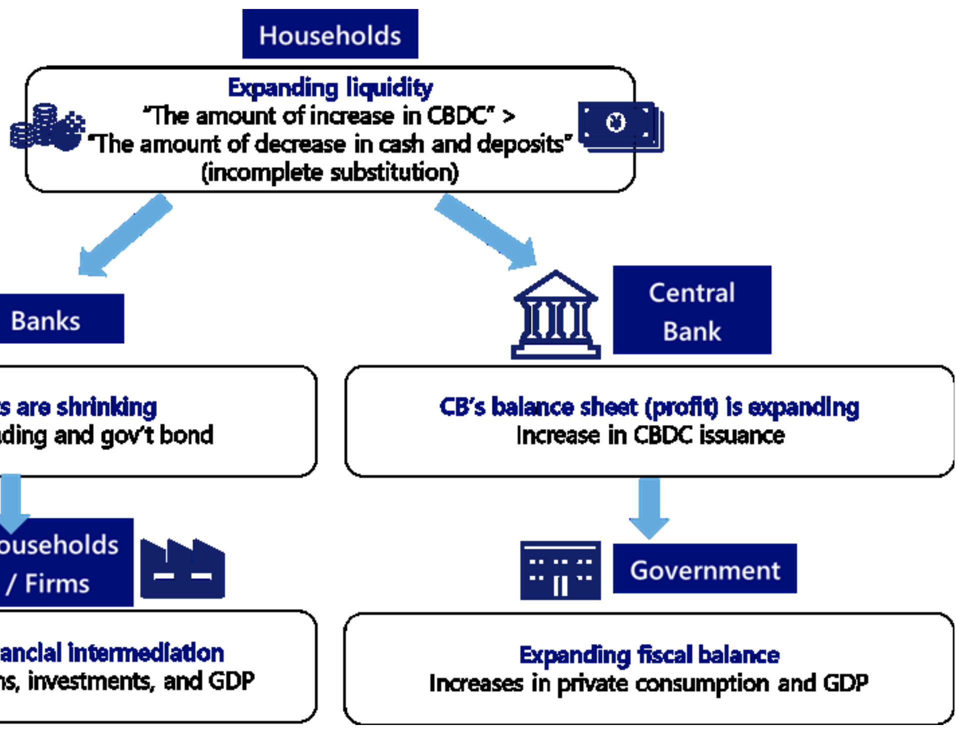


**Figure 13. The Impact of CBDC Issuance on GDP**

# 4 Data and Calibration

Calibration involves adjusting model parameters so that model-generated variables align with observed data. Using quarterly macroeconomic time series from 2000Q1 to 2023Q4, we

[18]Consequently, banks' equity capital decreases, and their holdings of risky assets—loans—decline further, given the assumption of zero risk weights for government bonds and reserves.

target the sample means and the standard deviations of detrended series. Specifically, we set parameters so that the steady-state values of the model variables closely correspond to the empirical data averages. Furthermore, we calibrate some parameters related to household demand for cash and deposits based on household survey results, reflecting potential CBDC demand in Japan.

Also, we choose the volatilities of exogenous shocks such that the standard deviations of the model variables closely align with those of the corresponding data series, measured as the standard deviations of deviations from linear trends over the sample period.[19] Details on the data sources are provided in Appendix 2.B.

## 4.1 Data

This section provides a detailed account of the macroeconomic time-series data used for calibration, emphasizing a comparison between Japan and the euro area. For comparison, we also report the euro area data employed in Burlon et al. (2024).

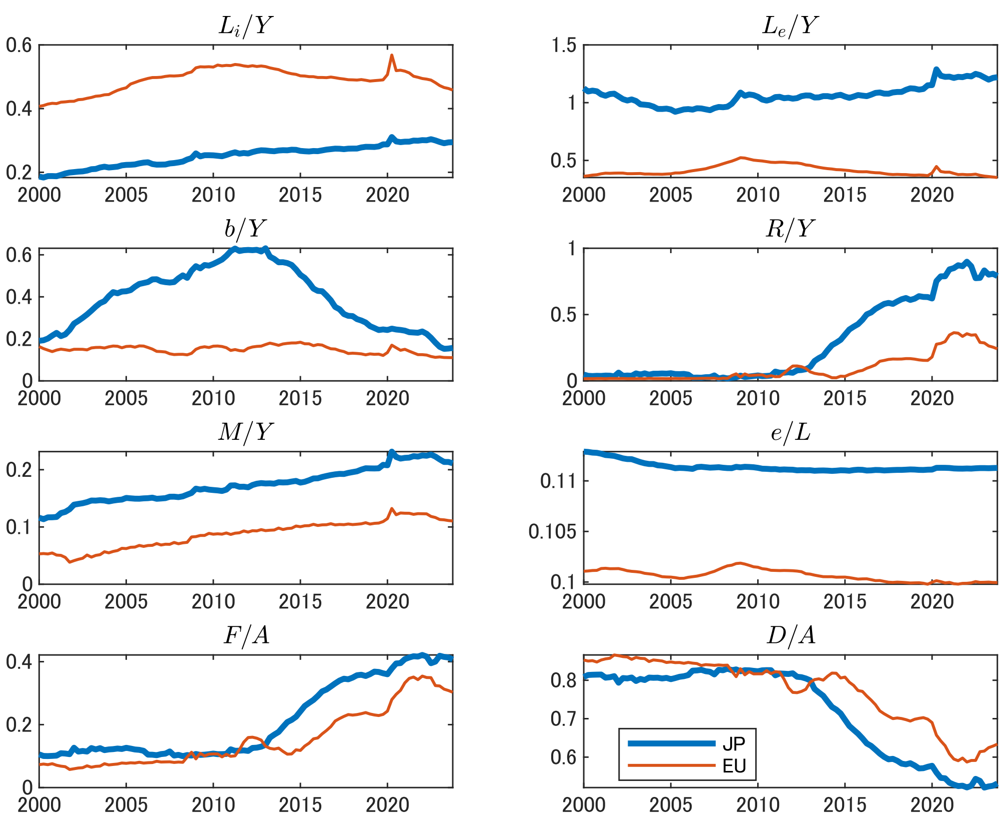


**Figure 14. Data Used for Calibration**

[19]As some of the data series exhibit growth trends, we use the standard deviations of detrended series—that is, deviations from linear trends—in the analysis.

The ratio of household bank loans to GDP ($L_i/Y$) has remained around 2 in the euro area since the 2000s (sample-period average: 1.9824), whereas it is approximately 1 in Japan (1.0097). In contrast, the ratio of corporate bank loans to GDP ($L_e/Y$) exceeds 1 in both economies but is substantially higher in Japan (4.2673) than in the euro area (1.6821).

The ratio of reserves to GDP ($R/Y$) in Japan has trended upward, especially since the launch of large-scale monetary easing in 2013 (1.1596). In the euro area, it rose during the 2011 sovereign debt crisis, declined thereafter, increased again following the introduction of negative interest rates in 2014, and surged during the COVID-19 pandemic, but has recently been declining (0.3326). The ratio of government bonds held by domestic banks to GDP ($b/Y$) rose in Japan until 2012 but declined after 2013 as the Bank of Japan's purchases reduced market holdings (1.5824). In the euro area, it has been on a downward trend since 2014, despite a temporary rise during the pandemic (0.6099). Overall, the central bank balance sheet has expanded more markedly in Japan.

The ratio of currency in circulation (banknotes + coins) to GDP ($M/Y$) has continued to rise in the euro area, exceeding 40% despite a slight decline following a temporary increase during the COVID-19 pandemic (0.3326). In Japan, the ratio has increased from about 50% in the early 2000s to nearly 80% in recent years (0.6929). The bank capital-to-loan ratio ($e/L$)—where capital is defined by the model's capital adequacy constraint as $e = (1-\gamma_i)L_i + (1-\gamma_e)L_e$ and total loans as $L = L_i + L_e$—has remained broadly stable in both the euro area (0.1060) and Japan (0.1114).[20]

In the model, central bank lending ($F$) is defined as the sum of cash ($M$) and reserves ($R$), such that $F = M + R$. From private banks' balance sheets, total assets are given by $A = L_i + L_e + R + b$, while total liabilities consist of deposits ($D$), equity ($e$), and central bank borrowing ($F$).[21] Because total assets equal total liabilities, deposits are obtained as $D = A - e - F$.[22] Using these definitions, the ratio of central bank lending to total assets ($F/A$) has increased in both the euro area (0.0892) and Japan (0.2118), whereas the ratio of deposits to total assets ($D/A$) has declined in both economies (0.8047 and 0.7210, respectively).

The deposit rate ($r_d$) has remained extremely low in Japan but was somewhat higher in the early part of the sample in the euro area. The corporate lending rate ($r_e$) declined in both regions, though it has recently risen in the euro area with monetary tightening. The

[20]In this analysis, we compute the risk weights for household and corporate loans, $(1-\gamma_i, 1-\gamma_e)$, from the calibration results, whereas we assume that the risk weights for reserves and government bonds are zero.

[21]See also Figure 12 for the balance sheets of private banks and the central bank.

[22]While we can obtain deposit data can in principle directly from the balance sheets of private banks, many balance sheet items of both private and central banks are abstracted from the model. Therefore, in this paper, we compute deposits as a residual derived from the balance sheet identity.

average spread ($r_e - r_d$) is 1.18% in Japan and 3.05% in the euro area (annualized).

The central bank lending rate ($r_f$) exceeds the interest rate on reserves ($r_R$), with an average spread ($r_f - r_R$) of 0.24% in Japan and 1.4% in the euro area. In the model, the central bank lending rate is determined by a Taylor-type rule, and the spread between the lending rate and the reserve rate is kept constant.

Early in the sample, the reserve rate was above the deposit rate, but under the negative interest rate policy it fell below it; the average spread ($r_R - r_d$) is 0.03% in Japan and 0.25% in the euro area. Consumption and investment have been relatively stable, at about 60% and 20% of GDP, respectively, in both economies.

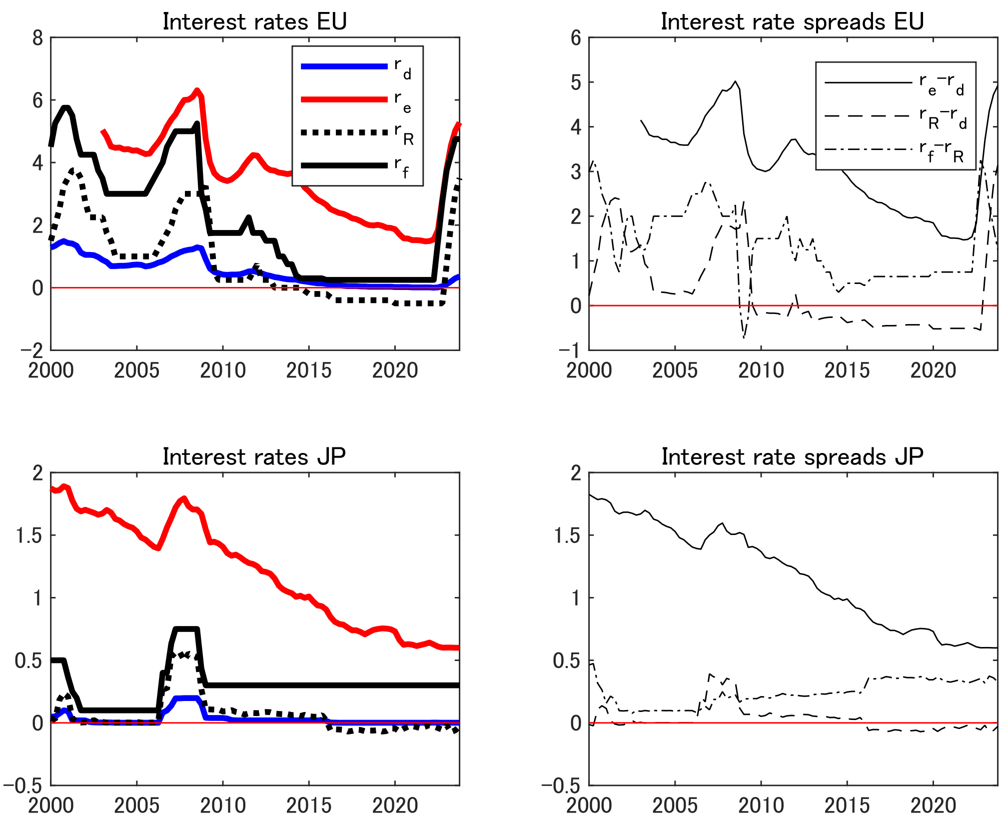


**Figure 15. Data Used for Calibration: Interest Rates**

## 4.2 Calibration

The model parameters can be broadly classified into three categories: (1) parameters that are predetermined; (2) parameters calibrated such that the steady-state values of model variables match the corresponding sample means in the data; and (3) parameters related to households' potential demand for CBDC, which are derived from household survey results.

In this study, we basically follow the procedure of Burlon et al. (2024), with some modifications to incorporate the survey results described in Section 2.3. Specifically, we follow a two-step procedure:

In the first step, taking the set of fixed parameters (1) as given, we calibrate the parameters (2) so that the model's steady-state values match the empirical data moments. In the second step, we re-estimate the household money-demand parameters (3) by solving the household utility maximization problem with respect to liquidity services.

Table 2 summarizes the parameters that are predetermined. For reference, it reports both the parameter values calibrated for Japan (JP) and those for the euro area (EU) from Burlon et al. (2024). In Japan, the prolonged period of near-zero interest rates has led to an overall lower level of interest rates. Reflecting this, the discount factors of patient and impatient households ($\beta_p, \beta_i$) are set at relatively high values. The parameters of the shares of capital and housing ($\alpha, \nu$), the collateral requirement on central bank lending ($\theta_b^{ss}$), and the intertemporal elasticity of substitution in dividends ($\sigma$) are set equal to those used for the euro area.

Given that inflation in Japan has been persistently low, the steady-state net inflation rate in the model ($\pi^{ss} - 1$) is set to zero. Since CBDC has not yet been issued in Japan, the parameter representing the CBDC-to-GDP ratio ($\phi_y$) is set to zero when we calibrate the model.

| Parameter | Description | JP | EU |
|---|---|---|---|
| $\beta_p$ | Saver's discount factor | 0.9990 | 0.9930 |
| $\beta_i$ | Borrowers' discount factor | 0.9950 | 0.9800 |
| $\alpha$ | Capital share in production | 0.3300 | 0.3300 |
| $\nu$ | Real estate share in production | 0.0100 | 0.0100 |
| $\theta_b^{ss}$ | Central bank funding collateral requirement | 0.9950 | 0.9950 |
| $\pi^{ss}$ | Gross inflation target | 1.0000 | 1.0050 |
| $\sigma$ | EIS dividends | 6.4000 | 6.4000 |
| $\phi_y$ | CBDC quantity rule | 0.0000 | 0.0000 |

**Table 2. Fixed Parameters**

Next, taking these fixed parameters as given, we calibrate the remaining parameters so that the steady-state values of the model variables match, as closely as possible, the sample means of the corresponding data series. The results of this calibration are summarized in Table 3.

For the household and corporate sector parameters, the utility weight on housing services for borrowers ($j_i^{ss}$) is smaller in Japan than in the euro area, reflecting the lower ratio of household loans to GDP ($L_i/Y$) observed in the Japanese data. Similarly, the utility weight on liquidity services for savers ($\chi_i^{ss}$) is also smaller in Japan. In contrast, the loan-to-collateral (LTC) ratio on productive capital for corporate borrowing ($m_K^{ss}$) is larger in Japan, consistent with the higher ratio of corporate loans to GDP ($L_e/Y$) relative to the euro area.

For the banking sector parameters, the calibrated risk weights on household and corporate loans ($1-\gamma_i, 1-\gamma_e$) indicate that the weight for corporate loans is smaller than that for household loans in Japan. This reflects the fact that Japanese banks extend a larger volume of loans to corporations than to households and tend to be more cautious toward the relatively riskier household lending. The reserve requirement ratio ($\theta_R^{ss}$), which specifies the required reserves as a fraction of deposits, is higher in Japan. In addition, the spread between the central bank lending rate and the reserve rate ($\mu$) is smaller in Japan, consistent with observed data.

| Parameter | | | | Target | | |
|---|---|---|---|---|---|---|
| | Description | JP | EU | | JP | EU |
| $j_p^{ss}$ | Savers' housing services weight | 0.0089 | 0.0100 | $(1+r_d)^4$ | 1.0003 | 1.0230 |
| $j_i^{ss}$ | Borrowers' housing services weight | 0.4245 | 8.7902 | $400(r_e - r_d)$ | 1.1752 | 3.0474 |
| $\chi_z^{ss}$ | Savers' liquidity services weight | 0.0146 | 0.0541 | $400(r_R - r_d)$ | 0.0334 | 0.2650 |
| $\omega_d$ | Deposits weight in liquidity services | 0.4003 | 0.7100 | $400(r_f - r_R)$ | 0.2394 | 1.3860 |
| $\eta_z^{ss}$ | Elast. of subst. liquidity services | 3.9366 | 3.5800 | $L_i/Y$ | 1.0097 | 1.9824 |
| $\psi_m$ | Cash storage cost parameter | 0.0004 | 0.0020 | $L_e/Y$ | 4.2673 | 1.6820 |
| $m_K^{ss}$ | LTC ratio on NFC physical capital | 0.4255 | 0.2140 | $R/Y$ | 1.1596 | 0.3326 |
| $\gamma_e$ | Debt-to-assets, NFC risk-adjusted | 0.9588 | 0.8950 | $M/Y$ | 0.6929 | 0.3326 |
| $\gamma_i$ | Debt-to-assets, HH risk-adjusted | 0.9450 | 0.9200 | $b/Y$ | 1.5824 | 0.6099 |
| $\delta_e$ | Erosion rate of bank capital | 0.0709 | 0.0710 | $F/A$ | 0.2118 | 0.0892 |
| $\theta_R^{ss}$ | Bank's liquidity (reserves) requirement | 0.1739 | 0.0874 | $L_i/A$ | 0.1268 | 0.4303 |
| $\phi_p$ | Fiscal rule: HH gov. bonds response | 0.4029 | 0.4010 | $D/A$ | 0.7211 | 0.8047 |
| $\phi_b$ | Fiscal rule: Banks' gov. bonds response | 0.2281 | 0.2300 | $C/Y$ | 0.5542 | 0.5479 |
| $\rho$ | Public consumption-to-GDP ratio | 0.1912 | 0.2070 | $I/Y$ | 0.2533 | 0.2124 |
| $\mu$ | Lending-deposit facility corridor parameter | 0.0007 | 0.0059 | $G/Y$ | 0.1912 | 0.2070 |

**Table 3. Calibration: Steady State**

Note: In the model, the following represent steady-state values: $Y$: GDP; $C$: consumption; $I$: investment; $M$: cash; $R$: reserves; $F$: central bank lending; $L$: private bank lending (to households and firms); $e$: bank equity; $D$: deposits; $L_i$: household loans; $L_e$: corporate loans; $A$: total bank assets; $b$: bank holdings of government bonds; $G$: public consumption; $\pi$: gross inflation rate; $r_d$: deposit rate; $r_e$: corporate lending rate; $r_f$: central bank lending rate; $r_R$: interest rate on reserves.

Finally, taking the previously calibrated parameters as given, we re-estimate a subset of parameters related to household money demand using the results of the household survey. In this analysis, we assume imperfect substitutability among the three components of household liquidity services—cash, deposits, and CBDC. Under this assumption, when a CBDC is introduced, households shift part of their holdings from cash and deposits into CBDC. To quantify this degree of substitutability, we consider the following utility maximization problem for household liquidity services:

$$\max_{(m,\, cbdc,\, d)} \left[ m^{\frac{\eta-1}{\eta}} + \omega_d\, d^{\frac{\eta-1}{\eta}} + I_{cbdc}\, cbdc^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$\text{subject to} \quad m + q_{cbdc}\, cbdc + q_d\, d + \frac{\psi_m}{2} m^2 \leq w,$$

where, $m$, $d$, and $cbdc$ denote household holdings of cash, deposits, and CBDC, respectively. $q_{cbdc}$ and $q_d$ represent the prices of CBDC and deposits, which are the inverses of their respective gross interest rates (cash, by contrast, bears no interest). $I_{cbdc}$ is an indicator variable that takes a value of 0 or 1, where $I_{cbdc} = 0$ indicates the pre-CBDC period and $I_{cbdc} = 1$ indicates the post-CBDC period. $\eta$ is the elasticity of substitution parameter among liquidity instruments.[23] $\omega_d$ represents the preference weight for deposits, while $\psi_m$ is a parameter associated with the holding cost of cash. Finally, $w$ denotes household income, which is treated here as an exogenous variable.[24]

By solving this utility-maximization problem, the household's desired holdings of cash, deposits, and CBDC can be expressed as functions of the parameters $\theta = (\eta, \omega_d, \psi_m, w, q_{cbdc}, q_d)$ and the indicator $I_{cbdc}$. That is, after the introduction of CBDC, the fractions shifted from cash and from deposits into CBDC are given by:

$$\frac{cbdc(\theta; I_{cbdc} = 1)}{m(\theta; I_{cbdc} = 0)}, \quad \frac{cbdc(\theta; I_{cbdc} = 1)}{d(\theta; I_{cbdc} = 0)}.$$

Using the parameter values set to match macro data $(\eta, \omega_d, \psi_m) = (3.9366, 0.4003, 0.0004)$, the implied substitution shares—i.e., the fractions shifted into CBDC from cash and from deposits—are 3.982 and 1.362, respectively.[25] By contrast, the household survey yields much smaller values, 0.156 (from cash) and 0.127 (from deposits), suggesting that the macro-data calibration may overstate households' preference for CBDC.[26] Accordingly, we recali-

[23]That is,

$$\eta = \frac{\partial \log(\text{cbdc}/m)}{\partial \log(q_{\text{cbdc}})} = \frac{\partial \log(d/m)}{\partial \log(q_d)}$$

(when $I_{\text{cbdc}} = \omega_d = 1$), which represents the price elasticity of the relative holdings of CBDC and deposits with respect to cash.

[24]In the model presented in Section 2, household income is determined endogenously by factors such as wages, interest rates, labor supply, and the amount of capital owned.

[25]The value of $w$ is calibrated so that the model-implied ratio $m(I_{\text{cbdc}} = 0)/d(I_{\text{cbdc}} = 0)$ matches the data value of 0.1664. The values of $(q_{\text{cbdc}}, q_d)$ are taken from the steady-state values of the model described in Section 2.

[26]The mean ratio—calculated for respondents who answered "I would like to hold CBDC" ($n = 924$)—of

brate the preference/holding-cost parameters $(\omega_d, \psi_m)$ so that the model-implied substitution shares align as closely as possible with the survey evidence, which yields $(\omega_d, \psi_m) = (1.5352, 0.000008)$ (Table 4). In other words, relative to the macro-calibrated values, households exhibit a stronger preference for deposits and a much lower holding cost of cash.

| | Description | JP |
|---|---|---|
| $\omega_d$ | Deposits weight in liquidity services | 1.5352 (0.4003) |
| $\psi_m$ | Cash storage cost parameter | 0.000008 (0.0004) |

**Table 4. Parameters Recalibrated Using Survey Results**

Note: Figures in parentheses indicate the values before recalibration (i.e., those obtained from the initial calibration).

When setting the parameters for the standard deviations of exogenous shocks in the model—each assumed to follow a normal distribution—we target, on the data side, the standard deviations of deviations from linear trends.[27] The parameters are chosen such that the standard deviations of the model variables match as closely as possible the standard deviations of deviations from linear trends in the corresponding data over the sample period.[28]

Figure 16 displays these detrended series—i.e., deviations from linear trends—for each variable (GDP, consumption, fixed investment, currency in circulation, reserves, central bank lending, private bank lending (to households and to firms), bank equity capital, deposit volume, and the deposit interest rate). All variables except the deposit rate are expressed in real terms using the GDP deflator.

Comparing these movements between Japan and the euro area, GDP, consumption, and investment are broadly positively correlated across the two economies, whereas financial variables such as currency and reserves exhibit markedly different dynamics around their trends. Moreover, examining the standard deviations reveals notable cross-regional differences. In Japan, the volatility of variables such as cash, loans, and bank equity is smaller than in the euro area, whereas reserves and central bank lending exhibit greater volatility. The deposit interest rate shows extremely low variability in the Japanese data.

the amount of CBDC they wish to hold (numerator) to their existing holdings of cash and bank deposits (ordinary deposits) (denominator), respectively.

[27] It is assumed that the trends differ across macroeconomic variables. Therefore, the balanced growth path (BGP) assumption is not satisfied in this analysis.

[28] In a real business cycle (RBC) model, the standard deviation of exogenous productivity shocks is set so that the standard deviation of GDP in the model matches the standard deviation of the cyclical component of GDP in the data—that is, the deviation of GDP from its trend.

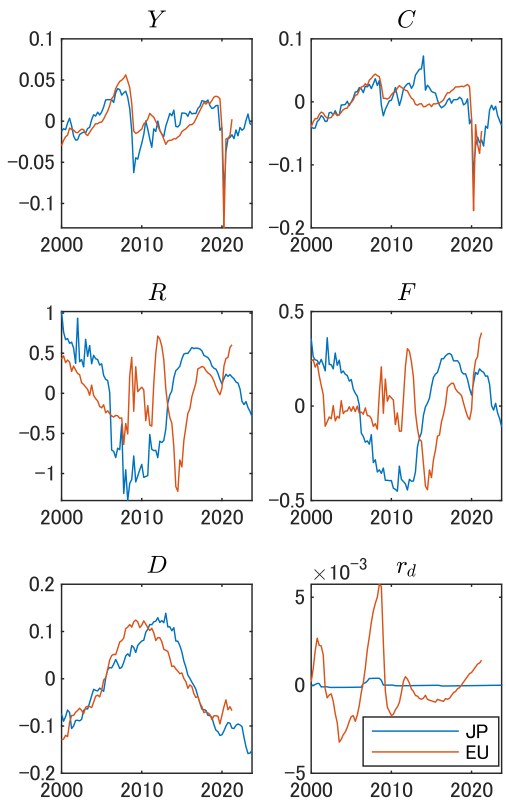


**Figure 16. Data Used for Calibration: Deviations from Linear Trends**

Table 5 summarizes the calibration results for the standard deviations of shocks. The standard deviation of productivity shocks ($\sigma_A$) is lower for Japan than for the euro area. Consistent with the smaller overall volatility of lending, the standard deviation of the collateral constraint shock for nonfinancial firms ($\sigma_{m_K}$) is also lower. Conversely, the relatively large volatility of reserves and central bank lending translates into a higher standard deviation of the liquidity requirement shock ($\sigma_{\theta_R}$). Finally, the standard deviation of the interest rate shock ($\sigma_r$) is smaller in Japan, reflecting its prolonged period of low interest rates.

| | Parameter | | | | Target | |
|---|---|---|---|---|---|---|
| | Description | JP | EU | | JP | EU |
| $\psi_I$ | Investment adj. cost parameter | 0.0112 | 0.0920 | $\sigma_I/\sigma_Y$ | 2.611 | 3.137 |
| $\sigma_A$ | Std. productivity shock | 0.0007 | 0.0023 | $100\sigma_Y$ | 2.199 | 2.631 |
| $\sigma_h$ | Std. housing pref. shock | 0.0201 | 0.0090 | $\sigma_C/\sigma_Y$ | 1.366 | 1.169 |
| $\sigma_\eta$ | Std. elast. of subst. liquidity services shock | 0.0037 | 0.0012 | $\sigma_D/\sigma_Y$ | 3.815 | 3.123 |
| $\sigma_z$ | Std. liquidity pref. shock | 0.0057 | 0.0043 | $\sigma_M/\sigma_Y$ | 1.700 | 3.408 |
| $\sigma_{mh}$ | Std. HH collateral shock | 0.0003 | 0.0076 | $\sigma_L/\sigma_Y$ | 1.467 | 3.138 |
| $\sigma_{mk}$ | Std. NFC collateral shock | 0.0005 | 0.0237 | $\sigma_e/\sigma_Y$ | 1.606 | 3.656 |
| $\sigma_{\theta_R}$ | Std. reserves requirement shock | 0.1911 | 0.1540 | $\sigma_R/\sigma_Y$ | 26.910 | 15.011 |
| $\sigma_{\theta_b}$ | Std. central bank funding collateral shock | 0.0032 | 0.0015 | $\sigma_F/\sigma_Y$ | 11.221 | 6.375 |
| $\sigma_r$ | Std. interest rate shock | 0.0004 | 0.0006 | $\sigma_{r_d}/\sigma_Y$ | 0.006 | 0.043 |

**Table 5. Calibration: Standard Deviations of Shocks**

Note: $\sigma_x$, $x \in \{Y, C, I, M, R, F, L, e, D, r_d, \Omega\}$ denotes the standard deviation of deviations from linear trends for GDP, consumption, investment, cash, reserves, central bank lending, private bank lending (to households and firms), bank equity, deposits, the deposit rate, and bank profits, respectively.

# 5 Numerical Results

Using the calibrated parameters from the previous section, we analyze the macroeconomic effects of CBDC introduction in Japan. We first examine the steady-state responses of macro variables to changes in CBDC circulation. In evaluating alternative policy rules within the quantitative DSGE framework, we assess how effectively each rule stabilizes variables such as consumption, GDP, and inflation in response to exogenous shocks. Social welfare is then derived from the utility functions of net savers and net borrowers.[29]

**Steady State** Figure 17 presents the steady-state ratios of total liquidity services and their components—cash, deposits, and CBDC—to GDP. We examine how variations in the CBDC-to-GDP ratio ($\phi_Y$ = CBDC$/Y$) affect these steady-state values. As the share of CBDC rises, the ratios of cash and deposits to GDP decline. Given that cash, deposits, and CBDC are imperfect substitutes, total liquidity services increase with CBDC introduction, while the share of deposits in total liquidity services declines.

Figure 18 presents the steady-state ratios to GDP of the central bank and private banks' balance-sheet items—–central bank lending, reserves, central bank profits, government bond

[29]Specifically, when we compute a second-order approximation of household utility around the steady state, the approximated utility becomes a function of the squared terms (variances) of consumption and inflation (Schmitt-Grohe and Uribe, 2007).

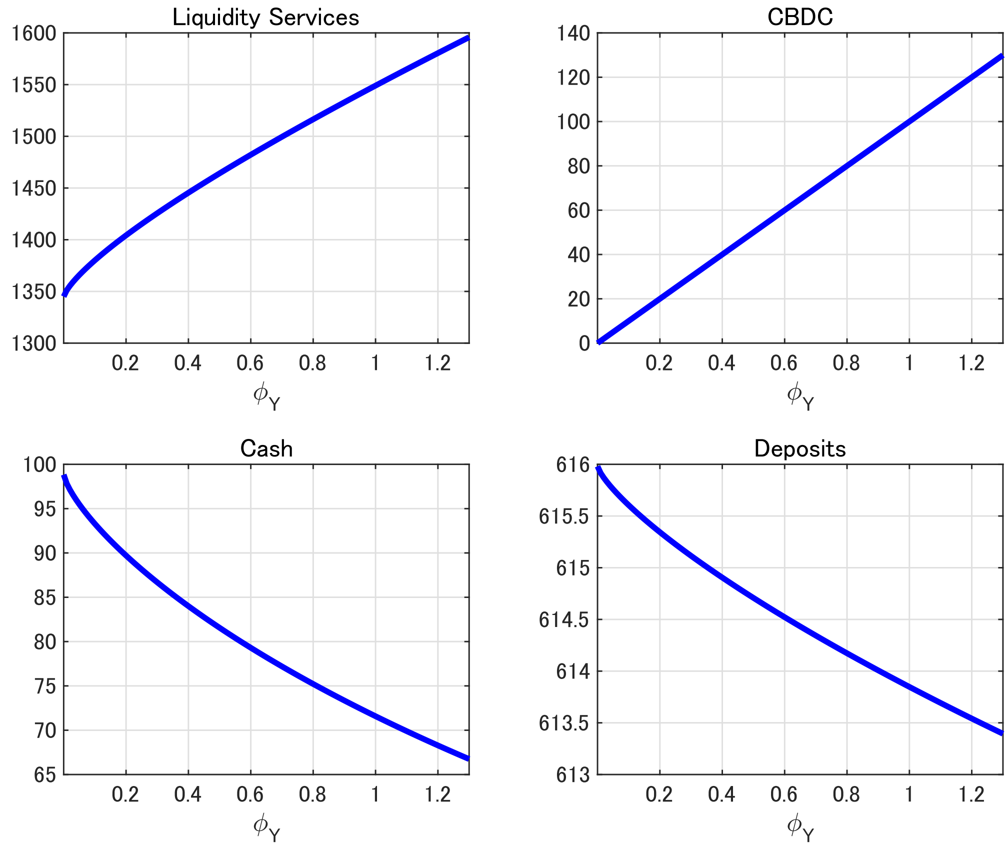


**Figure 17. Steady State: Liquidity Services (Cash, Deposits, and CBDC)**

holdings, equity capital, and bank loans.

As CBDC circulation increases, the central bank's balance sheet expands; since its only asset in the model is lending to banks, both central bank lending and profits (seigniorage) rise accordingly, while required reserves decline with lower household deposit demand.

For private banks, a shift on the liability side from deposits to central bank borrowing raises government bond holdings (as eligible collateral) on the asset side and reduces loans to households and firms. Consequently, bank profitability and equity capital decrease.

Figure 19 shows the steady-state ratios of investment, tax revenue, and consumption to GDP, as well as the steady-state level of GDP, where the value of GDP is normalized to 100 when $\phi_Y = 0$. A decline in corporate lending suppresses investment. At the same time, an increase in seigniorage income due to the expansion of the central bank's balance sheet and a rise in demand for government bonds as eligible collateral by private banks strengthen the government's fiscal capacity. This, in turn, boosts consumption and GDP through tax reductions. Overall, however, as the circulation of CBDC increases, GDP gradually declines.

Given household demand for CBDC, when the central bank sets its supply, the market determines the corresponding interest rate. The CBDC circulation level consistent with a

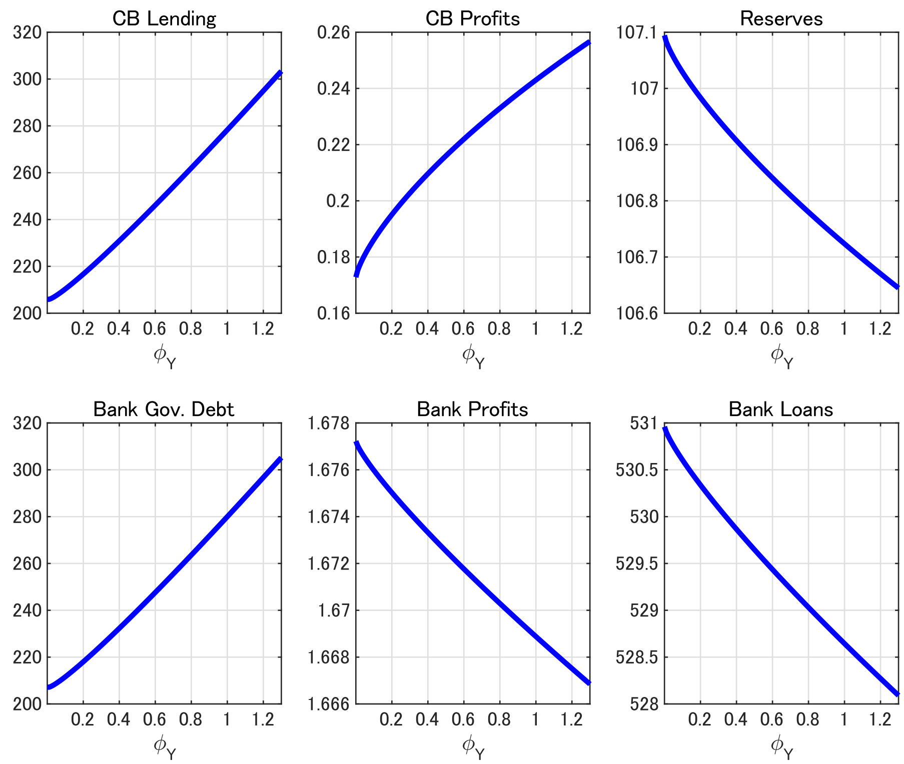


**Figure 18. Steady State: Balance Sheets of the Central Bank and Private Banks**

zero interest rate equals 92% of quarterly GDP, substantially higher than the 64.4% estimated for the euro area.

**Social Welfare under Shocks** Figure 20 compares the utilities of net savers and net borrowers, and their weighted average (social welfare), when the model is exposed to shocks under different CBDC issuance rules. Three types of quantity rules are considered:

(i) issuance linked to real GDP: $CBDC_t = \phi_Y Y_t$,

(ii) a fixed ratio to steady-state real GDP: $CBDC_t = \phi_Y Y$, and

(iii) issuance linked to past CBDC circulation, the steady-state level of real GDP, and deviations from the steady-state real GDP: $CBDC_t = \rho CBDC_{t-1} + (1-\rho)(\phi_Y Y + \phi_X \log(Y_t/Y))$.

The utility of net savers increases with CBDC circulation due to its liquidity service effect, while that of net borrowers follows an inverted U-shaped relationship, reflecting the trade-off between stabilization gains and reduced bank intermediation. As circulation rises, the welfare gains from liquidity and stabilization effects outweigh the costs from weaker intermediation.

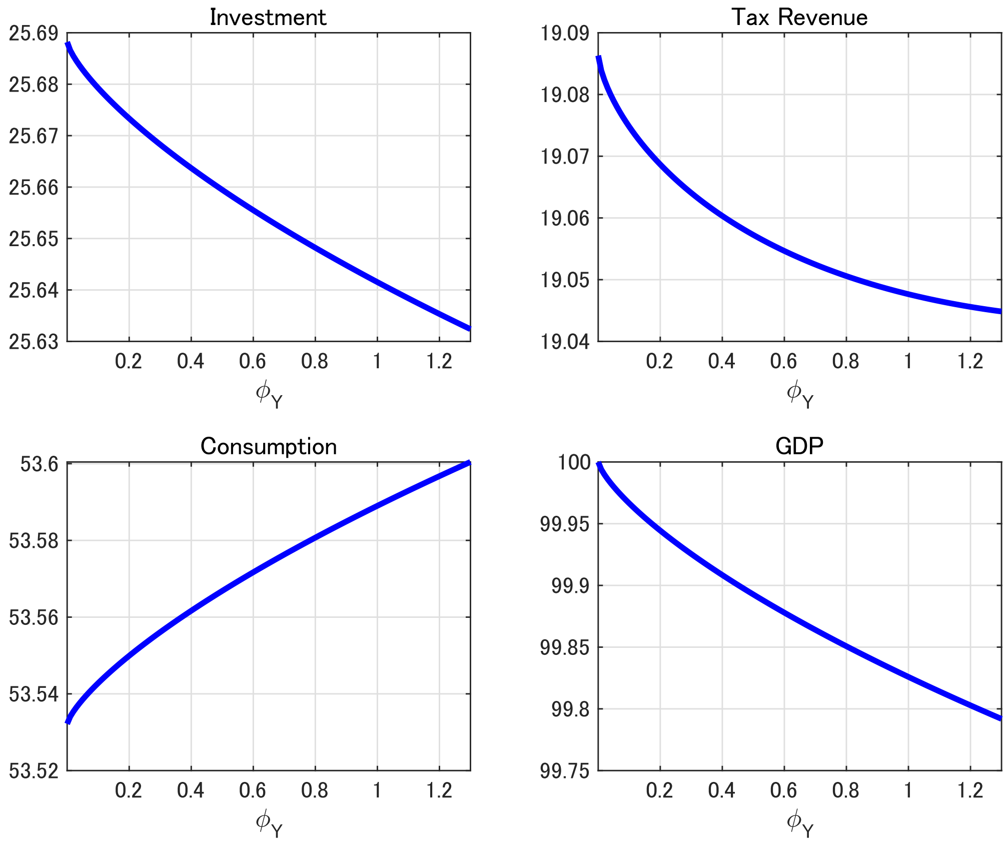


**Figure 19. Steady State: Real Economy**

Social welfare is evaluated under two weighting schemes. The first approach (Case A) assigns equal weights to the utilities of net savers and net borrowers, whereas the second approach (Case B) assigns a lower (higher) weight for net savers (borrowers) according to their high (low) discount factors. Under Case A, the welfare-maximizing CBDC circulation is about 129% of GDP across all issuance rules (i)–(iii). Under Case B, it is 28% of GDP. Given household demand, the corresponding market-determined CBDC interest rates are 0.04% (Case A) and –0.17% (Case B).

Figure 21 compares different issuance rules regarding the CBDC interest rate. We consider three rules:[30]

(i) a zero interest rate: $r_{cbdc,t} = 0$,

(ii) a fixed rate proportional to the steady-state interest rate on reserves: $r_{cbdc,t} = \phi_r r_R$, and

(iii) a rate linked to the current interest rate on reserves: $r_{cbdc,t} = \phi_r r_{R,t}$.

[30] Rule (i) can be regarded as a special case of rules (ii) and (iii) when $\phi_r = 0$.

In rules (ii) and (iii), it is optimal for savers that the CBDC interest rate moves more strongly in the same direction as the reserve rate ($\phi_r > 1$), whereas for borrowers, it is optimal that the CBDC rate moves in the opposite direction ($\phi_r < 0$). This is because savers directly benefit from a positive CBDC rate, while borrowers prefer a lower CBDC rate, which encourages the central bank to reduce CBDC issuance, increase private bank lending, and thereby enable greater borrowing.

In rule (i), the CBDC circulation is 92% of GDP, and the CBDC interest rate is 0%. In rule (ii), under the optimal interest rate rule, the market-determined CBDC circulation equals 203% of GDP under Case A and 65% under Case B, with corresponding CBDC interest rates of 0.10% and –0.05%, respectively. In rule (iii), under the optimal interest rate rule, the market-determined CBDC circulation equals 172% of GDP under Case A and 62% under Case B, with corresponding interest rates of 0.08% and –0.05%, respectively.

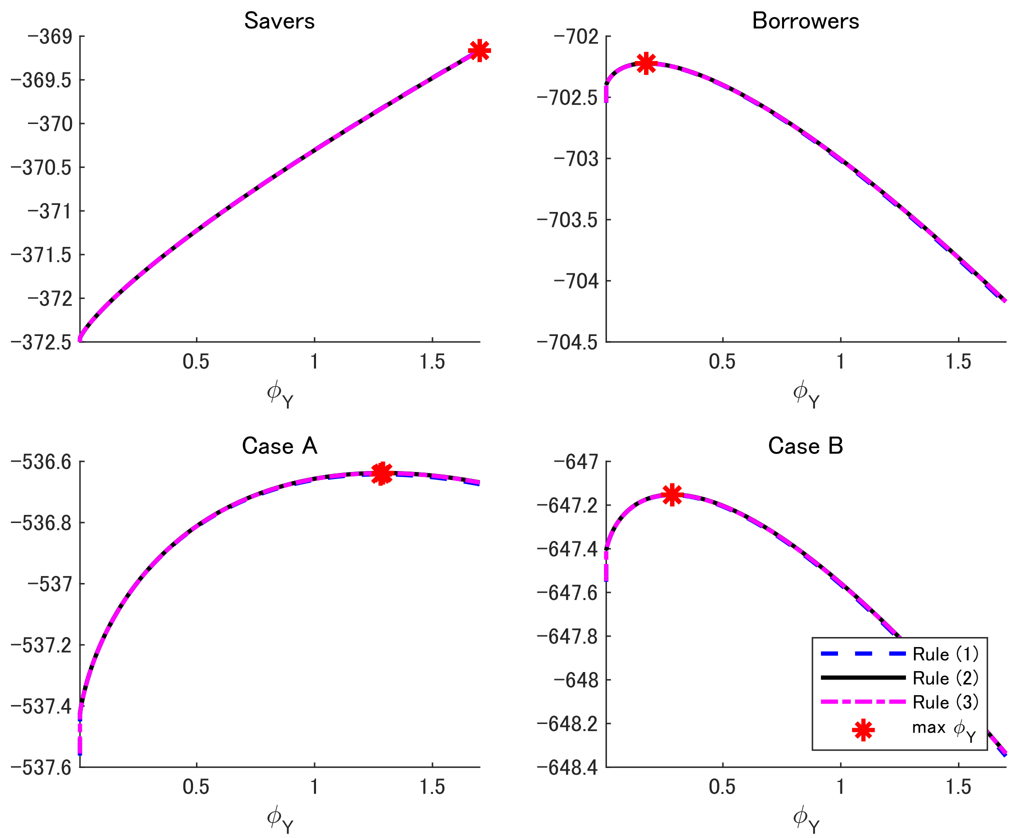


**Figure 20. Social Welfare under Shocks: Quantity Rules**

Figure 22 plots the optimal interest rate (x-axis) and optimal circulation (y-axis) under different quantity and interest rate rules. Depending on the weighting scheme of household utility, the welfare-maximizing CBDC circulation ranges between 28% and 203% of quarterly GDP, while the optimal CBDC interest rate lies between –0.17% and 0.10%.

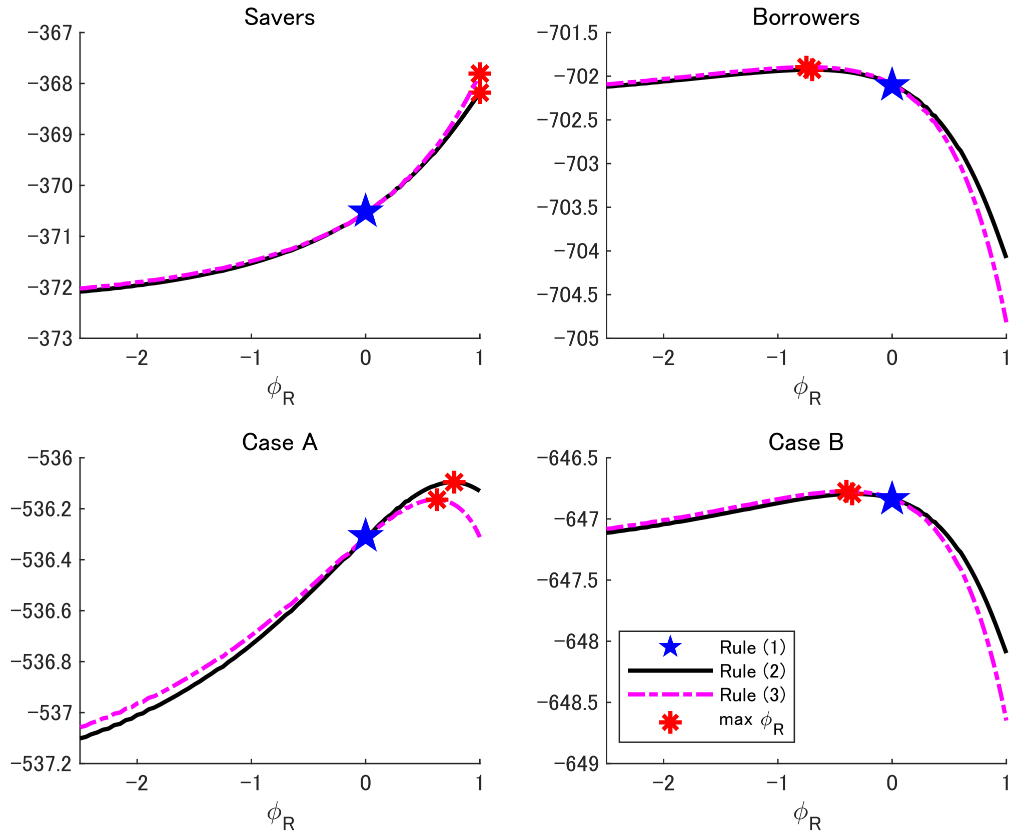


**Figure 21. Social Welfare under Shocks: Interest Rate Rules**

# 6 Concluding Remarks

This paper examined Japanese households' payment behavior and estimated the welfare-maximizing circulation of a central bank digital currency (CBDC) using a dynamic stochastic general equilibrium (DSGE) model. Based on a nationwide household survey conducted in 2024, we found that cashless payments have become more prevalent—particularly among younger generations—while cash usage remains deeply rooted. Only about 12 percent of respondents expressed a willingness to hold CBDC, indicating that they would convert approximately 15.6 percent of their cash and 12.7 percent of their deposits into CBDC. Incorporating the liquidity preferences derived from the survey into a calibrated DSGE model, we found that the welfare-maximizing CBDC circulation in Japan ranges between 28 and 203 percent of quarterly GDP, with the corresponding optimal CBDC interest rate between –0.17 and 0.10 percent. Assuming a zero CBDC interest rate, the optimal circulation is 92 percent of GDP—higher than the 64 percent benchmark for the euro area.

These results imply that Japan's structural and behavioral characteristics—including its large central bank balance sheet, high corporate lending-to-GDP ratio, and stronger household preference for liquidity—may justify a higher welfare-enhancing level of CBDC

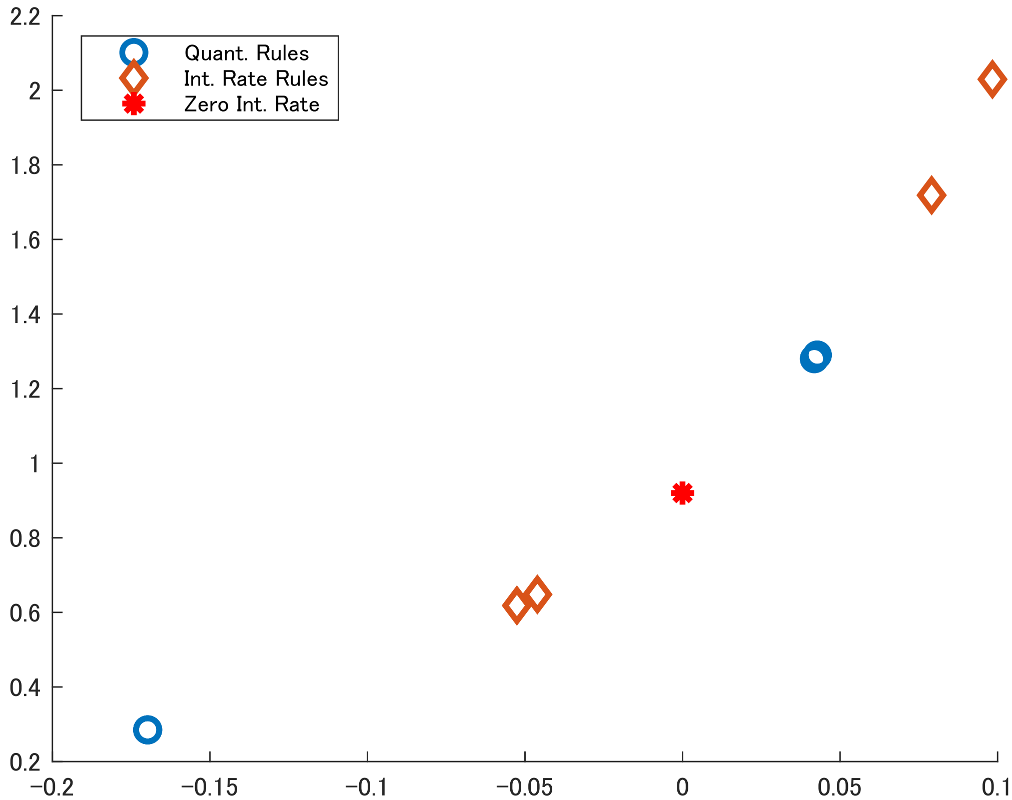


**Figure 22. Social Welfare under Shocks: Quantity vs. Interest Rate Rules**

circulation compared with other economies. A promising direction for future research is to extend the current model by incorporating the heterogeneity of household preferences revealed in the survey—such as differences in age, income, and financial literacy—in order to better understand how CBDC adoption and welfare effects may differ across these groups.

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# Appendix A Data Sources

**Household loans** ($L_i$): The sum of bank loans (to households) and shinkin bank loans (to households). Bank loans (to households) are taken from the Bank of Japan, "Loans by Industry", defined as the sum of: (1) Housing funds / Balance / Loans to individuals / Total of banking accounts, trust accounts, and overseas branches / Domestic banks (LA01'DLHLLKG71_DLHL2DSSL); (2) Consumer goods and services purchase funds / Balance / Loans to individuals / Total of banking accounts, trust accounts, and overseas branches / Domestic banks (LA01'DLCLLKG71_DLCL2DSTSL).

Shinkin bank loans (to households) are obtained from Shinkin Central Bank, "Loans by Industry", under the item "Individuals."

**Loans to non-financial corporations** ($L_e$): The sum of bank loans (to firms) and shinkin bank loans (to firms). Bank loans (to firms) are taken from the Bank of Japan, "Loans by Industry", Total loans / Balance / Banking accounts, trust accounts, and overseas branches / Domestic banks (LA01'DLLILKG90_DLLI5DS2T). Shinkin bank loans (to firms) are from Shinkin Central Bank, "Loans by Industry", Total loans / Balance / Banking accounts / Shinkin banks (LA01'DLLILKG90_DLLI5KB2T).

**Bank holdings of government bonds** ($b$): From the Bank of Japan, "Flow of Funds Accounts", the sum of two items under "Government bonds and FILP bonds (banks + postal savings)": (1) Assets – Government and FILP bonds / Banks / Stock (FF'FOF_FFAS121A311); (2) Assets – Government and FILP bonds / Postal savings / Stock (FF'FOF_FFAS126A311).

**Currency in circulation** ($M$): From the Bank of Japan, "Currency in Circulation" (MD05'MACCV1). Monthly data are converted to quarterly data using end-of-quarter values and seasonally adjusted by X13.

**Reserves** ($R$): From the Bank of Japan, "Assets and Liabilities of Domestic Banks (Banking Accounts)", item "Deposits with the Bank of Japan / Assets" (BS02'FAABK_FAAB2DBEA06). Data prior to September 2007 include deposits of Japan Post and Japan Post Bank.

**Total bank assets** ($A$): Calculated as the sum of household loans, corporate loans, government bond holdings, and reserves based on the model's private bank balance sheet: $A = L_i + L_e + b + R$.

**Central bank lending** ($F$): Calculated as the sum of currency in circulation and reserves, based on the central bank balance sheet: $F = M + R$.

**GDP** ($Y$), **consumption** ($C$), **investment** ($I$), and **government spending** ($G$): From the National Accounts of Japan. We use nominal, seasonally adjusted series for GDP (expenditure side), private final consumption, gross fixed capital formation, and government final consumption expenditure. Real values are obtained using the GDP deflator.

**Deposit rate** ($r_d$): From the Bank of Japan, "Average Interest Rates on Deposits by Type", ordinary deposit rate (IR02'DLDR120). Monthly data are averaged and converted to quarterly frequency.

**Corporate loan rate** ($r_e$): From the Bank of Japan, "Average Contract Interest Rates on Loans", computed as a weighted average of: (1) Stock / Short-term / Domestic banks (IR04'DLLR2CIDBST2), (2) Stock / Short-term / Shinkin banks (IR04'DLLR2CICR31).

Monthly data are averaged and converted to quarterly frequency. The weights are based on outstanding loan amounts from Shinkin Central Bank, "Loans by Industry".

**Central bank lending rate** ($r_f$): From the Bank of Japan, "Basic Discount Rate and Basic Loan Rate (formerly Official Discount Rate)" (IR01'MADR1M). Monthly averages are converted to quarterly data.

**Interest rate on reserves** ($r_R$): From the Bank of Japan, "Call Market Statistics (Daily)", uncollateralized overnight call rate / average daily rate (FM01'STRDCLUCON). Daily data are averaged to monthly and then converted to quarterly frequency.

# Appendix B Model Appendix

This section summarize the optimization problems and first-order conditions (FOCs) in the model. Subsections B.1–B.6 describe the optimization problems; Subsection B.7 provides derivations of the FOCs.

## B.1 Patient households

Patient households maximize expected utility and solve the following optimization problem:

$$\max_{c_{p,t},h_{p,t},d_{p,t},m_t,cbdc_t,b_{p,t},n_{p,t}} \quad E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \frac{1}{1-\sigma_h} \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{1+\phi} \right)^{(1-\sigma_h)} + j_{p,t} \log h_{p,t} + \chi_{z,t} \log z_t \right\}$$

$$\text{s.t.} \quad c_{p,t} + q_t(h_{p,t} - h_{p,t-1}) + m_t + f(m_t) + cbdc_t + d_t + b_{p,t} + \omega_T T_t$$
$$= \frac{m_{t-1}}{\pi_t} + R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} + R_{d,t-1}\frac{d_{t-1}}{\pi_t} + R_{g,t-1}\frac{b_{p,t-1}}{\pi_t} + w_t n_{p,t} + \Omega_t$$

where liquidity aggregator $z_t(m_t, cbdc_t, d_t)$ is given by

$$z_t(m_t, cbdc_t, d_t) = \left[ m_t^{(\eta_{z,t}-1)/\eta_{z,t}} + cbdc_t^{(\eta_{z,t}-1)/\eta_{z,t}} + \omega_d d_t^{(\eta_{z,t}-1)/\eta_{z,t}} \right]^{\eta_{z,t}/(\eta_{z,t}-1)} \tag{1}$$

and overall profits from owning entrepreneurial firms and banks are given by

$$\Omega_t \equiv \Omega_{b,t} + \Omega_{e,t} \tag{2}$$

Consequently, the Lagrangian would be:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_p^t \Bigg\{ & \frac{1}{1-\sigma_h} \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{1+\phi} \right)^{(1-\sigma_h)} + j_{p,t} \log h_{p,t} + \chi_{z,t} \log z_t \\ & - \lambda_t^p \Bigg[ c_{p,t} + q_t(h_{p,t} - h_{p,t-1}) + m_t + f(m_t) + cbdc_t + d_t + b_{p,t} \\ & - \frac{m_{t-1}}{\pi_t} - R_{cbdc,t-1}\frac{cbdc_{t-1}}{\pi_t} - R_{d,t-1}\frac{d_{t-1}}{\pi_t} - R_{g,t-1}\frac{b_{p,t-1}}{\pi_t} - w_t n_{p,t} - \Omega_t \Bigg] \Bigg\} \end{aligned} \tag{3}$$

The list of optimality conditions is as follows:

- FOC w.r.t. $c_{p,t}$ (C.2):

$$\lambda_t^p = \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} \tag{4}$$

- FOC w.r.t. $n_{p,t}$ (C.8):

$$\frac{j_{p,t}}{h_{p,t}} + \beta_p E_t \left( \lambda_{t+1}^p q_{t+1} \right) = \lambda_t^p q_t \tag{5}$$

- FOC w.r.t. $m_t$ (C.6):

$$\lambda_t^p (1 + \psi_m m_t) = \beta_p E_t \left( \frac{\lambda_{t+1}^p}{\pi_{t+1}} \right) + \frac{\chi_{z,t}}{z_t} \left( \frac{z_t}{m_t} \right)^{1/\eta_{z,t}} \tag{6}$$

- FOC w.r.t. $cbdc_t$ (C.5):

$$\frac{\chi_{z,t}}{z_t}\left(\frac{z_t}{cbdc_t}\right)^{1/\eta_{z,t}} + \beta_p E_t\left(\frac{\lambda^p_{t+1}R_{cbdc,t}}{\pi_{t+1}}\right) = \lambda^p_t \tag{7}$$

- FOC w.r.t. $d_t$ (C.4):

$$\frac{\chi_{z,t}}{z_t}\omega_d\left(\frac{z_t}{d_t}\right)^{1/\eta_{z,t}} + \beta_p E_t\left(\frac{\lambda^p_{t+1}R_{d,t}}{\pi_{t+1}}\right) = \lambda^p_t \tag{8}$$

- FOC w.r.t. $b_{p,t}$ (C.7):

$$\lambda^p_t = \beta_p E_t\left(\frac{\lambda^p_{t+1}R_{g,t}}{\pi_{t+1}}\right) \tag{9}$$

- FOC w.r.t. $\lambda^p_t$: patient household's budget constraint at period $t$ (C.1).

## B.2 Impatient households

Impatient households maximize expected utility and solve the following optimization problem:

$$\begin{aligned}
\max_{c_{i,t},h_{i,t},n_{i,t},l_{i,t}} \quad & E_0\sum_{t=0}^{\infty}\beta_i^t\left\{\frac{1}{1-\sigma_h}\left(c_{i,t}-\frac{n_{i,t}^{1+\phi}}{(1+\phi)}\right)^{1-\sigma_h} + j_{i,t}\log h_{i,t}\right\}\\
\text{s.t.} \quad & c_{i,t}+q_t(h_{i,t}-h_{i,t-1})+R_{i,t-1}\frac{l_{i,t-1}}{\pi_t}+(1-\omega_T)T_t = l_{i,t}+w_t n_{i,t}\\
\text{and} \quad & l_{i,t}\le m_{H,t}E_t\left(\frac{q_{t+1}}{R_{i,t}}h_{i,t}\pi_{t+1}\right)
\end{aligned}$$

Assuming borrowing constraint binds in the neighborhood of the steady state one can plug it into the budget constraint. Consequently, the Lagrangian would be:

$$\begin{aligned}
\mathcal{L} = E_0\sum_{t=0}^{\infty}\beta_i^t\Bigg\{&\frac{1}{1-\sigma_h}\left(c_{i,t}-\frac{n_{i,t}^{1+\phi}}{(1+\phi)}\right)^{1-\sigma_h} + j_{i,t}\log h_{i,t}\\
&-\lambda^i_t\Bigg[c_{i,t}+q_t(h_{i,t}-h_{i,t-1})+R_{i,t-1}\frac{m_{H,t-1}E_t\left(\frac{q_t}{R_{i,t-1}}h_{i,t-1}\pi_t\right)}{\pi_t}+\\
&(1-\omega_T)T_t - m_{H,t}E_t\left(\frac{q_{t+1}}{R_{i,t}}h_{i,t}\pi_{t+1}\right)-w_t n_{i,t}\Bigg]\Bigg\}
\end{aligned} \tag{10}$$

Simplify the Lagrangian:

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_i^t \Bigg\{ & \frac{1}{1-\sigma_h} \left( c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right)^{1-\sigma_h} + j_{i,t} \log h_{i,t} \\
& - \lambda_t^i \Big[ c_{i,t} + q_t(h_{i,t} - h_{i,t-1}) + (m_{H,t-1} q_t h_{i,t-1}) + \\
& (1-\omega_T)T_t - m_{H,t} E_t \left( \frac{q_{t+1}}{R_{i,t}} h_{i,t} \pi_{t+1} \right) - w_t n_{i,t} \Big] \Bigg\}
\end{aligned}
\tag{11}
$$

The list of optimality conditions is as follows:

- FOC w.r.t. $c_{i,t}$ (C.11):

$$
\lambda_t^i = \left( c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} \tag{12}
$$

- FOC w.r.t. $n_{i,t}$ (C.13):

$$
w_t = n_{i,t}^{\phi} \tag{13}
$$

- FOC w.r.t $h_{i,t}$ (C.12):

$$
\frac{j_{i,t}}{h_{i,t}} - \lambda_t^i \left( q_t - E_t \left[ \frac{m_{H,t} q_{t+1} \pi_{t+1}}{R_{i,t}} \right] \right) + \beta_i E_t \left[ \lambda_{t+1}^i q_{t+1} (1 - m_{H,t}) \right] = 0 \tag{14}
$$

- FOC w.r.t. $\lambda_t^i$: impatient household's budget constraint at period $t$ (C.9).

- Impatient household's borrowing constraint at period $t$ (C.10).

### B.3 Banks

Banks maximize the present value of dividends transferred to patient households and solve the following optimization problem (subject to equations C.14-C.18 respectively):

$$
\begin{aligned}
\max_{\Omega_{b,t},L_{i,t},L_{e,t},b_{b,t},\tilde{R}_{b,t},D_t,f_t} \quad & E_0 \sum_{t=0}^{\infty} \Lambda_{t,t+1} f(\Omega_{b,t}) \\
\text{s.t.} \quad & L_{i,t} + L_{e,t} + b_{b,t} + \tilde{R}_{b,t} = e_t + D_t + f_t \\
& \Omega_{b,t} + e_t - (1-\delta^e)\frac{e_{t-1}}{\pi_t} \\
& = \frac{\Big(r_{i,t-1}L_{i,t-1} + r_{e,t-1}L_{e,t-1} + r_{g,t-1}b_{b,t-1} + r_{\tilde{R},t-1}\tilde{R}_{b,t-1} - r_{d,t-1}D_{t-1} - r_{f,t-1}f_{t-1}\Big)}{\pi_t} \\
& D_t + f_t \leq \gamma_i L_{i,t} + \gamma_e L_{e,t} + \gamma_b b_{b,t} + \gamma_{\tilde{R}} \tilde{R}_{b,t} \\
& \theta_{R,t} D_t \leq \tilde{R}_{b,t} \\
& f_t \leq \theta_{b,t} E_t \left( \frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right)
\end{aligned}
$$

From the bank's balance sheet identity express $e_t$:

$$
e_t = L_{i,t} + L_{e,t} + b_{b,t} + \tilde{R}_{b,t} - D_t - f_t \tag{15}
$$

Plug into bank's cash flow constraint:

$$
\begin{aligned}
\Omega_{b,t} = & -\Big(L_{i,t} + L_{e,t} + b_{b,t} + \tilde{R}_{b,t} - D_t - f_t\Big) + (1-\delta^e)\frac{\Big(L_{i,t-1} + L_{e,t-1} + b_{b,t-1} + \tilde{R}_{b,t-1} - D_{t-1} - f_{t-1}\Big)}{\pi_t} \\
& + \frac{\Big(r_{i,t-1}L_{i,t-1} + r_{e,t-1}L_{e,t-1} + r_{g,t-1}b_{b,t-1} + r_{\tilde{R},t-1}\tilde{R}_{b,t-1} - r_{d,t-1}D_{t-1} - r_{f,t-1}f_{t-1}\Big)}{\pi_t}
\end{aligned} \tag{16}
$$

Reintroduce a stochastic discount factor in a more intuitive way (as in Gerali et al., 2010). Instead of the original one $\left(\Lambda_{t,t+1} = \beta_p \frac{\lambda^p_{t+1}}{\lambda^p_t}\right)$ assume the following:

$$
\begin{aligned}
\Lambda_{0,t} &= \beta_p^t \frac{\lambda^p_t}{\lambda^p_0} \\
\Lambda_{t,t+1} &= \beta_p \frac{\lambda^p_{t+1}}{\lambda^p_t}
\end{aligned} \tag{17}
$$

Given that $f(\Omega_{b,t}) \equiv \frac{\Omega_{b,t}^{1-1/\sigma}}{1-1/\sigma}$ we can rewrite the optimization problem as follows:

$$\begin{aligned}
\max_{L_{i,t},L_{e,t},b_{b,t},\tilde{R}_{b,t},D_t,f_t} \quad & E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \frac{\Omega_{b,t}^{1-1/\sigma}}{1-1/\sigma} \\
\text{s.t.} \quad & D_t + f_t \leq \gamma_i L_{i,t} + \gamma_e L_{e,t} + \gamma_b b_{b,t} + \gamma_{\tilde{R}} \tilde{R}_{b,t} \\
& \theta_{R,t} D_t \leq \tilde{R}_{b,t} \\
& f_t \leq \theta_{b,t} E_t \left( \frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right)
\end{aligned}$$

Consequently, the Lagrangian would be:

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \Bigg\{ & \frac{\Omega_{b,t}^{1-1/\sigma}}{1-1/\sigma} \\
& - \mu_{e,t} \Big[ D_t + f_t - \gamma_i L_{i,t} - \gamma_e L_{e,t} - \gamma_b b_{b,t} - \gamma_{\tilde{R}} \tilde{R}_{b,t} \Big] \\
& - \mu_{\tilde{R},t} \Big[ \theta_{R,t} D_t - \tilde{R}_{b,t} \Big] \\
& - \mu_{f,t} \Big[ f_t - \theta_{b,t} E_t \left( \frac{b_{b,t}}{R_{f,t}} \pi_{t+1} \right) \Big] \Bigg\}
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\Omega_{b,t} = - \Big( L_{i,t} + L_{e,t} + b_{b,t} + \tilde{R}_{b,t} - D_t - f_t \Big) + (1-\delta^e) \frac{\Big( L_{i,t-1} + L_{e,t-1} + b_{b,t-1} + \tilde{R}_{b,t-1} - D_{t-1} - f_{t-1} \Big)}{\pi_t} \\
+ \frac{\Big( r_{i,t-1} L_{i,t-1} + r_{e,t-1} L_{e,t-1} + r_{g,t-1} b_{b,t-1} + r_{\tilde{R},t-1} \tilde{R}_{b,t-1} - r_{d,t-1} D_{t-1} - r_{f,t-1} f_{t-1} \Big)}{\pi_t}
\end{aligned}$$

The list of optimality conditions is as follows:

- FOC w.r.t. $L_{i,t}$ (C.21):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} \gamma_i = E_t \left\{ \Lambda_{t,t+1} \Omega_{b,t+1}^{-1/\sigma} \left[ \frac{r_{i,t} + 1 - \delta^e}{\pi_{t+1}} \right] \right\} \tag{19}$$

- FOC w.r.t. $L_{e,t}$ (C.20):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} \gamma_e = E_t \left\{ \Lambda_{t,t+1} \Omega_{b,t+1}^{-1/\sigma} \left[ \frac{r_{e,t} + 1 - \delta^e}{\pi_{t+1}} \right] \right\} \tag{20}$$

- FOC w.r.t. $b_{b,t}$ (C.23):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t}\gamma_b - \mu_{f,t}\theta_{b,t}E_t\left(\frac{\pi_{t+1}}{R_{f,t}}\right) = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{g,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{21}$$

- FOC w.r.t. $\tilde{R}_{b,t}$ (C.22):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t}\gamma_{\tilde{R}} - \mu_{\tilde{R},t} = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{\tilde{R},t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{22}$$

- FOC w.r.t. $D_t$ (C.24):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{\tilde{R},t}\theta_{R,t} = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{d,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{23}$$

- FOC w.r.t. $f_t$ (C.25):

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{f,t} = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{f,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{24}$$

- Set of optimality conditions also includes constraints C.15-C.18.

## B.4 Entrepreneurial Managers

Entrepreneurial managers maximize the present value of profits and solve:

$$\begin{aligned}
\max_{\Omega_{e,t},l_{e,t},k_{e,t},h_{e,t},u_t} \quad & E_0\sum_{t=0}^{\infty}\Lambda_{t,t+1}f(\Omega_{e,t}) \\
\text{s.t.} \quad & \Omega_{e,t} + R_{e,t-1}\frac{l_{e,t-1}}{\pi_t} + q_{k,t}\left[k_{e,t} - (1-\delta_t^k)k_{e,t-1}\right] + q_t\left(h_{e,t} - h_{e,t-1}\right) \\
& = r_{h,t}h_{e,t-1} + r_{k,t}u_t k_{e,t-1} + l_{e,t} + J_{er,t} \\
\text{and} \quad & l_{e,t} \leq m_{K,t}E_t\left[\frac{q_{k,t+1}}{R_{e,t}}(1-\delta_{t+1}^k)k_{e,t}\pi_{t+1}\right]
\end{aligned}$$

where

$$\delta_t^k(u_t) = \delta_0^k + \delta_1^k(u_t - 1) + \frac{\delta_2^k}{2}(u_t-1)^2 \tag{25}$$

Assuming borrowing constraint binds in the neighborhood of the steady state one can plug it into the budget constraint. Next, express $\Omega_{e,t}$ from the budget constraint and plug it into the entrepreneurs' objective function. Given that $f(\Omega_{e,t}) \equiv \frac{\Omega_{e,t}^{1-1/\sigma}}{1-1/\sigma}$ we can rewrite the maximization problem in the following way (also, use same convention for stochastic discount rate as for banks):

$$\max_{l_{e,t},k_{e,t},h_{e,t},u_t} \quad E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \frac{\Omega_{e,t}^{1-1/\sigma}}{1-1/\sigma} \tag{26}$$

where

$$\begin{aligned} \Omega_{e,t} = r_{h,t}h_{e,t-1} + r_{k,t}u_t k_{e,t-1} + J_{er,t} - q_t\left(h_{e,t} - h_{e,t-1}\right) + \\ m_{K,t}E_t\left[\frac{q_{k,t+1}}{R_{e,t}}\left(1-\delta_0^k-\delta_1^k(u_{t+1}-1)-\frac{\delta_2^k}{2}(u_{t+1}-1)^2\right)k_{e,t}\pi_{t+1}\right] - \\ m_{K,t-1}E_t\left[q_{k,t}\left(1-\delta_0^k-\delta_1^k(u_t-1)-\frac{\delta_2^k}{2}(u_t-1)^2\right)k_{e,t-1}\right] - \\ q_{k,t}\left[k_{e,t}-\left(1-\delta_0^k-\delta_1^k(u_t-1)-\frac{\delta_2^k}{2}(u_t-1)^2\right)k_{e,t-1}\right] \end{aligned} \tag{27}$$

The list of optimality conditions is as follows:

- FOC w.r.t. $h_{e,t}$ (C.29):

$$E_t\left\{\beta_p\frac{\lambda_{t+1}^p}{\lambda_t^p}\Omega_{e,t+1}^{1/\sigma}\left(r_{h,t+1}+q_{t+1}\right)\right\} = q_t\Omega_{e,t}^{1/\sigma} \tag{28}$$

- FOC w.r.t. $k_{e,t}$ (C.30):

$$\begin{aligned} \Omega_{e,t}^{1/\sigma}\left\{m_{K,t}E_t\left[\frac{q_{k,t+1}}{R_{e,t}}\left(1-\delta_{t+1}^k\right)\pi_{t+1}\right]-q_{k,t}\right\}+ \\ E_t\left\{\beta_p\frac{\lambda_{t+1}^p}{\lambda_t^p}\Omega_{e,t+1}^{1/\sigma}\left[r_{k,t+1}u_{t+1}+q_{k,t+1}\left(1-\delta_{t+1}^k\right)\left(1-m_{K,t}\right)\right]\right\}=0 \end{aligned} \tag{29}$$

- FOC w.r.t. $u_t$ (C.31):

$$r_{k,t} = q_{k,t}(1-m_{K,t-1})\left(\delta_1^k+\delta_2^k[u_t-1]\right) \tag{30}$$

## B.5 Entrepreneurial Retailers

Entrepreneurial retailers solve a two-stage optimization problem. In the first stage, they take input prices as given and rent factors of production in a perfectly competitive market.

Hence, retailers solve:

$$\min_{k_{e,t-1}(j),h_{e,t-1}(j),N_t(j)} \quad r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_tN_t(j)$$
$$\text{s.t.} \quad Y_t(j) = A_t\left[k_{e,t-1}(j)\right]^{\alpha} h_{e,t-1}(j)^{\nu} N_t(j)^{1-\alpha-\nu}$$

- FOC w.r.t $k_{e,t-1}(j)$ (where $\bar{\lambda}_t$ is the Lagrange multiplier on the production constraint):

$$r_{k,t} - \bar{\lambda}_t\left(A_t\alpha\left[k_{e,t-1}(j)\right]^{\alpha-1} h_{e,t-1}(j)^{\nu} N_t(j)^{1-\alpha-\nu}\right) = 0$$

$$r_{k,t} - \bar{\lambda}_t\left(\alpha\left[k_{e,t-1}(j)\right]^{\alpha-1} \frac{Y_t(j)}{k_{e,t-1}(j)^{\alpha}}\right) = 0$$

  Finally:

$$\bar{\lambda}_t = \frac{r_{k,t}}{\alpha\frac{Y_t(j)}{k_{e,t-1}(j)}} \tag{31}$$

- Similarly, FOC w.r.t. $h_{e,t-1}(j)$:

$$r_{h,t} - \bar{\lambda}_t\left(\nu\frac{Y_t(j)}{h_{e,t-1}}\right) = 0 \tag{32}$$

- Similarly, FOC w.r.t. $N_t(j)$:

$$w_t - \bar{\lambda}_t\left((1-\alpha-\nu)\frac{Y_t(j)}{N_t}\right) = 0 \tag{33}$$

Plug FOC w.r.t. $k_{e,t-1}(j)$ into two other FOCs to get:

$$\frac{r_{h,t}}{r_{k,t}} = \frac{\nu k_{e,t-1}(j)}{\alpha h_{e,t-1}(j)} \tag{34}$$

$$\frac{w_t}{r_{k,t}} = \frac{(1-\alpha-\nu)k_{e,t-1}(j)}{\alpha N_t(j)} \tag{35}$$

Note that all firms have the same optimal input ratios. Assume unit mass of firms. Using market clearing conditions (as in Fernández-Villaverde and Rubio-Ramírez, 2006):

$$\int_0^1 k_{e,t-1}(j)dj = u_tk_{e,t-1} \tag{36}$$

and

$$\int_0^1 h_{e,t-1}(j)dj = h_{e,t-1} \tag{37}$$

and

$$\int_0^1 N_t(j)dj = N_t \tag{38}$$

obtain the following equations:

- Equation C.34:

$$\frac{r_{h,t}}{r_{k,t}} = \frac{\nu u_t k_{e,t-1}}{\alpha h_{e,t-1}} \tag{39}$$

- Equation C.33:

$$\frac{w_t}{r_{k,t}} = \frac{(1-\alpha-\nu)u_t k_{e,t-1}}{\alpha N_t} \tag{40}$$

Plug returns ratios into the total cost (objective) function to derive total real costs for firm $j$:

$$TC_t(j) \equiv \frac{1}{\alpha} r_{k,t} k_{e,t-1}(j) \tag{41}$$

Given CRS property of a production function $Y_t(j)$, let us find real $mc_t$. Normalize output to 1 (i.e. $Y_t(j) = 1$) and plug optimality conditions for firm's input demand:

$$Y_t(j) = A_t \left[k_{e,t-1}(j)\right]^\alpha h_{e,t-1}(j)^\nu N_t(j)^{1-\alpha-\nu} = k_{e,t-1} A_t u_t^\alpha \left(\frac{\nu}{\alpha}\frac{r_{k,t}}{r_{h,t}}\right)^\nu \left(\frac{1-\alpha-\nu}{\alpha}\frac{r_{k,t}}{w_t}\right)^{1-\alpha-\nu} = 1$$

$$k_{e,t-1} = \left[A_t \left(\frac{\nu}{\alpha}\frac{r_{k,t}}{r_{h,t}}\right)^\nu \left(\frac{1-\alpha-\nu}{\alpha}\frac{r_{k,t}}{w_t}\right)^{1-\alpha-\nu}\right]^{-1} \tag{42}$$

Plug expression for $k_{e,t-1}$ into the expression for $TC_t$ to derive the function for $mc_t$ (C.35):

$$mc_t = \frac{r_{k,t}^\alpha r_{h,t}^\nu w_t^{1-\alpha-\nu}}{A_t \alpha^\alpha \nu^\nu (1-\alpha-\nu)^{1-\alpha-\nu}} \tag{43}$$

In the second stage, firms choose price to maximize discounted real profits subject to Calvo type price rigidity and solve the following problem:

$$\begin{aligned} \max_{p_{i,t}} \quad & E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \left( \prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{i,t}}{p_{i,t+\tau}} - mc_{t+\tau} \right) y_{i,t+\tau} \right\} \\ \text{s.t.} \quad & y_{i,t+\tau} = \left( \prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{i,t}}{p_{t+\tau}} \right)^{-\varepsilon} y_{t+\tau}^d \end{aligned}$$

By plugging the constraint into the objective function and taking the derivative with respect to $p_{i,t}$ one can obtain standard optimality conditions: C.36–C.40. Representative firm's real total costs might be obtained by using clearing condition for any input market. For example:

$$TC_t \equiv \frac{1}{\alpha} r_{k,t} \cdot u_t k_{e,t-1} \tag{44}$$

Rewriting the equation C.41 while dropping the index $j$ implies:

$$\begin{aligned} J_{er,t} &= \int_0^1 J_{er,t}(j)dj = \int_0^1 \Big\{ Y_t(j) - [r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)] \Big\} dj \\ &= \int_0^1 Y_t(j)dj + \int_0^1 [r_{k,t}k_{e,t-1}(j) + r_{h,t}h_{e,t-1}(j) + w_t N_t(j)]\, dj = \frac{Y_t}{v_t} - \frac{1}{\alpha} r_{k,t} \cdot u_t k_{e,t-1} \end{aligned} \tag{45}$$

## B.6 Capital and Final Goods Producers

Final goods producers maximize profits and solve the following problem:

$$\begin{aligned} \max_{Y_t(j)} \quad & P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj \\ \text{s.t.} \quad & Y_t = \left[ \int_0^1 Y_t(j)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)} \end{aligned}$$

Profit maximization problem yields the optimal demand for an intermediate good $j$ given the aggregate demand:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t \tag{46}$$

Capital goods producers solve the following problem:

$$\begin{aligned} \max_{\bar{x}_t, I_t} \quad & E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left( q_t^k \Delta \bar{x}_t - I_t \right) \\ \text{s.t.} \quad & \bar{x}_t = \bar{x}_{t-1} + I_t \left[ 1 - \frac{\psi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \end{aligned}$$

where $\Delta \bar{x}_t = K_t - (1 - \delta_t^k) K_{t-1}$. FOCs yield standard optimality conditions: C.42–C.43.

## B.7 First order conditions (FOCs) derivations

### B.7.1 Patient households

- FOC w.r.t. $c_{p,t}$ (C.2):

$$\beta_p^t \left\{ \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} - \lambda_t^p \right\} = 0 \tag{47}$$

Simplify:

$$\lambda_t^p = \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} \tag{48}$$

- FOC w.r.t. $n_{p,t}$ (C.8):

$$\beta_p^t \left\{ -n_{p,t} \left( c_{p,t} - \frac{n_{p,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} + \lambda_t^p w_t \right\} = 0 \tag{49}$$

Plug FOC w.r.t. $c_{p,t}$ and simplify:

$$w_t = n_{p,t}^{\phi} \tag{50}$$

- FOC w.r.t. $h_{p,t}$ (C.3):

$$\beta_p^t \left\{ \frac{j_{p,t}}{h_{p,t}} - \lambda_t^p q_t \right\} + \beta_p^{t+1} E_t \left( -\lambda_{t+1}^p \cdot -q_{t+1} \right) = 0 \tag{51}$$

Simplify and rearrange:

$$\frac{j_{p,t}}{h_{p,t}} + \beta_p E_t \left( \lambda_{t+1}^p q_{t+1} \right) = \lambda_t^p q_t \tag{52}$$

- FOC w.r.t. $m_t$ (C.6):

$$\beta_p^t \left\{ \frac{\chi_{z,t}}{z_t} z'_{m,t} - \lambda_t^p \left( 1 + f'_{m,t} \right) \right\} + \beta_p^{t+1} E_t \left( \lambda_{t+1}^p \frac{1}{\pi_{t+1}} \right) = 0 \tag{53}$$

where

$$
\begin{aligned}
z'_{m,t} &\equiv \frac{\partial z_t}{\partial m_t} \\
&= \frac{\eta_{z,t}-1}{\eta_{z,t}} \cdot m_t^{-1/\eta_{z,t}} \cdot \frac{\eta_{z,t}}{\eta_{z,t}-1} \cdot \left[ m_t^{(\eta_{z,t}-1)/\eta_{z,t}} + cbdc_t^{(\eta_{z,t}-1)/\eta_{z,t}} + \omega_d d_t^{(\eta_{z,t}-1)/\eta_{z,t}} \right]^{1/(\eta_{z,t}-1)} \\
&= m_t^{-1/\eta_{z,t}} z_t^{1/\eta_{z,t}}
\end{aligned} \tag{54}
$$

and

$$
f'_{m,t} \equiv \frac{\partial f(m_t)}{\partial m_t} = \psi_m m_t \tag{55}
$$

Plug in both derivative into the FOC:

$$
\chi_{z,t} \frac{1}{z_t} m_t^{-1/\eta_{z,t}} z_t^{1/\eta_{z,t}} - \lambda_t^p (1 + \psi_m m_t) + \beta_p E_t \left( \lambda_{t+1}^p \frac{1}{\pi_{t+1}} \right) = 0 \tag{56}
$$

Rearrange:

$$
\lambda_t^p (1 + \psi_m m_t) = \beta_p E_t \left( \frac{\lambda_{t+1}^p}{\pi_{t+1}} \right) + \frac{\chi_{z,t}}{z_t} \left( \frac{z_t}{m_t} \right)^{1/\eta_{z,t}} \tag{57}
$$

- FOC w.r.t. $cbdc_t$ (C.5):

$$
\beta_p^t \left\{ \frac{\chi_{z,t}}{z_t} z'_{m,t} - \lambda_t^p \right\} + \beta_p^{t+1} E_t \left( \frac{\lambda_{t+1}^p R_{cbdc,t}}{\pi_{t+1}} \right) = 0 \tag{58}
$$

Simplify and rearrange:

$$
\frac{\chi_{z,t}}{z_t} \left( \frac{z_t}{cbdc_t} \right)^{1/\eta_{z,t}} + \beta_p E_t \left( \frac{\lambda_{t+1}^p R_{cbdc,t}}{\pi_{t+1}} \right) = \lambda_t^p \tag{59}
$$

- FOC w.r.t. $d_t$ (C.4):

$$
\frac{\chi_{z,t}}{z_t} \omega_d \left( \frac{z_t}{d_t} \right)^{1/\eta_{z,t}} + \beta_p E_t \left( \frac{\lambda_{t+1}^p R_{d,t}}{\pi_{t+1}} \right) = \lambda_t^p \tag{60}
$$

- FOC w.r.t. $b_{p,t}$ (C.7):

$$
\beta_p^t (-\lambda_t^p) + \beta_p^{t+1} E_t \left\{ -\lambda_{t+1}^p \left( -R_{g,t} \frac{1}{\pi_{t+1}} \right) \right\} = 0 \tag{61}
$$

Rearrange:

$$
\lambda_t^p = \beta_p E_t \left( \frac{\lambda_{t+1}^p R_{g,t}}{\pi_{t+1}} \right) \tag{62}
$$

- FOC w.r.t. $\lambda_t^p$: patient household's budget constraint at period $t$.

### B.7.2 Impatient households

- FOC w.r.t. $c_{i,t}$ (C.11):

$$\beta_i^t \left\{ \left( c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} - \lambda_t^i \right\} = 0 \tag{63}$$

Simplify:

$$\lambda_t^i = \left( c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} \tag{64}$$

- FOC w.r.t. $n_{i,t}$ (C.13):

$$\beta_i^t \left\{ -n_{i,t} \left( c_{i,t} - \frac{n_{i,t}^{1+\phi}}{(1+\phi)} \right)^{-\sigma_h} - \lambda_{i,t} \cdot (-w_t) \right\} = 0 \tag{65}$$

Plug FOC w.r.t. $c_{i,t}$ and simplify:

$$w_t = n_{i,t}^{\phi} \tag{66}$$

- FOC w.r.t $h_{i,t}$ (C.12):

$$\begin{aligned} &\beta_i^t \left\{ \frac{j_{i,t}}{h_{i,t}} - \lambda_t^i \left[ q_t - m_{H,t} E_t \left( \frac{q_{t+1}\pi_{t+1}}{R_{i,t}} \right) \right] \right\} \\ &\quad + \beta_i^{t+1} E_t \left\{ -\lambda_{t+1}^i \left[ -q_{t+1} + R_{i,t} \frac{1}{\pi_{t+1}} m_{H,t} E_t \left( \frac{q_{t+1}\pi_{t+1}}{R_{i,t}} \right) \right] \right\} = 0 \end{aligned} \tag{67}$$

Open brackets and simplify:

$$\frac{j_{i,t}}{h_{i,t}} + \lambda_t^i \left( -q_t + E_t \left[ \frac{m_{H,t} q_{t+1} \pi_{t+1}}{R_{i,t}} \right] \right) + \beta_i E_t \left[ \lambda_{t+1}^i q_{t+1} (1 - m_{H,t}) \right] = 0 \tag{68}$$

$$\frac{j_{i,t}}{h_{i,t}} - \lambda_t^i \left( q_t - E_t \left[ \frac{m_{H,t} q_{t+1} \pi_{t+1}}{R_{i,t}} \right] \right) + \beta_i E_t \left[ \lambda_{t+1}^i q_{t+1} (1 - m_{H,t}) \right] = 0 \tag{69}$$

- FOC w.r.t. $\lambda_t^i$: impatient household's budget constraint at period $t$.

#### B.7.3 Banks

- FOC w.r.t. $L_{i,t}$ (C.21):

$$\Lambda_{0,t}\left\{-\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}(-\gamma_i)\right\}+\Lambda_{0,t+1}E_t\left\{\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{i,t}}{\pi_{t+1}}+(1-\delta^e)\frac{1}{\pi_{t+1}}\right]\right\}=0 \tag{70}$$

Simplify and rearrange:

$$\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}\gamma_i=E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{i,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{71}$$

- FOC w.r.t. $L_{e,t}$ (C.20):

$$\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}\gamma_e=E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{e,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{72}$$

- FOC w.r.t. $b_{b,t}$ (C.23):

$$\begin{aligned}\Lambda_{0,t}\left\{-\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}(-\gamma_b)-\mu_{f,t}\left(-\theta_{b,t}E_t\left[\frac{\pi_{t+1}}{R_{f,t}}\right]\right)\right\}+\\ E_t\left\{\Lambda_{0,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{g,t}}{\pi_{t+1}}+(1-\delta^e)\frac{1}{\pi_{t+1}}\right]\right\}=0\end{aligned} \tag{73}$$

Simplify and rearrange:

$$-\Omega_{b,t}^{-1/\sigma}+\mu_{e,t}\gamma_b+\mu_{f,t}\theta_{b,t}E_t\left[\frac{\pi_{t+1}}{R_{f,t}}\right]+E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{g,t}+1-\delta^e}{\pi_{t+1}}\right]\right\}=0 \tag{74}$$

$$\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}\gamma_b-\mu_{f,t}\theta_{b,t}E_t\left[\frac{\pi_{t+1}}{R_{f,t}}\right]=E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{g,t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{75}$$

- FOC w.r.t. $\tilde{R}_{b,t}$ (C.22):

$$\Lambda_{0,t}\left\{-\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}(-\gamma_{\tilde{R}})-\mu_{\tilde{R},t}(-1)\right\}+E_t\left\{\Lambda_{0,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{\tilde{R},t}}{\pi_{t+1}}+(1-\delta^e)\frac{1}{\pi_{t+1}}\right]\right\}=0 \tag{76}$$

Simplify and rearrange:

$$-\Omega_{b,t}^{-1/\sigma}+\mu_{e,t}\gamma_{\tilde{R}}+\mu_{\tilde{R},t}+E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{\tilde{R},t}+1-\delta^e}{\pi_{t+1}}\right]\right\}=0 \tag{77}$$

$$\Omega_{b,t}^{-1/\sigma}-\mu_{e,t}\gamma_{\tilde{R}}-\mu_{\tilde{R},t}=E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{\tilde{R},t}+1-\delta^e}{\pi_{t+1}}\right]\right\} \tag{78}$$

- FOC w.r.t. $D_t$ (C.24):

$$\Lambda_{0,t}\left\{\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{\tilde{R},t}\theta_{R,t}\right\} + E_t\left\{\Lambda_{0,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{-r_{d,t} - (1-\delta^e)}{\pi_{t+1}}\right]\right\} = 0 \tag{79}$$

Simplify and rearrange:

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{\tilde{R},t}\theta_{R,t} = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{d,t} + 1 - \delta^e}{\pi_{t+1}}\right]\right\} \tag{80}$$

- FOC w.r.t. $f_t$ (C.25):

$$\left\{\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{f,t}\right\} + E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{-r_{f,t} - (1-\delta^e)}{\pi_{t+1}}\right]\right\} = 0 \tag{81}$$

$$\Omega_{b,t}^{-1/\sigma} - \mu_{e,t} - \mu_{f,t} = E_t\left\{\Lambda_{t,t+1}\Omega_{b,t+1}^{-1/\sigma}\left[\frac{r_{f,t} + 1 - \delta^e}{\pi_{t+1}}\right]\right\} \tag{82}$$